Frustration and Anger in Games*

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Abstract

Frustration, anger, and blame have important consequences for economic and social behavior, concerning for example monopoly pricing, contracting, bargaining, violence, and politics. Drawing on insights from psychology, we develop a formal approach to exploring how frustration and anger, via blame and aggression, shape interaction and outcomes in economic settings.

KEYWORDS: frustration, anger, blame, belief-dependent preferences, psychological games.

JEL codes: C72, D03

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1 Introduction

Anger can shape economic outcomes. Consider three cases:

> **Case 1:** In 2015 Turing Pharmaceuticals raised the price of Daraprim, a therapeutic drug, from $12 to $750 per dose. The company was subsequently accused of price gouging. Should Turing have considered the consequences of customer anger before determining the new price for the drug?
>
> **Case 2:** When local football teams favored to win instead lose, the police get more reports of husbands assaulting wives (Card & Dahl 2011). Do unexpected losses spur vented frustration?
>
> **Case 3:** Following Sovereign Debt Crises (2009–), some EU countries embarked on austerity programs. Was it because citizens lost benefits that some cities experienced riots?

Pricing, domestic violence, political landscapes: these are important themes, and we propose that others (involving—say—recessions, contracting, arbitration, terrorism, road rage, or support for populist political candidates) could plausibly be imagined. However, to carefully assess the impact of anger on social and economic interactions, one needs a theory that predicts outcomes based on the decision-making of anger-prone individuals and that accounts for the strategic considerations by their co-players. We develop such a theory.

Insights from psychology about the triggers and repercussions of anger are evocative. The behavioral consequences of emotions are called "action tendencies," and the action tendency associated with anger is aggression and the urge to retaliate. Angry players may be willing to forego material gains to punish others, or be predisposed to aggression when this serves as a credible threat. But while insights of this nature can be gleaned from psychologists' writings, their analysis usually stops with the individual rather than going on to assess overall economic and social implications. We take the basic insights about anger that psychology has produced as input and inspiration for our theory.[1]

Anger is typically anchored in frustration, which occurs when someone is unexpectedly denied something he or she cares about.[2] We assume (admittedly restrictively; cf. Section 7) that people are frustrated when they get less material rewards than expected. They then

[1]The psychology literature is huge. One source of inspiration for us is the *International Handbook of Anger* (Potegal, Spielberger & Stemmler 2010) which offers cross-disciplinary perspective reflecting "affective neuroscience, business administration, epidemiology, health science, linguistics, political science, psychology, psychophysiology, and sociology" (p. 3). The absence of "economics" in the list may indicate that our approach is original!

[2]Psychologists often refer to this as "goal-blockage;" cf. p. 3 of the (*op. cit.*) *Handbook*.

become hostile towards whomever they blame. There are a number of ways that blame may be assigned (see e.g. Alicke 2000 in psychology, and Chockler & Halpern 2004 for structural modeling) and we present three approaches, captured by distinct utility functions. While players motivated by simple anger (SA) become generally hostile when frustrated, those motivated by anger from blaming behavior (ABB) or anger from blaming intentions (ABI) go after others discriminately, asking who caused, or who intended to cause, their frustration.[3] Because a player's frustration depends on his beliefs about others' choices, and the blame a player attributes to another may depend on his beliefs about others' choices or belief, all our models find their intellectual home in the framework of psychological game theory; see Geanakoplos, Pearce & Stacchetti (1989), and Battigalli & Dufwenberg (2009).

We examine two equilibrium notions. The first is an extension of the sequential equilibrium for psychological games developed in Battigalli & Dufwenberg (2009), adapted to our framework and to capture beliefs about a player's own action. In addition, we also develop a notion of polymorphic sequential equilibrium (PSE). In this approach, players correctly anticipate how others behave on average, yet different types of the same player may have different plans in equilibrium. This yields meaningful updating of players' views of others' intentions as various subgames are reached, which is crucial for a sensible treatment of how players consider intentionality as they blame others. We apply this solution concept to the aforementioned utility functions, explore properties, and compare predictions.

A small literature exists examining the role of anger in economic behavior. Early work exploring the role of anger in solving commitment problems includes Hirshleifer (1987), Frank (1988), and Elster (1998). Most recent studies are empirical or experimental, indicative of hostile action occurring in economic situations, based on either observational or experimental data.[4] A few studies do present theory, though largely with a focus on applications (Akerlof 2016; Passarelli & Tabellini 2016; Rotemberg 2005, 2008, 2011).

Our approach differs from the previous literature in that we do not start with data, but with notions from psychology which we incorporate into general games, and we are led to use assumptions which differ substantially from previous work (Section 7 elaborates, in regards to Rotemberg's approach). Winter *et al.* (2009) and Winter (2014) model anger and other emotions in games with a version of the indirect evolutionary approach:[5] Like us, Winter *et al* assume that preferences over outcomes are "emotional" and endogenous, but we differ in the way we model emotions and make them endogenous. We assume that emotions depend on

[3]Because frustration is based upon expected payoffs, all three models can capture the non-consequential behavior observed by, e.g. Brandts & Solá (2001) and Falk, Fehr, & Fischbacher (2003, 2008).

[4]See Anderson & Simester (2010) and Rotemberg (2005, 2011) on pricing, Card & Dahl (2011) and Munyo & Rossi (2013) on violence, and Carpenter & Matthews (2012), Gurdal, Miller & Rustichini (2014), and Gneezy & Imas (2014) for experiments.

[5]See, for example, Güth & Kliemt (1988).

endogenous beliefs, while Winter *et al* model the rest points of an adaptation process of belief-independent preferences. Brams (2011) studies anger in sequential interactions by modeling players who take turns changing the state of a 2×2 payoff matrix and receive payoffs at the end of the game. However, like Winter's, his model of anger is belief-independent, while we argue that beliefs are central to emotions.

We develop most of our analysis for a two-period setting described in Section 2. Sections 3 and 4 define our notions of frustration, blame, anger, and utility. Section 5 examines equilibria. Section 6 generalizes to multistage games. Section 7 concludes. Proofs are collected in the Appendix.

2 Setup

We first describe the rules of interaction (the game form), then define first- and second-order conditional belief systems.

2.1 Game form

Consider a finite two-stage game form describing the rules of interaction and the consequences of players' actions. The set of players is I. To ease notation, we assume that all players take actions simultaneously at each stage. Thus, nodes are histories h of action profiles $a^t = (a_i^t)_{i \in I}$; $h = \varnothing$ is the empty history (the root), $h = (a^1)$ a history of length one, which may be terminal or not, and $h = (a^1, a^2)$ a history of length 2, which is terminal. H is the set of nonterminal histories and Z is the set of terminal histories (end nodes). The set of feasible actions of i given $h \in H$ is $A_i(h)$. This set is a singleton if i is not active given h. Thus, for all $h \in H$, $I(h) = \{i \in I : |A_i(h)| > 1\}$ is the set of active players given h. In a perfect information game, $I(h)$ is a singleton for each $h \in H$. We omit parentheses whenever no confusion may arise. For example, we may write $h = a^1$ instead of $h = (a^1)$, and $h = (a_i^1, a_j^2)$ if i (resp. j) is the only first (resp. second) mover. Finally, we let $A(h) = \times_{i \in I} A_i(h)$ and $A_{-i}(h) = \times_{j \neq i} A_j(h)$.

We assume that the material consequences of players' actions are determined by a profile of monetary payoff functions $(\pi_i : Z \to \mathbb{R})_{i \in I}$. This completes the description of the game form, if there are no chance moves. If the game contains chance moves, we augment the player set with a dummy player c (with $c \notin I$), who selects a feasible action at random. Thus, we consider an augmented player set $I_c = I \cup \{c\}$, and the sets of first and second movers may include c: $I(\varnothing), I(a^1) \subseteq I_c$. If the chance player is active at $h \in H$, its move is described by a probability density function $\sigma_c(\cdot|h) \in \Delta(A_c(h))$.

The following example, to which we will return in our discussion of blame, is here employed to illustrate our notation:

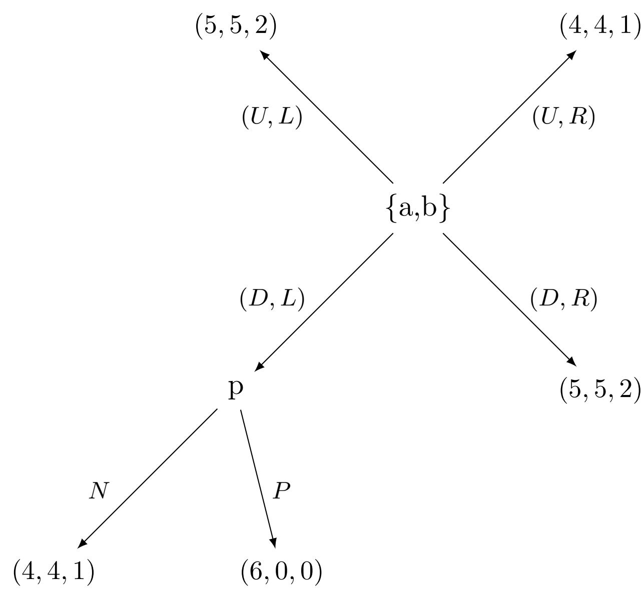

Figure A. Asymmetric punishment.

Example 1 Ann and Bob (a and b in Fig. A) move simultaneously in the first stage. Penny the punisher (p) may move in the second stage; by choosing P she increases π_a and decreases π_b. Profiles of actions and monetary payoffs are listed in players' alphabetical order. We have:

$$\begin{aligned} H &= \{\varnothing, (D,L)\}\,,\; Z = \{(U,L),(U,R),(D,R),((D,L),N)\,,((D,L),P)\}\,, \\ I(\varnothing) &= \{a,b\},\; I\left((D,L)\right) = \{p\}, \\ A_a(\varnothing) &= \{U,D\},\; A_b(\varnothing) = \{L,R\},\; A_p\left((D,L)\right) = \{N,P\}. \quad \blacktriangle \end{aligned}$$

2.2 Beliefs

It is conceptually useful to distinguish three aspects of a player's beliefs: beliefs about co-players' actions, beliefs about co-players' beliefs, and the player's plan, which we represent as beliefs about own actions. Beliefs are defined conditional on each history. Abstractly denote by Δ_{-i} the space of co-players' beliefs (the formal definition is given below). Player i's beliefs can be compactly described as conditional probability measures over paths and beliefs of others, *i.e.*, over $Z \times \Delta_{-i}$. Events, from i's point of view, are subsets of $Z \times \Delta_{-i}$. Events about behavior take form $Y \times \Delta_{-i}$, with $Y \subseteq Z$; events about beliefs take form $Z \times E_{\Delta_{-i}}$, with $E_{\Delta_{-i}} \subseteq \Delta_{-i}$.[6]

[6] Δ_{-i} turns out to be a compact metric space. Events are Borel measurable subsets of $Z \times \Delta_{-i}$. We do not specify terminal beliefs of i about others' beliefs, as they are not relevant for the models in this paper.

Personal histories To model how i determines the subjective value of feasible actions, we add to the commonly observed histories $h \in H$ also personal histories of the form (h, a_i), with $a_i \in A_i(h)$. In a game with perfect information, $(h, a_i) \in H \cup Z$. But if there are simultaneous moves at h, then (h, a_i) is not a history in the standard sense. As soon as i irreversibly chooses action a_i, he observes (h, a_i), and can determine the value of a_i using his beliefs conditional on this event (i knows in advance how he is going to update his beliefs conditional on what he observes). We denote by H_i the set of histories of i—standard and personal—and by $Z(h_i)$ the set of terminal successors of h_i.[7] The standard precedence relation $\prec$ for histories in $H \cup Z$ is extended to H_i in the obvious way: for all $h \in H$, $i \in I(h)$, and $a_i \in A_i(h)$, it holds that $h \prec (h, a_i)$ and $(h, a_i) \prec (h, (a_i, a_{-i}))$ if i is not the only active player at h. Note that $h \prec h'$ implies $Z(h') \subseteq Z(h)$, with strict inclusion if at least one player (possibly chance) is active at h.

First-order beliefs For each $h_i \in H_i$, player i holds beliefs $\alpha_i(\cdot|Z(h_i)) \in \Delta(Z(h_i))$ about the actions that will be taken in the continuation of the game. The system of beliefs $\alpha_i = (\alpha_i(\cdot|Z(h_i)))_{h_i \in H_i}$ must satisfy two properties. First, the rules of conditional probabilities hold whenever possible: if $h_i \prec h'_i$ then for every $Y \subseteq Z(h'_i)$

$$\alpha_i(Z(h'_i)|Z(h_i)) > 0 \Rightarrow \alpha_i(Y|Z(h'_i)) = \frac{\alpha_i(Y|Z(h_i))}{\alpha_i(Z(h'_i)|Z(h_i))}. \tag{1}$$

We use obvious abbreviations to denote conditioning events and the conditional probabilities of actions: for all $h \in H$, $a = (a_i, a_{-i}) \in A_i(h) \times A_{-i}(h)$,

$$\begin{aligned}
\alpha_i(a|h) &= \alpha_i(Z(h, a)|Z(h)), \\
\alpha_{i,i}(a_i|h) &= \sum_{a'_{-i} \in A_{-i}(h)} \alpha_i(a_i, a'_{-i}|h), \\
\alpha_{i,-i}(a_{-i}|h) &= \sum_{a'_i \in A_i(h)} \alpha_i(a'_i, a_{-i}|h).
\end{aligned}$$

Note that $\alpha_{i,i}(a_i|h) = \alpha_i(Z(h, a_i)|Z(h))$, and that (1) implies $\alpha_i(a^1, a^2|\varnothing) = \alpha_i(a^2|a^1)\,\alpha_i(a^1|\varnothing)$.

With this, we can write in a simple way our second requirement, that i's beliefs about the actions simultaneously taken by the co-players are independent of i's action: for all $h \in H$, $i \in I$, $a_i \in A_i(h)$, and $a_{-i} \in A_{-i}(h)$,

$$\alpha_{i,-i}(a_{-i}|h) = \alpha_{i,-i}(a_{-i}|h, a_i). \tag{2}$$

[7] That is, $H_i = H \cup \{(h, a_i) : h \in H, i \in I(h), a_i \in A_i(h)\}$. The definition of $Z(h_i)$ is standard for $h_i \in H$; for $h_i = (h, a_i)$ we have $Z(h, a_i) = \bigcup_{a_{-i} \in A_{-i}(h)} Z(h, (a_i, a_{-i}))$.

Properties (1)–(2) imply

$$\alpha_i(a_i, a_{-i}|h) = \alpha_{i,i}(a_i|h)\alpha_{i,-i}(a_{-i}|h).$$

Thus, α_i is made of two parts, what i believes about his own behavior and what he believes about the behavior of others. The array of probability measures $\alpha_{i,i} \in \times_{h\in H}\Delta\left(A_i(h)\right)$ is—technically speaking—a behavioral strategy, and we interpret it as the **plan** of i. The reason is that the result of i's contingent planning is precisely a system of conditional beliefs about what action he would take at each history. If there is only one co-player, also $\alpha_{i,-i} \in \times_{h\in H}\Delta\left(A_{-i}(h)\right)$ corresponds to a behavioral strategy. With multiple co-players, $\alpha_{i,-i}$ corresponds instead to a "correlated behavioral strategy." Whatever the case, $\alpha_{i,-i}$ gives i's conditional beliefs about others' behavior, and these beliefs may not coincide with the plans of others. We emphasize: a player's plan does not describe actual choices, actions on the path of play are the only actual choices.

A system of conditional probability measures $\alpha_i = \left(\alpha_i(\cdot|Z(h_i))\right)_{h_i\in H_i}$ satisfying (1)–(2) is a **first-order** belief of i. Let Δ_i^1 denote the space of such beliefs. It can be checked that Δ_i^1 is a compact metric space, so the same holds for $\Delta_{-i}^1 = \times_{j\neq i}\Delta_j^1$, the space of co-players' first-order beliefs profiles.

Second-order beliefs Players do not only hold beliefs about paths, they also hold beliefs about the beliefs of co-players. In the following analysis, the only co-players' beliefs affecting the values of actions are their first-order beliefs. Therefore, we limit our attention to **second-order** beliefs, *i.e.*, systems of conditional probability measures $\left(\beta_i(\cdot|h_i)\right)_{h_i\in H_i} \in \times_{h_i\in H_i}\Delta\left(Z(h_i)\times\Delta_{-i}^1\right)$ that satisfy properties analogous to (1)–(2).[8] First, if $h_i \prec h_i'$ then

$$\beta_i(h_i'|h_i) > 0 \Rightarrow \beta_i\left(E|h_i'\right) = \frac{\beta_i\left(E|h_i\right)}{\beta_i(h_i'|h_i)} \tag{3}$$

for all $h_i, h_i' \in H_i$ and every event $E \subseteq Z(h_i') \times \Delta_{-i}^1$. Second, i realizes that his choice cannot influence the first-order beliefs of co-players and their simultaneous choices, so i's beliefs satisfy an independence property:

$$\beta_i\left(Z\left(h,(a_i,a_{-i})\right)\times E_\Delta|(h,a_i)\right) = \beta_i\left(Z\left(h,(a_i',a_{-i})\right)\times E_\Delta|(h,a_i')\right), \tag{4}$$

for every $h \in H$, $a_i, a_i' \in A_i(h)$, $a_{-i} \in A_{-i}(h)$, and event $E_\Delta \subseteq \Delta_{-i}^1$ about co-players' first-order beliefs. The space of i's second-order beliefs is denoted by Δ_{-i}^2.

It can be checked that starting from $\beta_i \in \Delta_i^2$ and letting $\alpha_i\left(Y|h_i\right) = \beta_i\left(Y\times\Delta_{-i}^1|h_i\right)$ for all $h_i \in H_i$ and $Y \subseteq Z$, we obtain a system α_i satisfying (1)–(2), *i.e.*, an element of Δ_i^1. This

[8] We use obvious abbreviations, such as writing h for event $Z(h)\times\Delta_{-i}^1$, whenever this causes no confusion.

α_i is the first-order belief implicit in β_i. Whenever we write in a formula beliefs of different orders for a player, we assume that first-order beliefs are derived from second-order beliefs, otherwise beliefs of different orders would not be mutually consistent. Also, we write initial beliefs omitting the empty history, as in $\beta_i(E) = \beta_i(E|\varnothing)$ or $\alpha_i(a) = \alpha_i(a|\varnothing)$, whenever this causes no confusion.

Conditional expectations Let ψ_i be any real-valued measurable function of variables that i does not know, *e.g.*, the terminal history or the co-players' first-order beliefs. Then i can compute the expected value of ψ_i conditional on any history $h_i \in H_i$ by means of his belief system β_i, denoted $\mathbb{E}[\psi_i|h_i;\beta_i]$. If ψ_i depends only on actions, *i.e.*, on the path z, then $\mathbb{E}[\psi_i|h_i;\beta_i]$ is determined by the α_i derived from β_i, and we can write $\mathbb{E}[\psi_i|h_i;\alpha_i]$. In particular, α_i gives the conditional expected material payoffs:

$$\mathbb{E}[\pi_i|h;\alpha_i] = \sum_{z\in Z(h)} \alpha_i(z|h)\pi_i(z),$$
$$\mathbb{E}[\pi_i|\,(h,a_i)\,;\alpha_i] = \sum_{z\in Z(h,a_i)} \alpha_i(z|h,a_i)\pi_i(z)$$

for all $h \in H$, $a_i \in A_i(h)$. $\mathbb{E}[\pi_i|h;\alpha_i]$ is what i expects to get conditional on h given α_i, which also specifies i's plan. $\mathbb{E}[\pi_i|(h,a_i);\alpha_i]$ is i's expected payoff of action a_i. If a_i is what i planned to choose at h, $\alpha_{i,i}(a_i|h) = 1$, and then $\mathbb{E}[\pi_i|h;\alpha_i] = \mathbb{E}[\pi_i|\,(h,a_i)\,;\alpha_i]$. For initial beliefs, we omit $h = \varnothing$ from such expressions; in particular, the initially expected material payoff is $\mathbb{E}[\pi_i;\alpha_i]$.

3 Frustration

Anger is triggered by frustration. While we focus on anger as a social phenomenon—frustrated players blame, become angry with, and care for the payoffs of *others*—our account of frustration refers to own payoffs only. In Section 7 (in hindsight of definitions to come) we discuss this approach in depth. Here, we define player i's **frustration**, in stage 2, given a^1, as

$$\mathrm{F}_i(a^1;\alpha_i) = \left[\mathbb{E}[\pi_i;\alpha_i] - \max_{a_i^2\in A_i(a^1)} \mathbb{E}[\pi_i|(a^1,a_i^2);\alpha_i]\right]^+,$$

where $[x]^+ = \max\{x,0\}$. In words, frustration is given by the gap, if positive, between i's initially expected payoff and the currently best expected payoff he believes he can obtain. Diminished expectation—$\mathbb{E}[\pi_i|a^1;\alpha_i] < \mathbb{E}[\pi_i;\alpha_i]$—is only a necessary condition for frustration. For i to be frustrated it must also be the case that i cannot close the gap.

$F_i(a^1;\alpha_i)$ expresses stage-2 frustration. One could also define frustration at the root, or at end nodes, but neither would matter for our purposes. At the root nothing has happened, so frustration equals zero. Frustration is possible at the end nodes, but can't influence subsequent choices as the game is over. One might allow the anticipated frustration to be felt at end nodes to influence earlier decisions; however, the assumptions we make in the analysis below rule this out. Furthermore, a player is influenced by the frustrations of co-players only insofar as their behavior is affected.

Example 2 Return to Fig. A. Suppose Penny initially expects \$2: $\alpha_p(U,L)+\alpha_p(D,R)=1$, $\mathbb{E}[\pi_p;\alpha_p]=2$. After (D,L) we have

$$\mathrm{F}_p((D,L);\alpha_p)=[\mathbb{E}[\pi_p;\alpha_p]-\max\{\pi_p((D,L),N),\pi_p((D,L),P)\}]^+=2-1=1.$$

This is independent of her plan, because she is initially certain she will not move. If instead $\alpha_p((U,L)|\varnothing)=\alpha_p((D,L)|\varnothing)=\frac{1}{2}$ then

$$\mathrm{F}_p((D,L);\alpha_p)=\frac{1}{2}\cdot 2+\frac{1}{2}\alpha_p(N|(D,L))\cdot 1-1=\frac{1}{2}\alpha_p(N|(D,L));$$

Penny's frustration is highest if she initially plans not to punish Bob. ▲

4 Anger

A player's preferences over actions at a given node—his action tendencies—depend on expected material payoffs and frustration. A frustrated player tends to hurt others, if this is not too costly (cf. Dollard *et al.* 1939, Averill 1983, Berkowitz 1989). We consider different versions of this frustration-aggression hypothesis related to different cognitive appraisals of blame. In general, player i moving at history h chooses action a_i to maximize the expected value of a belief-dependent "decision utility" of the form

$$u_i(h,a_i;\beta_i)=\mathbb{E}\left[\pi_i|(h,a_i);\alpha_i\right]-\theta_i\sum_{j\neq i}\mathrm{B}_{ij}(h;\beta_i)\,\mathbb{E}\left[\pi_j|(h,a_i);\alpha_i\right], \tag{5}$$

where α_i is the first-order belief system derived from second-order belief β_i, and $\theta_i\geq 0$ is a sensitivity parameter. $\mathrm{B}_{ij}(h;\beta_i)\geq 0$ measures how much of i's frustration is blamed on j, and the presence of $\mathbb{E}[\pi_j|(h,a_i);\alpha_i]$ in the formula translates this into a tendency to hurt j. We assume that

$$\mathrm{B}_{ij}(h;\beta_i)\leq \mathrm{F}_i(h;\alpha_i). \tag{6}$$

Therefore, the decision utility of a first-mover coincides with expected material payoff, because there cannot be any frustration in the first stage: $u_i(\varnothing,a_i;\beta_i)=\mathbb{E}[\pi_i|a_i;\alpha_i]$. When i is the only active player at $h = a^1$, he determines the terminal history with his choice $a_i = a^2$, and decision utility has the form

$$u_i(h, a_i; \beta_i) = \pi_i(h, a_i) - \theta_i \sum_{j \neq i} \mathrm{B}_{ij}(h; \beta_i)\, \pi_j(h, a_i).$$

We next consider functional forms that capture different notions of blame.

4.1 Simple Anger (SA)

Our most rudimentary hypothesis, **simple anger (SA)**, is that i's tendency to hurt others is proportional to i's frustration. This affect heuristic is unmodulated by the cognitive appraisal of blame, so $\mathrm{B}_{ij}(h; \beta_i) = \mathrm{F}_i(h; \alpha_i)$:

$$u_i^{SA}(h, a_i; \alpha_i) = \mathbb{E}\left[\pi_i | (h, a_i); \alpha_i\right] - \theta_i \sum_{j \neq i} \mathrm{F}_i(h; \alpha_i) \mathbb{E}\left[\pi_j | (h, a_i); \alpha_i\right]. \qquad (7)$$

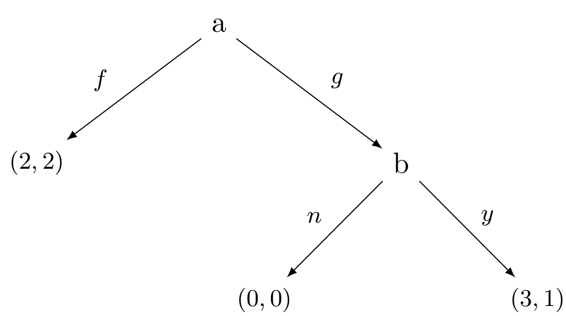

Figure B. Ultimatum Minigame.

Example 3 *(Ultimatum Minigame)* Ann and Bob (a and b in Fig. B) negotiate: Ann can make fair offer f, which is automatically accepted, or greedy offer g, which Bob accepts or rejects. His frustration following g is

$$\mathrm{F}_b(g; \alpha_b) = \left[(1 - \alpha_b(g)) \cdot 2 + \alpha_b(g)\alpha_b(y|g) \cdot 1 - 1\right]^+.$$

Therefore

$$u_b^{SA}(g, n; \alpha_b) - u_b^{SA}(g, y; \alpha_b) = 3\theta_b \left[2\left(1 - \alpha_b(g)\right) + \alpha_b(g)\alpha_b(y|g) - 1\right]^+ - 1.$$

For Bob to be frustrated he must not expect g with certainty. If frustrated, the less he expects g, and—interestingly—the less he plans to reject, the more prone he is to reject once

g materializes. The more resigned Bob is to getting a low payoff, the less frustrated and prone to aggression he is. Furthermore, it is readily seen from the example how our model captures non-consequential behavior (cf. the references in footnote 3). Holding beliefs and other payoffs constant, increasing Bob's payoff from the fair offer f will lead to greater frustration after g. This greater frustration will increase the disutility Bob receives from Ann's material payoff, making rejection of the greedy offer (and punishment of Ann) more attractive. ▲

4.2 Anger from blaming behavior (ABB)

Action tendencies may depend on a player's cognitive appraisal of how to blame others. When a frustrated player i blames co-players for their behavior, he examines the actions chosen in stage 1, without considering others' intentions. How much i blames j is determined by a continuous function $\mathrm{B}_{ij}(a^1;\alpha_i)$ that depends only on first-order belief α_i and is such that

$$\mathrm{B}_{ij}(a^1;\alpha_i) = \begin{cases} 0, & \text{if } j \notin I(\varnothing), \\ \mathrm{F}_i(a^1;\alpha_i), & \text{if } \{j\} = I(\varnothing). \end{cases} \tag{8}$$

According to (8), if j is not active in the first stage, he cannot be blamed by i. If instead j is the *only* active player, he is fully blamed.[9] We consider below specific versions of $\mathrm{B}_{ij}(h;\alpha_i)$ that satisfy (6) and (8). With this, i's decision utility with **anger from blaming behavior (ABB)** is

$$u_i^{ABB}(h,a_i;\alpha_i) = \mathbb{E}\left[\pi_i|\left(h,a_i\right);\alpha_i\right] - \theta_i \sum_{j\neq i} \mathrm{B}_{ij}(h;\alpha_i)\mathbb{E}\left[\pi_j|\left(h,a_i\right);\alpha_i\right].$$

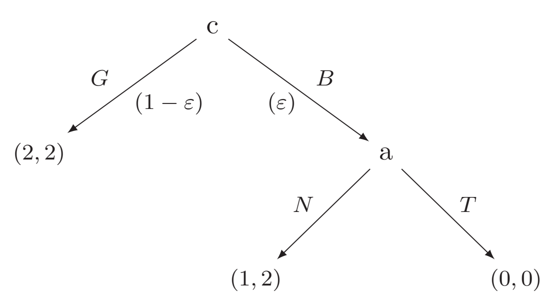

Figure C. Hammering one's thumb.

[9]Recall that $I(h)$ is the set of active players at h, possibly including chance. For example, $I(\varnothing) = \{c\}$ in the game form of Figure C.

Example 4 *(Inspired by Frijda 1993)* To illustrate the difference between SA and ABB, consider Fig. C. Andy the handyman (a) uses a hammer. His apprentice, Bob (b), is inactive. On a bad day (determined by chance, c) Andy hammers his thumb and can then take it out on Bob, or not. Assuming $\alpha_a(B) = \varepsilon < 1/2$, we have

$$\mathrm{F}_a(B; \alpha_a) = (1-\varepsilon) \cdot 2 + \varepsilon \alpha_a(N|B) \cdot 1 - 1 > 0.$$

With SA and with θ_a sufficiently high, on a bad day Andy chooses T. But, since Bob is passive, with ABB Andy chooses N regardless of θ_a. ▲

SA and ABB yield the same behavior in the Ultimatum Minigame and similar game forms. Say that a game form is a **leader-followers game** if there is only one active player in the first stage, who does not move in stage two: $I(\varnothing) = \{j\}$ and $I(\varnothing) \cap I(a^1) = \emptyset$ for some $j \in I$ and every a^1. Let us write u_{i,θ_i} to make the dependence of u_i on θ_i explicit; then (8) implies:

Remark 1 *In leader-followers games, SA and ABB coincide, that is,* $u^{SA}_{i,\theta_i} = u^{ABB}_{i,\theta_i}$ *for all* θ_i.

Next, we contrast two specific functional forms for ABB.

Could-have-been blame When frustrated i considers, for each j, what he would have obtained at most, in expectation, had j chosen differently:

$$\max_{a'_j \in A_j(\varnothing)} \mathbb{E}\left[\pi_i | (a^1_{-j}, a'_j); \alpha_i\right].$$

If this could-have-been payoff is more than what i currently expects (that is, $\mathbb{E}[\pi_i|a^1; \alpha_i]$), then i blames j, up to i's frustration (so (6) holds):

$$\mathrm{B}_{ij}(a^1; \alpha_i) = \min\left\{ \left[\max_{a'_j \in A_j(\varnothing)} \mathbb{E}\left[\pi_i|(a^1_{-j}, a'_j); \alpha_i\right] - \mathbb{E}[\pi_i|a^1; \alpha_i] \right]^+, \mathrm{F}_i(a^1; \alpha_i) \right\}. \tag{9}$$

Blame function (9) satisfies (8) (cf. Remark 4 below).

Example 5 Consider Penny at $a^1 = (D, L)$ in Fig. A. Her could-have-been payoff—wrt both Ann and Bob—is $2 \geq \mathbb{E}[\pi_p; \alpha_p]$, her updated expected payoff is $\mathbb{E}[\pi_p|(D, L); \alpha_p] \leq 1$, and her frustration is $[\mathbb{E}[\pi_p; \alpha_p] - 1]^+$. Therefore

$$\mathrm{B}_{pa}((D, L); \alpha_p) = \mathrm{B}_{pb}((D, L); \alpha_p) =$$

$$\min\left\{ [2 - \mathbb{E}[\pi_p|(D, L); \alpha_p]]^+, [\mathbb{E}[\pi_p; \alpha_p] - 1]^+ \right\} = [\mathbb{E}[\pi_p; \alpha_p] - 1]^+,$$

i.e., each of Ann and Bob is fully blamed by Penny for her frustration. ▲

Blaming unexpected deviations When frustrated after a^1, i assesses, for each j, how much he would have obtained had j behaved as expected:

$$\sum_{a'_j \in A_j(\varnothing)} \alpha_{ij}(a'_j) \mathbb{E}\left[\pi_i | (a^1_{-j}, a'_j); \alpha_i\right],$$

where $\alpha_{ij}(a'_j)$ is the marginal probability of action a'_j according to i's belief α_i. With this, the blame formula is

$$\mathrm{B}_{ij}(a^1; \alpha_i) =$$
$$\min\left\{\left[\sum_{a'_j \in A_j(\varnothing)} \alpha_{ij}(a'_j) \mathbb{E}\left[\pi_i | (a^1_{-j}, a'_j); \alpha_i\right] - \mathbb{E}[\pi_i | a^1; \alpha_i]\right]^+, \mathrm{F}_i(a^1; \alpha_i)\right\}. \quad (10)$$

If j is not active in the first stage, we get

$$\mathrm{B}_{ij}(a^1; \alpha_i) = \min\left\{\left[\mathbb{E}[\pi_i | a^1; \alpha_i] - \mathbb{E}[\pi_i | a^1; \alpha_i]\right]^+, \mathrm{F}_i(a^1; \alpha_i)\right\} = 0;$$

that is, j cannot have deviated and cannot be blamed. If, instead, j is the only active player in the first stage, then

$$\sum_{a'_j \in A_j(\varnothing)} \alpha_{ij}(a'_j) \mathbb{E}\left[\pi_i | (a^1_{-j}, a'_j); \alpha_i\right] = \sum_{a' \in A(\varnothing)} \alpha_i(a') \mathbb{E}\left[\pi_i | a'; \alpha_i\right] = \mathbb{E}\left[\pi_i; \alpha_i\right],$$

and (10) yields

$$\mathrm{B}_{ij}(a^1; \alpha_i) = \min\left\{\left[\mathbb{E}[\pi_i; \alpha_i] - \mathbb{E}[\pi_i | a^1; \alpha_i]\right]^+, \mathrm{F}_i(a^1; \alpha_i)\right\} = \mathrm{F}_i(a^1; \alpha_i).$$

Therefore, like blame function (9), also (10) satisfies (8).

If a^1_j is what i expected j to do in the first stage ($\alpha_{ij}(a^1_j) = 1$) then

$$\mathrm{B}_{ij}(a^1; \alpha_i) = \min\left\{\left[\mathbb{E}[\pi_i | a^1; \alpha_i] - \mathbb{E}[\pi_i | a^1; \alpha_i]\right]^+, \mathrm{F}_i(a^1; \alpha_i)\right\} = 0.$$

In other words, j did not deviate from what i expected and j is not blamed by i. This is different from "could-have-been" blame (9).

Example 6 Suppose that, in Fig. A, Penny is initially certain of (U, L): $\alpha_p(U, L) = 1$ and $\mathbb{E}[\pi_p; \alpha_p] = 2$. Upon observing (D, L) her frustration is $\mathrm{F}_p((D, L); \alpha_p) = [\mathbb{E}[\pi_p; \alpha_p] - 1]^+ = 1$. Using eq. (10), at $a^1 = (D, L)$, Penny fully blames Ann, who deviated from U to D. Since

$$\sum_{a'_a \in A_a(\varnothing)} \alpha_{pa}(a'_a) \mathbb{E}\left[\pi_p | (a^1_{-a}, a'_a); \alpha_p\right] = \pi_p(U, L) = 2,$$

we get that Penny's blame of Ann equals Penny's frustration

$$\mathrm{B}_{pa}((D,L);\alpha_p) = \min\left\{\left[2 - \mathbb{E}[\pi_p|a^1;\alpha_p]\right]^+, 1\right\} = 1.$$

On the other hand, Penny does not blame Bob, who played L as expected. To verify this, note that when frustrated after (D,L) Penny assesses how much she would have obtained had Bob behaved as expected:

$$\sum_{a'_b \in A_b(\varnothing)} \alpha_{pb}(a'_b)\mathbb{E}\left[\pi_p|(a^1_{-b}, a'_b);\alpha_p\right] = \mathbb{E}[\pi_p|(D,L);\alpha_p]$$

and

$$\mathrm{B}_{pb}((D,L);\alpha_p) = \min\left\{\left[\mathbb{E}[\pi_p|(D,L);\alpha_p] - \mathbb{E}[\pi_p|(D,L);\alpha_p]\right]^+, 1\right\} = 0,$$

in contrast to could-have-been blame under which, as we saw, Penny fully blames Bob (Example 5). ▲

Formulae (9) and (10) each credit the full frustration on the first-mover of a leader-followers game, because each satisfies (8) (see Remark 1).

4.3 Anger from blaming intentions (ABI)

A player i prone to **anger from blaming intentions (ABI)** asks himself, for each co-player j, whether j intended to give him a low expected payoff. Since such intention depends on j's first-order beliefs α_j (which include j's plan, $\alpha_{j,j}$), how much i blames j depends on i's second-order beliefs β_i, and the decision utility function has the form (5).

The maximum payoff that j, initially, can expect to give to i is

$$\max_{a^1_j \in A_j(\varnothing)} \sum_{a^1_{-j} \in A_{-j}(\varnothing)} \alpha_{j,-j}(a^1_{-j})\mathbb{E}\left[\pi_i|\left(a^1_j, a^1_{-j}\right);\alpha_j\right].$$

Note that

$$\begin{aligned}&\max_{a^1_j \in A_j(\varnothing)} \sum_{a^1_{-j} \in A_{-j}(\varnothing)} \alpha_{j,-j}(a^1_{-j})\mathbb{E}\left[\pi_i|\left(a^1_j, a^1_{-j}\right);\alpha_j\right] \\ &\geq \sum_{a^1 \in A(\varnothing)} \alpha_j(a^1)\mathbb{E}\left[\pi_i|a^1;\alpha_j\right] = \mathbb{E}\left[\pi_i|\alpha_j\right],\end{aligned}$$

where the inequality holds by definition, and the equality is implied by the chain rule (3). Note also that $\alpha_j(\cdot|a^1)$ is kept fixed under the maximization; we focus on what j initially believes he could achieve, taking the view that at the root he cannot control a_j^2 but predicts how he will choose in stage 2. We assume that i's blame on j at a^1 equals i's expectation, given second-order belief β_i and conditional on a^1, of the difference between the maximum payoff that j can expect to give to i and what j actually plans/expects to give to i, capped by i's frustration:

$$\mathrm{B}_{ij}(a^1;\beta_i) = \tag{11}$$

$$\min\left\{\mathbb{E}\left[\max_{a_j^1}\sum_{a_{-j}^1}\alpha_{j,-j}(a_{-j}^1)\mathbb{E}\left[\pi_i|\left(a_j^1,a_{-j}^1\right);\alpha_j\right]-\mathbb{E}[\pi_i;\alpha_j]\middle| a^1;\beta_i\right],\mathrm{F}_i(a^1;\alpha_i)\right\},$$

where α_i is derived from β_i. The expression is nonnegative as per the previously highlighted inequality. Now, i's decision utility after $h = a^1$ is

$$u_i^{ABI}\left(h,a_i;\beta_i\right) = \mathbb{E}\left[\pi_i|\left(h,a_i\right);\alpha_i\right] - \theta_i\sum_{j\neq i}\mathrm{B}_{ij}(h;\beta_i)\mathbb{E}\left[\pi_j|\left(h,a_i\right);\alpha_i\right],$$

with $\mathrm{B}_{ij}(a^1;\beta_i)$ given by (11).

Example 7 Return to Fig. B. The maximum payoff Ann can expect to give to Bob is 2, independently of α_a. Suppose Bob, upon observing g, is certain Ann "randomized" and planned to offer g with probability p: $\beta_b(\alpha_a(g) = p|g) = 1$, with $p < 1$. Also, Bob is certain after g that Ann expected him to accept with probability q, *i.e.*, $\beta_b(\alpha_a(y|g) = q|g) = 1$. Finally, suppose Bob initially expected to get the *fair* offer ($\alpha_b(f) = 1$), so his frustration after g is $F_b(a^1;\alpha_b) = 2 - 1 = 1$. Bob's blame of Ann's intentions is

$$\mathrm{B}_{ba}(g;\beta_b) = \min\left\{2 - [2(1-p) + qp], 1\right\} = \min\left\{p(2-q), 1\right\}.$$

If p is low enough, or q high enough, Bob does not blame all his frustration on Ann. He gives her some credit for the initial intention to make the fair offer with probability $1 - p > 0$, and the degree of credit depends on q. ▲

5 Equilibrium analyses

While in this paper we depart from traditional game-theoretic analysis in using belief-dependent decision utility, our analysis is otherwise traditional.

We consider two equilibrium notions. The first is an adaptation of Battigalli & Dufwenberg's (2009) sequential equilibrium (SE) concept.[10] For simplicity, we adopt a complete

[10]Battigalli & Dufwenberg (2009) extend Kreps & Wilson's (1982) classic notion of sequential equilibrium to psychological games; we here consider the version (Section 6) for preferences with own-plan dependence and "local" psychological utility functions.

information framework where the rules of the game and players' (psychological) preferences are common knowledge.[11] We interpret a (sequential) equilibrium as a profile of strategies and beliefs representing a "commonly understood" way to play the game by rational (utility maximizing) agents. Our approach allows us to investigate the implications of our belief-dependent utility model within a standard framework. This is a choice of focus more than an endorsement of sequential equilibrium as a solution concept.[12]

SE requires that each player i is certain and never changes his mind about the true beliefs and plans, hence intentions, of his co-players. We find this feature questionable. Therefore, we also explore a generalization — "polymorphic sequential equilibrium" (PSE)— that allows for meaningful updating about others' intentions.

5.1 Sequential Equilibrium (SE)

The SE concept gives equilibrium conditions for infinite hierarchies of conditional probability systems. In our particular application, utility functions only depend on first- or second-order beliefs, so we define SE for assessments comprising beliefs up to only the second order. Since, technically, first-order beliefs are features of second-order beliefs (see 2.2), we provide definitions that depend only on second-order beliefs, which give SEs for games where psychological utility functions depend only of first-order beliefs as a special case. Finally, although so far we have restricted our analysis of frustration and anger to two-stage game forms, our abstract definitions of equilibrium for games with belief-dependent preferences (and the associated existence theorem) apply to all multistage game forms.

Fix a game form and decision utility functions $u_i(h,\cdot;\cdot) : A_i(h) \times \Delta_i^2 \to \mathbb{R}$ ($i \in I$, $h \in H$). This gives a **psychological game** in the sense of Battigalli & Dufwenberg (2009, Section 6). An **assessment** is a profile of behavioral strategies and beliefs $(\sigma_i, \beta_i)_{i\in I} \in \times_{i\in I}(\Sigma_i \times \Delta_i^2)$ such that $\Sigma_i = \times_{h\in H}\Delta\left(A_i(h)\right)$ and σ_i is the plan $\alpha_{i,i}$ entailed by second-order belief β_i:

$$\sigma_i(a_i|h) = \alpha_{i,i}(a_i|h) = \beta_i\left(Z(h,a_i) \times \Delta_{-i}^1|h\right) \tag{12}$$

for all $i \in I$, $h \in H$, $a_i \in A_i(h)$. Eq. (12) implies that the behavioral strategies contained in an assessment are implicitly determined by players' beliefs about paths; therefore, they could be dispensed with. We follow Battigalli & Dufwenberg (2009) and make behavioral strategies explicit in assessments only to facilitate comparisons with the equilibrium refinements literature.

[11] For an equilibrium analysis of incomplete-information psychological games see Attanasi, Battigalli & Manzoni (2016), for a non-equilibrium analysis see Battigalli, Charness & Dufwenberg (2013).

[12] Battigalli & Dufwenberg (2009, Sections 2, 5) argue that, with belief-dependent preferences, alternatives to SE like rationalizability, forward induction, and self-confirming equilibrium may be plausible.

Definition 1 *An assessment* $(\sigma_i, \beta_i)_{i \in I}$ *is* ***consistent*** *if, for all* $i \in I$*,* $h \in H$*, and* $a = (a_j)_{j \in I(h)} \in A(h)$*,*
(a) $\alpha_i(a|h) = \prod_{j \in I(h)} \sigma_j(a_j|h)$*,*
(b) $marg_{\Delta^1_{-i}} \beta_i(\cdot|h) = \delta_{\alpha_{-i}}$*,*
where α_i *is derived from* β_i *and* $\delta_{\alpha_{-i}}$ *is the Dirac probability measure that assigns probability* 1 *to the singleton* $\{\alpha_{-i}\} \subseteq \Delta^1_{-i}$*.*

Condition (a) requires that players' beliefs about actions satisfy independence across co-players (on top of own-action independence), and—conditional on each h—each i expects each j to behave in the continuation as specified by j's plan $\sigma_j = \alpha_{j,j}$, even though j has previously deviated from $\alpha_{j,j}$. All players thus have the same first-order beliefs. Condition (b) requires that players' beliefs about co-players' first-order beliefs (hence their plans) are correct and never change, on or off the path. Thus all players, essentially, have the same second-order beliefs (considering that they are introspective and therefore know their own first-order beliefs). These conditions capture the "trembling-hand" interpretation of deviations implicit in Kreps & Wilson's (1982) original definition of equilibrium.

Definition 2 *An assessment* $(\sigma_i, \beta_i)_{i \in I}$ *is a* ***sequential equilibrium (SE)*** *if it is consistent and satisfies the following sequential rationality condition: for all* $h \in H$ *and* $i \in I(h)$*,* $\mathrm{Supp}\sigma_i(\cdot|h) \subseteq \arg\max_{a_i \in A_i(h)} u_i(h, a_i; \beta_i)$*.*

It can be checked that this definition of SE is equivalent to the traditional one when players have standard preferences, *i.e.*, when there is a profile of utility functions $(v_i : Z \to \mathbb{R})_{i \in I}$ such that $u_i(h, a_i; \beta_i) = \mathbb{E}[v_i|(h, a_i); \alpha_i]$.[13] A special case is the **material-payoff game**, where $v_i = \pi_i$ for each $i \in I$.

Theorem 1 *If* $u_i(h, a_i; \cdot)$ *is continuous for all* $i \in I$*,* $h \in H$ *and* $a_i \in A_i(h)$*, then there is at least one SE.*

Battigalli & Dufwenberg (2009) prove a version of this existence result where first-order beliefs are modeled as belief systems over pure strategy profiles rather than paths. But their "trembling-hand" technique can be used here with straightforward adaptations. We omit the details.[14]

What we said so far about equilibrium does not assume specific functional forms. From now on, we focus on u_i^{SA}, u_i^{ABB}, and u_i^{ABI}. Since frustration and blame are continuous in beliefs, decision utility is also continuous, and we obtain existence in all cases of interest:

[13] According to the standard definition of SE, sequential rationality is given by global maximization over (continuation) strategies at each $h \in H$. By the One-Shot-Deviation principle, in the standard case this is equivalent to "local" maximization over actions at each $h \in H$.

[14] A similar technique is used in the proof of Prop. 1 (first part) in the appendix.

Corollary 1 *Every game with SA, ABB, or ABI has at least one SE.*

Remark 2 *Let $(\sigma_i, \beta_i)_{i\in I}$ be an SE assessment of a game with SA, ABB, or ABI; if a history $h \in H$ has probability 1 under profile $(\sigma_i)_{i\in I}$, then*

$$\mathrm{F}_i(h'; \alpha_i) = 0 \text{ and } \mathrm{Supp}\sigma_i(\cdot|h') \subseteq \arg\max_{a'_i \in A_i(h')} \mathbb{E}[\pi_i|h'; \alpha_i]$$

for all $h' \preceq h$ and $i \in I$, where α_i is derived from β_i. Therefore, an SE strategy profile of a game with SA, ABB, or ABI with randomization (if any) only in the last stage is also a Nash equilibrium of the agent form of the corresponding material-payoff game.

To illustrate, in the Ultimatum Minigame (Fig. B), (f, n) can be an SE under ABB, and is a Nash equilibrium of the agent form[15] with material-payoff utilities. With (counterfactual) anger, n becomes a credible threat. Corollary 1 and Remark 2 also hold for the multistage extension of Section 6.

Recall that two assessments are **realization-equivalent** if the corresponding strategy profiles yield the same probability distribution over terminal histories:

Proposition 1 *In every perfect-information (two-stage) game form with no chance moves and a unique SE of the material-payoff game, this unique material-payoff equilibrium is realization-equivalent to an SE of the psychological game with ABI, ABB, or—with only two players—SA.*

If unique, the material-payoff SE of a perfect-information game must be in pure strategies. By Remark 2, players must maximize their material payoff on the path even if they are prone to anger. As for off-equilibrium path decision nodes, deviations from the material-payoff SE strategies can only be due to the desire to hurt the first-mover, which can only increase his incentive to stick to the material-payoff SE action.

It is quite easy to show by example that without perfect information, or with chance moves, a material-payoff SE need not be an SE with frustration and anger. The same holds for some multistage game forms (cf. Section 6). Disregarding chance moves, randomization, and ties, the common feature of material-payoff equilibria that are not realization-equivalent to equilibria with frustration and anger is this (see Fig. A and Ex. 8 below): Start with a material-payoff equilibrium and add anger to decision utility; now, at an off-path node after Ann deviates, frustrated Penny wants to hurt Bob, which implies rewarding Ann; this makes it impossible to incentivize both Ann not to deviate and Penny to punish Bob after Ann's deviation.

We close this section with three examples, which combine to illustrate how the SE works and how SA, ABB (both versions), and ABI may alter material incentives and produce different predictions.

[15] In the agent form of a game, each h where player i is active corresponds to a copy (i, h) of i with strategy set $A_i(h)$ and the same utility function as i.

Example 8 Consider Fig. A., Asymmetric Punishment. Can material-payoff equilibrium outcome (U, L) be part of a SE with frustration and anger? The answer is yes under ABI and the blaming-unexpected-deviations version of ABB. To see this note that Ann and Bob act as-if selfish (as they are not frustrated). Hence they would deviate if they could gain materially. In the SE, they would expect 5 if not deviating, making Ann the sole deviation candidate (she would get $6 > 5$ were Penny to choose P; for Bob, 5 is the best he could hope for). But Ann deviating can be dismissed since if (D, L) were reached Penny would not blame Bob (her only punishable co-player) under either relevant blame function, so she would choose N (regardless of θ_p). Under SA and the could-have-been version of ABB, however, it may be impossible to sustain a SE with (U, L); at (D, L) Penny would blame each of Ann and Bob (as explained). By choosing P she hurts Bob more than she helps Ann and would do so if

$$u_p^{ABB}((D,L),P;\alpha_p) > u_p^{ABB}((D,L),N;\alpha_p)$$

$$\Longleftrightarrow$$

$$0 - 6\theta_p \mathrm{B}_{pa}((D,L);\alpha_p) > 1 - 8\theta_p \mathrm{B}_{pa}((D,L);\alpha_p).$$

The rhs of the last inequality uses $\mathrm{B}_{pb}((D,L);\alpha_p) = \mathrm{B}_{pa}((D,L);\alpha_p)$. Since $\mathrm{B}_{pa}((D,L);\alpha_p) = \mathrm{F}_p((D,L);\alpha_p) = 1 > 0$, Penny would choose P if $-6\theta_p > 1 - 8\theta_p \Longleftrightarrow \theta_p > 1/2$, so Ann would want to deviate and choose D. ▲

Example 9 Consider Fig. B., the Ultimatum Minigame. By Proposition 1, every utility function discussed admits (g, y) as a SE, regardless of anger sensitivity. To check this directly, just note that, if Bob expects g, he cannot be frustrated, so—when asked to play—he maximizes his material payoff. Under SA and ABB (both versions), (f, n) qualifies as another SE if $\theta_b \geq 1/3$; following g, Bob would be frustrated and choose n, so Ann chooses f. Under ABI (f, n) cannot be a SE. To verify, assume it were, so $\alpha_a(f) = 1$. Since the SE concept does not allow for players revising beliefs about beliefs, we get $\beta_b(\alpha_a(f) = 1|g) = 1$ and $\mathrm{B}_{ba}(g;\beta_b) = 0$; Bob maintains his belief that Ann planned to choose f, hence she intended to maximize Bob's payoff. Hence, Bob would choose y, contradicting that (f, n) is a SE. Next, note that (g, n) is not a SE under any concept: Given SE beliefs Bob would not be frustrated and so he would choose y. The only way to observe rejected offers with positive probability in a SE is with non-deterministic plans. To find such a SE, note that we need $\alpha_a(g) \in (0, 1)$; if $\alpha_a(g) = 0$ Bob would not be reached and if $\alpha_a(g) = 1$ he would not be frustrated, and hence, he would choose y. Since Ann uses a non-degenerate plan she must be indifferent, so $\alpha_b(y) = 2/3$, implying that Bob is indifferent too. In SE, Bob's frustration is $\left[2\left(1 - \alpha_a(g)\right) + \frac{2}{3}\alpha_a(g) - 1\right]^+ = \left[1 - \frac{4}{3}\alpha_a(g)\right]^+$, which equals his blame of Ann under SA and

ABB. Hence we get the indifference condition

$$1-\theta_b\left[1-\frac{4}{3}\alpha_a(g)\right]^+\cdot 3=0-\theta_b\left[1-\frac{4}{3}\alpha_a(g)\right]^+\cdot 0$$
$$\Longleftrightarrow$$
$$\alpha_a(g)=\frac{3}{4}-\frac{1}{4\theta_b},$$

where $\theta_b \geq 1/3$. The more prone to anger Bob is the more likely he is to get the low offer, so Bob's initial expectations, and hence his frustration and blame, is kept low. Under ABI we get another indifference condition:

$$1-\theta_b \mathrm{B}_{ba}(g;\beta_b)\cdot 3=0-\theta_b \mathrm{B}_{ba}(g;\beta_b)\cdot 0$$
$$\Longleftrightarrow$$
$$1-\theta_b \min\left\{1-\frac{4}{3}\alpha_a(g),\frac{4}{3}\alpha_a(g)\right\}\cdot 3=0.$$

The left term in braces is Bob's frustration, while

$$\frac{4}{3}\alpha_a(g)=2-\left[2(1-\alpha_a(g))+\frac{2}{3}\alpha_a(g)\right]$$

is the difference between the maximum payoff Ann could plan for Bob and that actually planned. The first term is lower if $\alpha_a(g)\geq 3/8$; so, if we can solve the equation for such a number, we duplicate the SA/ABB-solution; again, this is doable if $\theta_b>1/3$. If $\theta_b\geq 2/3$, with ABI, there is a second non-degenerate equilibrium plan with $\alpha_a(g)\in(0,\frac{3}{8})$ such that $\alpha_a(g)=1/4\theta_b$; to see this, solve the ABI indifference condition assuming $\frac{4}{3}\alpha_a(g)\leq 1-\frac{4}{3}\alpha_a(g)$. This SE exhibits starkly different comparative statics: The higher is θ_b, the less likely Bob is to get a low offer and the less he blames Ann following g in light of her intention to choose f with higher probability. ▲

The reason why (f,n) in Example 10 cannot be a SE under ABI is that if Bob initially expects Ann to choose f, and she doesn't, so that Bob is frustrated, then he would rate her choice an unintended mistake and not blame her. We emphasize that this is due to assumptions that underlie the SE concept, *i.e.*, the "trembling-hand" interpretation of deviations, not to the formulation of ABI utility. According to other solution concepts, or allowing for incomplete information (cf. footnote 10), Bob would revise his belief about Ann's plan.

Example 10 Consider Fig. C, Hammering One's Thumb. With u_a^{ABB} (either version), or u_a^{ABI}, Andy will not blame Bob so his SE-choice is the material-payoff equilibrium, N. But with u_a^{SA} Andy may choose T. Recall that $\mathrm{F}_a(B;\alpha_a) = 2(1-\varepsilon) + \varepsilon\alpha_a(N|B) - 1$, so the more likely Andy believes it to be that he will take it out on Bob, the less he expects initially and the less frustrated he is after B. Yet, in SE, the higher is θ_a the more likely Andy is to take it out on Bob: Andy's utility from N and T is

$$u_a^{SA}(B, N; \alpha_a) = 1 - \theta_a[2(1-\varepsilon) + \varepsilon\alpha_a(N|B) - 1] \cdot 2,$$
$$u_a^{SA}(B, T; \alpha_a) = 0 - \theta_a[2(1-\varepsilon) + \varepsilon\alpha_a(N|B) - 1] \cdot 0 = 0.$$

Sequential rationality of SE implies that one possibility is $\alpha_a(N|B) = 1$ and $u_a^{SA}(B, N; \alpha_a) \geq u_a^{SA}(B, T; \alpha_a)$, implying $\theta_a \leq \frac{1}{2(1-\varepsilon)}$. Another possibility is $\alpha_a(N|B) = 0$ and $u_a^{SA}(B, N; \alpha_a) \leq u_a^{SA}(B, T; \alpha_a)$, implying $\theta_a \geq \frac{1}{2(1-2\varepsilon)}$. That is, if Andy is sufficiently susceptible to simple anger, on bad days he takes his frustration out on Bob. If $\theta_a \in (\frac{1}{2(1-\varepsilon)}, \frac{1}{2(1-2\varepsilon)})$, we can solve for a SE where $u_a^{SA}(B, N; \alpha_a) = u_a^{SA}(B, T; \alpha_a)$ and $\alpha_a(N|B) = \frac{1}{2\varepsilon\theta_a} - \frac{1-2\varepsilon}{\varepsilon} \in (0, 1)$. ▲

The final case, where $\theta_a \in (\frac{1}{2(1-\varepsilon)}, \frac{1}{2(1-2\varepsilon)})$, illustrates how we cannot take for granted the existence of a SE in which players use deterministic plans (a point relevant also for u_i^{ABB} or u_i^{ABI} in other games). Here this happens in a game form with a single active player, highlighting that we deal with a psychological game, as this could not be the case in a standard game.

5.2 Polymorphic sequential equilibrium (PSE)

Suppose a game is played by agents drawn at random and independently from large populations, one for each player role $i \in I$. Different agents in the same population i have the same belief-dependent preferences,[16] but they may have different plans, hence different beliefs about paths, even if their beliefs agree about the behavior and beliefs of co-players $-i$. In this case, we say that the population is "polymorphic." Once an agent observes some moves of co-players, he makes inferences about their intentions.

Let λ_i be a finite support distribution over $\Sigma_i \times \Delta_i^2$, with $\mathrm{Supp}\lambda_i = \{(\sigma_{t_i^1}, \beta_{t_i^1}), (\sigma_{t_i^2}, \beta_{t_i^2}), ...\}$, where $t_i = t_i^1, t_i^2, ...$ is an index we refer to as "type" of i.[17] We interpret λ_i as a statistical distribution of plans and beliefs of agents playing in role i.[18] With a slight abuse of notation, we let $\lambda_i(t_i)$ denote the fraction of agents in population i with plan and beliefs $(\sigma_{t_i}, \beta_{t_i})$. Also, we denote by

$$T_i(\lambda_i) = \{t_i : (\sigma_{t_i}, \beta_{t_i}) \in \mathrm{Supp}\lambda_i\}$$

[16]Recall that we are not modelling incomplete information.

[17]These are "types" in the sense of epistemic game theory (*e.g.*, Battigalli, Di Tillio & Samet 2013).

[18]The marginal of λ_i on Σ_i is a behavior strategy mixture (see Selten 1975).

the set of possible types of i in distribution λ_i, and we write $T_{-i}(\lambda_{-i}) = \times_{j\neq i} T_j(\lambda_j)$ for the set of profiles of co-players' types.

Let us take the perspective of an agent of type t_i who knows that the distribution over co-players' types is $\lambda_{-i} = \prod_{j\neq i} \lambda_j$ and believes that the behavior of each t_j is indeed described by t_j's plan σ_{t_j} (in principle, t_i may otherwise believe that t_j behaves differently from his plan). Then it is possible to derive the conditional probability of a type profile t_{-i} given history h. Given that beliefs satisfy independence across players (everybody knows there is independent random matching), the distribution is independent of t_i and can be factorized. In the current two-stage setting we have $\lambda_{-i}(t_{-i}|\varnothing) = \prod_{j\neq i} \lambda_j(t_j)$ and

$$
\begin{aligned}
\lambda_{-i}(t_{-i}|a^1) &= \frac{\prod_{j\neq i} \sigma_{t_j}(a^1_j)\lambda_j(t_j)}{\sum_{t'_{-i}\in T_{-i}(\lambda_{-i})} \prod_{j\neq i} \sigma_{t'_j}(a^1_j)\lambda_j(t'_j)} \\
&= \frac{\prod_{j\neq i} \sigma_{t_j}(a^1_j)\lambda_j(t_j)}{\prod_{j\neq i} \sum_{t'_j\in T_j(\lambda_j)} \sigma_{t'_j}(a^1_j)\lambda_j(t'_j)} \\
&= \prod_{j\neq i} \frac{\sigma_{t_j}(a^1_j)\lambda_j(t_j)}{\sum_{t'_j\in T_j(\lambda_j)} \sigma_{t'_j}(a^1_j)\lambda_j(t'_j)}.
\end{aligned}
$$

for all t_{-i} and a^1, provided that $\sum_{t'_j} \sigma_{t'_j}(a^1_j)\lambda_j(t'_j) > 0$ for each $j \neq i$. Letting

$$
\lambda_j(t_j|a^1) = \frac{\sigma_{t_j}(a^1_j)\lambda_j(t_j)}{\sum_{t'_j\in T_j(\lambda_j)} \sigma_{t'_j}(a^1_j)\lambda_j(t'_j)},
$$

we get

$$
\lambda_{-i}(t_{-i}|a^1) = \prod_{j\neq i} \lambda_j(t_j|a^1).
$$

We say that λ_j is **fully randomized** if σ_{t_j} is strictly positive for every type $t_j \in T_j(\lambda_j)$. If each λ_j is fully randomized, then, for all $h \in H$, $\lambda_{-i}(\cdot|h)$ is well defined, with $\lambda_{-i}(t_{-i}|h) = \prod_{j\neq i} \lambda_j(t_j|h)$ for all $t_{-i} \in T_{-i}(\lambda_{-i})$.

Definition 3 *A* ***polymorphic assessment*** *is a profile of finite support probability measures $\lambda = (\lambda_i)_{i\in I} \in \times_{i\in I}\Delta(\Sigma_i \times \Delta^2_i)$ such that, for every $i \in I$ and $t_i \in T_i(\lambda_i)$, σ_{i,t_i} is the behavior strategy obtained from β_{t_i} as per (12). A polymorphic assessment λ is* ***consistent*** *if there is a sequence $(\lambda^n)^\infty_{n=1}$ of polymorphic assessments converging to λ such that, for all $j \in I$ and $n \in \mathbb{N}$, λ^n_j is fully randomized, and*
(a-p) *for all $h \in H$, $a \in A(h)$, and $t_i \in T_i(\lambda^n_i)$,*

$$
\alpha^n_{t_i,-i}(a_{-i}|h) = \prod_{j\neq i} \sum_{t_j\in T_j(\lambda^n_j)} \sigma^n_{t_j}(a_j|h)\lambda^n_j(t_j|h),
$$

(b-p) *for all* $h \in H$ *and* $t_i \in T_i(\lambda_i^n)$,

$$\mathrm{marg}_{\Delta^1_{-i}} \beta^n_{t_i}(\cdot|h) = \sum_{t_{-i} \in T_{-i}(\lambda^n_{-i})} \lambda^n_{-i}(t_{-i}|h) \delta_{\alpha^n_{t_{-i}}},$$

where, for all $j \in I$, $t_j \in T_j(\lambda_j^n)$ *and* $n \in \mathbb{N}$, $\alpha^n_{t_j}$ *is the first-order belief system derived from* $\beta^n_{t_j}$.

Condition (a-p) extends independence condition (a) of Definition 1 to the multiple-types setting. Condition (b-p) implies that, conditional on the co-players' types, everyone has correct beliefs about the others' beliefs, including their plans. Yet uncertainty about co-players' types allows for uncertainty and meaningful updating about such beliefs. Conditions (a-p) and (b-p) imply that different types of the same player share the same beliefs about co-players, but may have different plans. Definition 3 thus is a minimal departure from the notion of consistent assessment, allowing for uncertainty and meaningful updating about the plans, hence intentions, of co-players.

Definition 4 *A polymorphic assessment* λ *is a* ***polymorphic sequential equilibrium (PSE)*** *if it is consistent and satisfies the following sequential rationality condition: for all* $h \in H$, $i \in I$, *and* $t_i \in T_i(\lambda_i)$,

$$\mathrm{Supp}\sigma_{t_i}(\cdot|h) \subseteq \arg\max_{a_i \in A_i(h)} u_i(h, a_i; \beta_{t_i}).$$

Remark 3 *Every SE is a degenerate (or monomorphic) PSE. Therefore, Theorem 1 implies that, if every decision-utility function* $u_i(h, \cdot; \cdot)$ $(i \in I\ , h \in H)$ *is continuous, then there is at least one PSE. In particular, every game with SA, ABB, or ABI has at least one PSE.*

Finally, we demonstrate how the PSE alters predictions in the Ultimatum Minigame and in leader-followers games more generally.

Example 11 Consider again Figure B. If $|\mathrm{Supp}\lambda_i| = 1$ for all i, then our results for the SE analysis still hold as a special case of the more general PSE analysis. Interesting new possibilities arise if $|\mathrm{Supp}\lambda_a| = 2$. Recall that, in the SE with SA/ABB utility functions and non-degenerate plans (Example 9) we had $\alpha_a(g) = \frac{3}{4} - \frac{1}{4\theta_b}$ (with $\theta_b > 1/3$) to keep Bob indifferent. Suppose instead there are two types of Ann, a fraction of $\frac{3}{4} - \frac{1}{4\theta_b}$ of them planning to choose g while the others plan for f. There is a corresponding PSE where (naming Ann's types by planned choice) $\mathrm{Supp}\lambda_a = \{(f, \beta_f), (g, \beta_g)\}$, $\alpha_f(y|g) = \alpha_g(y|g) = \alpha_b(y|g) = 2/3$, and this holds for also for ABI, not only SA and ABB. The first-order belief of type f of Ann, α_f, is derived from β_f, etc. Bob initially believes Ann is either an f- or a g-type, assigning probability $\lambda_g = \frac{3}{4} - \frac{1}{4\theta_b}$ to the latter possibility. After action g he ceases to assign positive probability to being matched with an f-type, assigning instead probability 1 to the g-type, a form of updating about Ann's intentions implied by consistency (Def. 3). This inference makes ABI work as ABB (and SA). Bob's frustration is as in Example 9, so equal to his blame of Ann for each blaming function. Again Bob is indifferent between y and n, and sequentially rational if $\alpha_b(y|g) = 2/3$. Condition $\alpha_f(y|g) = \alpha_g(y|g) = 2/3$ implies both types of Ann are indifferent, hence sequentially rational. Thus, starting with the non-degenerate SE under ABB (and SA) we obtain a PSE, under every blaming function, where Ann's plan is purified. ▲

The insight of the previous example can be generalized:[19]

Proposition 2 *Consider a leader-followers game and an arbitrary parameter profile $(\theta_i)_{i\in I}$. Every SE with decision-utility functions $(u^{ABB}_{i,\theta_i})_{i\in I}$ [or $(u^{SA}_{i,\theta_i})_{i\in I}$] where the behavioral strategy of the leader has full support corresponds to a PSE with decision-utility functions $(u^{ABB}_{i,\theta_i})_{i\in I}$ [or $(u^{SA}_{i,\theta_i})_{i\in I}$] and also $(u^{ABI}_{i,\theta_i})_{i\in I}$ where the leader is purified.*

6 Multistage extension

In a multistage game form, a (nonempty) nonterminal history is a sequence of action profiles, $h = (a^1, ..., a^t)$ where $t \geq 1$. As in the two-stage case, we assume that actions are observable; hence, every non-terminal history is public. Our notation for the multistage setting is essentially the same as before. The set of sequences observable by player i also includes personal histories of the form (h, a_i): $H_i = H \cup \{(h, a_i) : h \in H, a_i \in A_i(h)\}$.

A belief system for i over paths and beliefs of others is an array of probability measures $\beta_i = (\beta_i(\cdot|h_i))_{h_i\in H_i}$ satisfying (3) and (4), which apply to the multistage setting as well. Also the notation on beliefs is as before: $\alpha_i \in \Delta^1_i$, $\beta_i \in \Delta^2_i$, and α_i is the first-order belief system derived from β_i when they appear in the same formula. The definition of SE can be applied without modifications.

We distinguish two extreme scenarios according to the behaviorally relevant periodization: In the **slow-play** scenario, stages correspond to periods, and player i's reference belief to determine frustration at the beginning of period (stage) $t + 1$ is given by his belief at the beginning of period t. In the **fast-play** scenario, the game's different stages occur in the same period and the relevant reference belief of i in stage t is given by his initial belief (at the root).[20] In either case, we maintain the assumption that blame is continuous in beliefs, capped by frustration, and equal to frustration in the case of SA.

[19] Recall that, by Remark 1, in leader-followers games SA is equivalent to both versions of ABB.

[20] Applications may involve intermediate cases, as in alternating-offer bargaining models where a period comprises two stages. The two extremes convey the main ideas.

6.1 Slow play

We start with this scenario because it allows for a relatively simple extension of the two-stage setting, with initial beliefs replaced by one-period-lagged beliefs: For any non-terminal history of the form $h = (\bar{h}, a)$ the frustration of i conditional on h given α_i is

$$\mathrm{F}_i\left(h;\alpha_i\right) = \left[\mathbb{E}[\pi_i|\bar{h};\alpha_i] - \max_{a_i\in A_i(h)} \mathbb{E}[\pi_i|\left(h,a_i\right);\alpha_i]\right]^+.$$

(When $\bar{h} = \varnothing$ and $h = a^1$, we are back to the two-period formula.) The decision utility of action $a_i \in A_i(h)$ has the general form (5), where the blame functions $\mathrm{B}_{ij}\left(h;\beta_i\right)$ are of the SA, ABB, or ABI type. Specifically: $\mathrm{B}_{ij}\left(h;\beta_i\right) = \mathrm{F}_i\left(h;\alpha_i\right)$ for SA, whereas the could-have-been blame, blaming deviations, and blaming intentions can be defined with straightforward adaptations of (9), (10), and (11) respectively; therefore we omit the details.

This extension of the two-stage setting has the stark feature that past frustrations do not affect current behavior. (A more nuanced version of the model might feature a decaying effect of past frustrations.)

A detail in modeling game forms becomes relevant in the slow play scenario: We have to explicitly allow for non-terminal histories after which no player (not even chance) is active, such as history g in Fig. D.

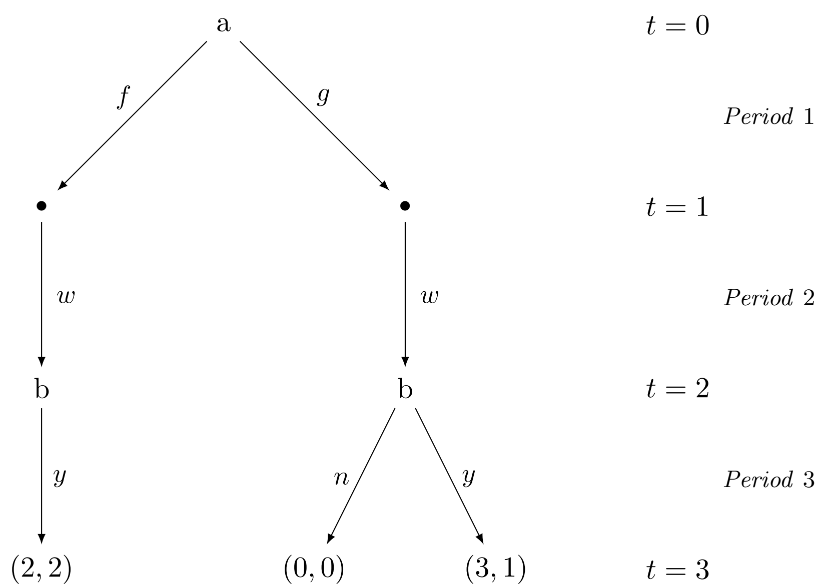

Figure D. Ultimatum Minigame with delayed reply.

At such histories there is only one feasible action profile, as each player has only one feasible action, to wait. In the two-periods setting this detail is irrelevant: If nobody is active at the root, play effectively starts (and ends) in the second period; if nobody is active at a^1, it can be modeled as a terminal history. With more than two periods, having to wait may affect behavior.

Example 12 Consider Fig. D. Suppose that Bob initially expects f with positive probability. Then, in period 2, after g, he is frustrated; however Bob cannot hurt Ann immediately because he has to wait. In period 3, Bob's lagged expectation has fully adapted downward, and he is not frustrated. According to our slow-play model, the frustration experienced by Bob in period 2 does not affect his decision utility in period 3: Bob fully "cools off" and behaves as-if selfish. Therefore the unique SE and PSE outcome of the game is (g, w, y), where w denotes waiting. ▲

6.2 Fast play

When play is fast, all stages belong to the same period, therefore the reference belief that determines player i's frustration conditional on any history is i's initially expected monetary payoff. Thus, i's frustration at h given α_i is

$$\mathrm{F}_i(h;\alpha_i) = \left[\mathbb{E}[\pi_i;\alpha_i] - \max_{a_i \in A_i(h)} \mathbb{E}[\pi_i|(h,a_i);\alpha_i]\right]^+ .$$

This implies that *there cannot be any "cooling off"* due to reference-point acclimatization. Formally, histories where nobody (not even chance) is active play no role and can be deleted from the game form without affecting the analysis. For example, in the fast-play scenario, the ultimatum game form of Fig. D is equivalent to the one of Fig. B.

The fast-play frustration formula can be plugged into the SA decision utility function (7). As for the ABB decision utility, property (8) of B_{ij} extends to the multistage setting as follows:

$$\mathrm{B}_{ij}(h;\alpha_i) = \begin{cases} 0, & \text{if } j \notin I(h') \text{ for all } h' \prec h, \\ \mathrm{F}_i(a^1;\alpha_i), & \text{if } \{j\} = I(h') \text{ for all } h' \prec h. \end{cases} \tag{13}$$

In words, co-player j cannot be blamed if he was never active in the past, and he is fully blamed if instead he was the only active player. A relatively simple extension of could-have-been blame satisfies this property:

$$\mathrm{B}_{ij}(h;\alpha_i) = \min\left\{\left[\max_{h' \prec h, a'_j \in A_j(h')} \mathbb{E}\left[\pi_i|(h',a'_j);\alpha_i\right] - \mathbb{E}[\pi_i|h;\alpha_i]\right]^+, \mathrm{F}_i(h;\alpha_i)\right\}. \tag{14}$$

We can follow a similar logic to extend ABI.

Remark 4 *If* B_{ij} *is defined by (14), then it satisfies (13).*

We now illustrate our definition, elucidating a modeling choice:

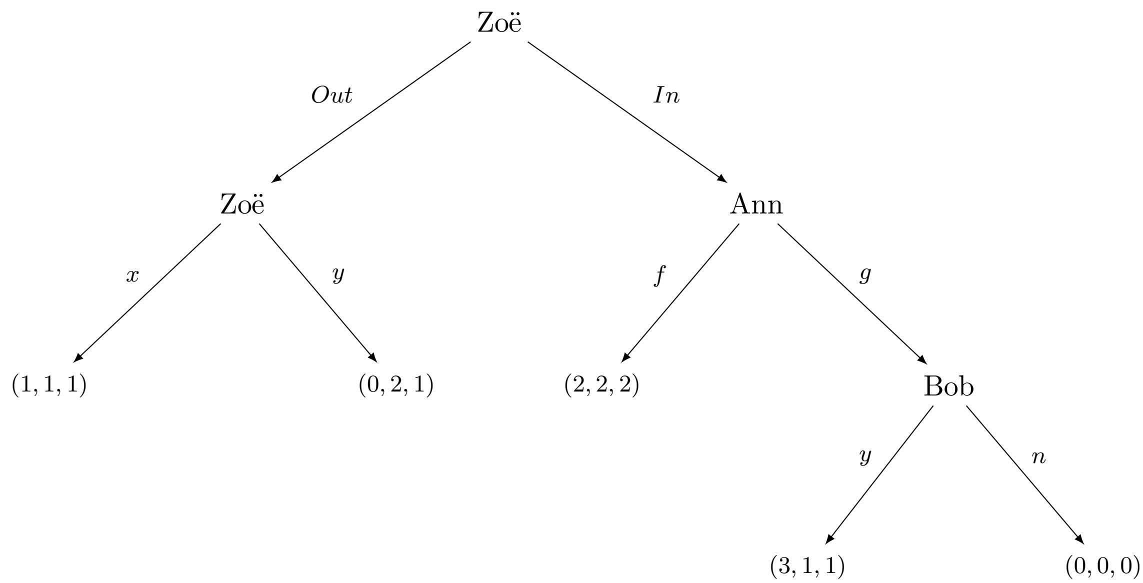

Figure E. Multistage Ultimatum featuring Zoë.

Example 13 Consider the game form in Fig. E (material payoffs are in alphabetical order). If Zoë chooses In, then Ann and Bob interact in an ultimatum minigame, but Zoë may instead exercise outside options and play (Out, x) or (Out, y). Zoë's payoffs equal Bob's, except following (Out, y) where a payoff transfer from Ann to Bob occurs, relative to (Out, x). Can strategy profile $(In\text{-}x, f, n)$ be a SE under ABB? Given equilibrium beliefs, this is the case if $0 - \theta_b \cdot 1 \cdot 0 \geq 1 - \theta_b \cdot 1 \cdot 3$, or $\theta_b \geq 1/3$. The calculation involves Bob blaming Ann, not Bob blaming Zoë, because if Zoë switched from In to Out (thus implementing (Out, x) instead of In) this would not improve Bob's payoff. This reflects a non-obvious modeling choice: Our definition assesses blame on the basis of single-agent deviations from the realized path, but if Bob alternatively assessed blame on the basis of multi-agent deviations, including off-realized-path deviations, he would consider that Zoë could have played (Out, y). She would then have increased Bob's payoff from 1 to 2, preventing his frustration of 1. If Bob's blame of Zoë were thus 1, then $(In\text{-}x, f, n)$ would be a SE under ABB if $0 - \theta_b \cdot 1 \cdot 0 \geq 1 - \theta_b \cdot 1 \cdot 3 - \theta_b \cdot 1 \cdot 1$, or $\theta_b \geq 1/4 \neq 1/3$. (This also shows that SE under ABB is not invariant with respect to coalescing sequential moves.) Finally, note that also $(In\text{-}y, f, n)$ is a SE under ABB in the fast-play scenario for $\theta_b \geq 1/4$, because at (In, g) Zoë would be blamed for not switching to Out (implementing (Out, y)); but it is a SE under ABB in the slow-play scenario for larger parameter values, $\theta_b \geq 1/3$, because Bob would be frustrated only in the third period, after (In, g), and Zoë—who played in the first—could not be blamed. ▲

The single- vs. multi-agent deviation issue illustrated here can arise also in two-stage games (with simultaneous moves), but the point is clearer, and perhaps more relevant, in games with more than two stages. We defend our chosen formulation thrice: It harmonizes well with how we define rational play, where players optimize only locally (although in equilibrium they predict correctly and choose as planned). The (hinted at) alternative definition would be formally convoluted. It is an open issue which formulation is empirically more relevant, so we stick with what is simpler.

6.3 Counterfactual anger and unique SE in hold-up

It is important to emphasize, worth a separate section, that anger (and in fact emotions more generally) can shape behavior without occurring. If anger is anticipated, this may steer behavior down alternative paths (cf. Remark 2). We already saw examples, *e.g.*, (f, n) is a SE in the Ultimatum Minigame, alongside (g, y). Our next example highlights how there may be circumstances where the SE is *unique* and has that property. It also illustrates a difference between fast and slow play.

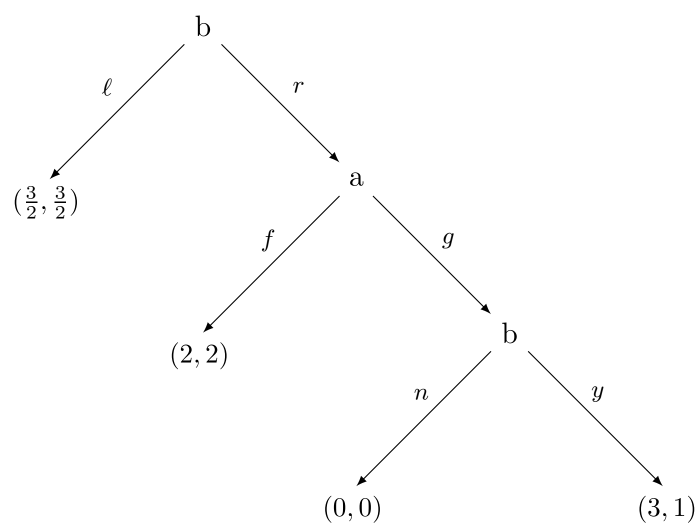

Figure F. Hold-up.

Example 14 Modify the Ultimatum Minigame by adding an initial move for Bob, as in Fig.

F, to get an illustration of a hold-up problem (cf. Dufwenberg, Smith & Van Essen 2013).[21] Under fast play, for each utility function seen so far,[22] if $\theta_b > 2/3$, there is a unique SE: Bob uses plan (r-n), Ann plans for f. To verify this, the key step is to check that if Bob plans for (ℓ, y) and Ann for g this is *not* a SE; if Bob initially expects \$1.5, off-path at (r, g), he would be frustrated and deviate to n. ▲

With slow play, by contrast, with $\theta_b > 2/3$, there are multiple SE, exactly as in the Ultimatum Minigame. In particular, both (r-n, f) and (ℓ-y, g) are SE; in the latter, Bob's updated expected payoff after (counterfactual) action r is only \$1, hence he cannot be frustrated by g.

7 Discussion

Incorporating the effects of emotions in economic analysis is a balancing act. One wants to focus on sentiments that make empirical sense, but human psychology is multi-faceted and there is no unambiguous yardstick. Our chosen formulation provides a starting point for exploring how anger shapes interaction, and experimental or other evidence will help to assess empirical relevance and suggest revised formulas. We conclude by discussing sundry topics that may help gain perspective on, build on, or further develop our work.

Frustration Consider substituting $\mathbb{E}[\pi_i; \alpha_i] - \mathbb{E}[\pi_i | a^1; \alpha_i]$ for $\mathrm{F}_i(a^1; \alpha_i)$ of Section 3. This alternative would measure i's *actual* diminished expectations at a^1, unlike $\mathrm{F}_i(a^1; \alpha_i)$ which captures diminished expectations relative to what i believes is the most he can get (which we think of as the adequate way to capture goal-blockage). To appreciate how dramatically this change would impact behavior, consider a two-player common-interest game: Ann chooses *Out* or *In*; in the former case the game ends with payoffs $(1, 1)$, in the latter case Bob chooses between $(0, 0)$ and $(2, 2)$. *Mutatis mutandis*, for high enough θ_b, with the alternative, under SA and ABB, there is a SE where Ann chooses *Out* and Bob would go for $(0, 0)$. Following *In*, Bob would be frustrated because he (so-to-say) sees himself as locked-in with his stage-2 planned action. Our formulation of $\mathrm{F}_i(a^1; \alpha_i)$ rules that out.

Consider a binary gamble where with probability $p > 0$ Ann wins $\$x > 0$, and otherwise gets \$0. Her frustration, using our definition, equals her initial expectation: $p \cdot x$. This embodies strong implications for how frustrations compare across contexts, *e.g.* the frustration

[21] Bob and Ann face a joint business opportunity worth $(2, 2)$ via path (r, f); however, r involves partnership-specific investment by Bob, which Ann can exploit choosing g (reneging), etc. As always, we list payoffs by alphabetical order of players: (π_a, π_b).

[22] Except the blaming-unexpected-deviations version of ABB, which we did not define explicitly for fast play.

of a highly expected failure to win the state lottery versus that of some unlikely small loss. We are agnostic as regards empirical relevance, but alert the reader to the issue.[23]

The psychological evidence (cited in Section 1) says a player becomes frustrated when his goals are unexpectedly thwarted. We addressed but one aspect: own material rewards. Cases 1-3 indicate broad applied potential. Yet our focus is restrictive, as one may imagine other sources of frustration:

> **Case 4:** In 2007 Apple launched its iPhone at $499. Two months later they introduced a new version at $399, re-priced the old model at $299, and caused outrage among early adopters. Apple paid back the difference. Did this help long run profit?
>
> **Case 5:** The 2008 TARP bank bail-out infuriated some US voters. Did this ignite Tea Party/Occupy-Wall Street movements?

In case 4, an early adopter is frustrated because he regrets he already bought, not because new information implies his expected rewards drop. In case 5, even an activist who is materially unaffected personally may be frustrated because of unexpected perceived unfairness. These examples are not exhaustive; further sources of frustration may *e.g.* involve shocks to self-esteem.[24] Techniques analogous to those we have developed may be applicable in these cases, but going in these directions is left for future research.

As regards the effects of frustration, we considered changes to a player's utility but neglected other plausible adjustments. Gneezy & Imas (2014) report data from an intriguing experiment involving two-player zero-sum material payoff games. In one game players gain if they are strong, in the other if they are smart. Before play starts, one subject may anger his opponent and force him to stay in the lab to do boring tasks. A thus frustrated player's performance is enhanced when strength is beneficial (possibly from increased adrenaline flow), but reduced when cool logic is called for (as if an angered player becomes cognitively impaired). Our model can capture the first consideration, but not the second. Specifically, we can let the consequences of actions depend also on beliefs, *e.g.*, because emotions affect strength or speed (cf. Rauh & Seccia 2006); this ultimately translates into belief-dependent utility (or cost) of actions. However, to capture the second effect, we would need a theory of endogenous cognitive abilities.

[23] The example involves one-player with a dummy-choice only to facilitate the frustration-calculation; interesting testable implications obviously arise more generally, *e.g.* in modified versions of the hammering one's thumb game.

[24] See Baumeister, Smart & Boden (1996) for an interesting discussion linking (threatened) self-esteem and violence.

Valence and action-tendency Psychologists classify emotions in multiple ways. Two prominent aspects are **valence**, the intrinsic pleasantness or aversiveness of an emotion, and **action-tendency**, or how behavior is shaped as the emotion occurs. Both notions have bearing on anger. For example, most psychologists believe anger has negative valence (see, *e.g.*, Harmon-Jones & Sigelman 2001, p. 978). Perhaps such considerations steer people to avoid frustrations, say by not investing in the stock market. That said, the distinguishing feature of anger that psychologists stress concerns its action-tendency of aggression, not its valence. In developing our theory, we have exaggerated this, abstracting away from frustration avoidance, while emphasizing frustration-induced aggression. This is reflected in the decision utility functions, which are shaped by current frustration, but not by the anticipation of the negative valence of future frustrations.[25]

Blame We explored various ways a player may blame others, but other notions are conceivable. For example, with anger from blaming behavior i's blame of j depends on what i believes he would truly get at counterfactual histories, rather than the most he could get there. We view this modeling choice as reflecting local agency; i's current agent views other agents of i as uncontrollable, and he has no direct care for their frustrations. Another example relates to how we model anger from blaming intentions: i's blame of j depends on β_i, his second-order beliefs. Recall that the interpretation concerns beliefs about beliefs about material payoffs, not beliefs about beliefs about frustration, which would be third- rather than second-order beliefs. Battigalli & Dufwenberg (2007), in a context which concerned guilt rather than anger, worked with such a notion.

Our blame concepts one way or another assess the *marginal* impact of other players. For example, consider a game where i exits a building while all $j \in I \backslash \{i\}$, unexpectedly to i, simultaneously hurl buckets of water at i, who gets soaked. According to our approach, i cannot blame any j as long as there are at least two hurlers. One could imagine alternatives where i blames, say, all the hurlers on the grounds that *collectively* they could thwart i's misery.

People may blame others in unfair ways, *e.g.* nominating scapegoats. Our notions of SA and ABB may embody related notions to some degree, but it has not been our intention to address such concerns systematically.

Several recent experiments explore interesting aspects of blame (Bartling & Fischbacher 2012, Gurdal *et al.* 2014, Celen, Schotter & Blanco 2014). We emphasize that our focus on

[25] In previous work we modeled another emotion: guilt; see, *e.g.*, Battigalli & Dufwenberg (2007), Chang, Smith, Dufwenberg & Sanfey (2011). To gain perspective note that in that work our approach to anticipation of valence and action-tendency was reversed. Guilt has valence (negative!) as well as action-tendency (say to engage in "repair behavior"; see, *e.g.*, Silfver 2007). In modeling guilt we highlighted the anticipation of its negative valence while neglecting action-tendencies.

blame is restricted to its relation to frustration, not reasons besides frustration that may lead people to blame each other.[26]

Kőszegi & Rabin and Card & Dahl Card & Dahl (2011) show that reports of domestic abuse go up when football home teams favored to win lose. They argue that this is in line with Kőszegi & Rabin's (2006, 2007, henceforth KR) theory of expectations-dependent reference points. KR model the loss felt when a player gets less than he expected, which one may think of as a form of disappointment with negative valence (cf. Bell 1985, Loomes & Sugden 1986). However, KR do not model other-regarding preferences directly: they focus on the consequences of their model for individual decisions. Our models study the social consequences of frustration: frustration results in lower weights on coplayer payoffs, and hence encourages costly punishment. Our simple anger model and the example of hammering-one's thumb captures Card & Dahl's result.

Another difference between this paper and KR is that in their work anticipation of the negative valence of future frustrations influences decision utility. Our decision makers are influenced by past frustrations, rather than future ones. Modeling details then distinguish how we define frustration and how KR define loss (*e.g.*, how we cap frustration using the highest attainable payoff).

Anger management People aware of their inclination to be angry may attempt to manage or contain their anger. Our players anticipate how frustrations shape behavior, and they may avoid or seek certain subgames because of that. However, there are interesting related phenomena we do not address: Can i somehow adjust θ_i say by taking an "anger management class?" If so, would rational individuals want to raise, or to lower, their θ_i? How might that depend on the game forms they play? These are potentially relevant questions related to how we have modeled action-tendency. Further issues would arise if we were to consider aspects involving anticipated negative valence of future frustrations, or bursts of anger.

Rotemberg's approach In a series of intriguing papers Rotemberg explores how consumer anger shapes firms' pricing (2005, 2011), as well as interaction in ultimatum games (2008). He proposes (versions of) a theory in which players are slightly altruistic, and consumers/responders also care about their co-players' degrees of altruism. Namely, they abruptly become very angry and punish a co-player whom they come to believe has an altruism parameter lower than some (already low) threshold. "One can thus think of individual i as acting

[26] For example, Celen *et al.* (2014) present a model where i asks how he would have behaved had he been in j's position and had j's beliefs. Then i blames j if j appears to be less generous to i than i would have been, and may blame j even if i is not surprised/frustrated. Or imagine a model where players blame those considered unkind, as defined in reciprocity theory (cf. subsection below), independently of frustration.

as a classical statistician who has a null hypothesis that people's altruism parameter is at least as large as some cutoff value. If a person acts so that i is able to reject this hypothesis, individual i gains ill-will towards this person" (Rotemberg 2008, p. 464).

As a literal statement of what upsets people, this assumption does not match well our reading of the relevant psychology. Recall that frustration arises when individuals are unexpectedly denied things they care about. Matters like "own payoff" (our focus) and "fairness" or "quality of past decisions" (which we mentioned) come to mind; a co-player's altruism being λ rather than $\lambda - \varepsilon$, where λ and ε are tiny numbers, hardly does. On the other hand, perhaps one may think of the approach as capturing a notion of scapegoating. Moreover, it is impressive how Rotemberg's model captures the action in his data sets. It is natural to wonder whether our models could achieve that too. As regards behavior in ultimatum (and some other) games, there is already some existing evidence that is consistent with our approach; see the discussion below. Regarding pricing, we leave for empirical economists the task of exploring the topic.

Negative reciprocity ... à la Rabin (1993), Dufwenberg & Kirchsteiger (2004), and Falk & Fischbacher (2006) joins anger as a motivation that can trigger hostility, but the two differ in key ways. The following sketched comparison is with Dufwenberg & Kirchsteiger's notion of sequential reciprocity equilibrium (SRE; refer to their article for formal definitions).

In the Hammering-One's-Thumb game (Fig. C), Andy may take it out on Bob if he is motivated by simple anger. Were he motivated by reciprocity, this could never happen: Bob's kindness, since he is a dummy-player, equals 0, implying that Andy chooses as-if selfish. Reciprocity here captures intuitions similar to the ABI concept, but that analogy only carries so far, as we show next.

Reciprocity allows for "miserable equilibria," where a player reciprocates expected unkindness before it occurs. For example, in the Ultimatum Minigame of Fig. B, (g, n) may be a SRE. Ann makes offer g despite believing that Bob will reject; given her beliefs about Bob's beliefs, Ann perceives Bob as seeing this coming, which makes him unkind, so she punishes by choosing g. Such self-fulfilling prophecies of destructive behavior have no counterpart under any of our anger notions. Since Ann moves at the root, she cannot be frustrated, and hence chooses as-if selfish.[27]

Our cooling-off effects (Section 6) have no counterpart in reciprocity theory, which makes the same prediction in Figures B and D. Reciprocal players do not cool off. "*La vengeance est un plat qui se mange froid.*"

[27] Another example is the hold-up game of Figure F. We gave conditions where $(r\text{-}n, f)$ was the unique SE. If Ann and Bob were reciprocal, $(\ell\text{-}n, g)$ and $(r\text{-}n, g)$ could be SRE, with miserable interaction, respectively, off and on the equilibrium path.

Experimental testing Our models tell stories of what happens when players prone to anger interact. It is natural to wonder about empirical relevance, and here experiments may be helpful.

A few experiments that measure beliefs, emotions, and behavior together provide support for the notion that anger and costly punishment result from outcomes which do not meet expectations. Pillutla & Murnighan (1996) find that reported anger predicted rejections better than perceived unfairness in ultimatum games. Fehr & Gächter (2002) elicit self-reports of the level of anger towards free riders in a public goods game, concluding that negative emotions including anger are the proximate cause of costly punishment.

Several other studies connect unmet expectations and costly punishment in ultimatum games. Falk, Fehr, & Fischbacher (2003) measure beliefs and behavior in ultimatum minigames. They report higher proportions of rejections of disadvantageous offers when responders' expected payoffs are higher, consistent with our models.[28] Schotter & Sopher (2007) also measure second-mover expectations, concluding that unfulfilled expectations drive rejections of low offers. Similarly, Sanfey (2009) finds that psychology students who are told that a typical offer in the ultimatum game is \$4-\$5 reject low offers more frequently than students who are told that a typical offer is \$1-\$2.

A series of papers by Frans van Winden and coauthors records both emotions and expectations in the power-to-take game (which resembles ultimatum games, but allows for partial rejections).[29] Second-mover expectations about first-mover "take rates" are a key factor in the decision to destroy income, and anger-like emotions are triggered by the difference between expected and actual take rates. The difference between the actual and reported "fair" take rate is not significant in determining anger-like emotions, suggesting that deviations from expectations, rather than from fairness benchmarks, drive both anger and the destruction of endowments in the games.

Apropos the cooling off effects discussed in Section 6, Grimm & Mengel (2011) run ultimatum games that force some responders to wait ten minutes before making their choice. Without delay, less than 20% of low offers were accepted while 60–80% were accepted if the acceptance decision were delayed.

A literature in neuroscience connects expectations with social norms to study the neural underpinnings of emotional behavior. In Xiang, Lohrenz & Montague (2013), subjects respond to a sequence of ultimatum game offers whilst undergoing fMRI imaging. Unbeknownst to subjects, the experimenter controls the distribution of offers in order to manipulate beliefs. Rejections occur more often when subjects expect high rather than low offers. The authors connect norm violations (i.e., lower than expected offers) with reward prediction errors from

[28]See the data in Figure 2 and Table 1 of their paper. Brandts & Solá (2001) find similar behavioral results (see Table 1). Compare also with our discussion of Example 3 in Section 4.

[29]Bosman & van Winden (2002), Bosman, Sutter & van Winden (2005), Reuben & van Winden (2008).

reinforcement learning, which are known to be the computations instantiated by the dopaminergic reward system. Xiang *et al.* note that "when the expectation (norm) is violated, these error signals serve as control signals to guide choices. They may also serve as the progenitor of subjective feelings."

It would be useful to develop tests specifically designed to target key features of our theory. For example, which version—SA, ABB, ABI—seems more empirically relevant, and how does the answer depend on context (*e.g.*, is SA more relevant for tired subjects)? Some insights may again be gleaned from existing studies. For example, Gurdal *et al.* (2014) study games where an agent invests on behalf of a principal, choosing between a safe outside option and a risky alternative. If the latter is chosen, then it turns out that many principals punish the agent if and only if by chance a poor outcome is realized. This seems to indicate some relevance of our ABB solution (relative to ABI). That said, Gurdal *et al.*'s intriguing design is not tailored to specifically test our theory (and beliefs and frustrations are not measured), so more work seems needed to draw clearer conclusions.

Applications Formulating, motivating, and elucidating the key definitions of our models is more than a mouthful, so we have not taken this paper in the direction of doing applied economics. Make no mistake about it though, the hope that our models will prove useful for such work has been a primary driving force. Our psychologically grounded models of frustration, anger, and blame may shed light on many of the themes (e.g. pricing, violence, politics, recessions, haggling, terror, and traffic) that we listed at the start of this paper. We hope to do some work in these directions ourselves.

A Appendix

This appendix contains proofs of the results stated in the main text.

A.1 Preliminaries

To ease exposition, some of the key definitions and equations contained in the main text are repeated below.

For each topological space X, we let $\Delta(X)$ denote the space of Borel probability measures on X endowed with the topology of weak convergence of measures. Every Cartesian product of topological spaces is endowed with the product topology. A topological space X is metrizable if there is a metric that induces its topology. A Cartesian product of a countable (finite, or denumerable) collection of metrizable spaces is metrizable.

$\Delta_i^1 \subseteq \times_{h_i \in H_i} \Delta\left(Z(h_i)\right)$ is the set of first-order beliefs, that is, the set of $\alpha_i = \left(\alpha_i\left(\cdot|Z(h_i)\right)\right)_{h_i \in H_i}$ such that:

- for all $h_i, h_i' \in H_i$, if $h_i \prec h_i'$ then for every $Y \subseteq Z(h_i')$

$$\alpha_i(Z(h_i')|Z(h_i)) > 0 \Rightarrow \alpha_i\left(Y|Z(h_i')\right) = \frac{\alpha_i(Y|Z(h_i))}{\alpha_i(Z(h_i')|Z(h_i))}; \tag{15}$$

- for all $h \in H$, $a_i \in A_i(h)$, $a_{-i} \in A_{-i}(h)$ (using obvious abbreviations)

$$\alpha_{i,-i}(a_{-i}|h) = \alpha_{i,-i}(a_{-i}|h, a_i). \tag{16}$$

$\Delta_i^2 \subseteq \times_{h_i \in H_i} \Delta\left(Z(h_i) \times \Delta_{-i}^1\right)$ —where $\Delta_{-i}^1 = \times_{j \neq i} \Delta_j^1$— is the set of second-order beliefs, that is, the set of $\beta_i = (\beta_i(\cdot|h_i))_{h_i \in H_i}$ such that:

- if $h_i \prec h_i'$ then
$$\beta_i(h_i'|h_i) > 0 \Rightarrow \beta_i\left(E|h_i'\right) = \frac{\beta_i\left(E|h_i\right)}{\beta_i(h_i'|h_i)} \tag{17}$$
for all $h_i, h_i' \in H_i$ and every event $E \subseteq Z(h_i') \times \Delta_{-i}^1$;

- i's beliefs satisfy an own-action independence property:
$$\beta_i\left(Z\left(h, (a_i, a_{-i})\right) \times E_\Delta | (h, a_i)\right) = \beta_i\left(Z\left(h, (a_i', a_{-i})\right) \times E_\Delta | (h, a_i')\right), \tag{18}$$
for every $h \in H$, $a_i, a_i' \in A_i(h)$, $a_{-i} \in A_{-i}(h)$, and (measurable) $E_\Delta \subseteq \Delta_{-i}^1$. The space of second-order beliefs of i is denoted Δ_{-i}^2.

Note that (16) and (18) are given by equalities between marginal measures (on $A_{-i}(h)$ and $A_{-i}(h) \times \Delta_{-i}^1$ respectively).

Lemma 2 *For each player $i \in I$, Δ_i^2 is a compact metrizable space.*

Proof Let Θ be a non-empty, compact metrizable space. Lemma 1 in Battigalli & Siniscalchi (1999) (B&S) establishes that the set of arrays of probability measures $(\mu(\cdot|h_i))_{h_i \in H_i} \in \times_{h_i \in H_i} \Delta\left(Z(h_i) \times \Theta\right)$ such that

$$h_i \prec h_i' \wedge \mu(h_i'|h_i) > 0 \Rightarrow \mu\left(E|h_i'\right) = \frac{\mu\left(E|h_i\right)}{\mu(h_i'|h_i)}$$

is closed. Note that, in the special case where Θ is a singleton, each $\Delta\left(Z(h_i) \times \Theta\right)$ is isomorphic to $\Delta\left(Z(h_i)\right)$; hence, the set of first-order beliefs satisfying (15) is closed. Letting $\Theta = \Delta_{-i}^1$, we obtain that the set of second-order beliefs satisfying (17) is closed.

Since $\times_{h_i \in H_i} \Delta(Z(h_i))$ is a compact subset of a Euclidean space and eq. (16) is a closed condition (equalities between marginal measures are preserved in the limit), Lemma 1 in B&S implies that Δ_i^1 is a closed subset of a compact metrizable space. Hence, Δ_i^1 is a compact metrizable space.

It is well known that if $X_1, ..., X_K$ are compact metrizable, so is $\times_{k=1}^K \Delta(X_k)$ (see Aliprantis & Border 2006, Theorem 15.11). Hence, by Lemma 1 in B&S, the set of second-order beliefs satisfying (17) is a closed subset of a compact metrizable space. Since eq. (18) is a closed condition (equalities between marginal measures are preserved in the limit), this implies that Δ_i^2 is compact metrizable. ■

Lemma 3 *For each profile of behavioral strategies $\sigma = (\sigma_i)_{i \in I}$ there is a unique profile of second-order beliefs $\beta^\sigma = (\beta_i^\sigma)_{i \in I}$ such that (σ, β^σ) is a consistent assessment. The map $\sigma \mapsto \beta^\sigma$ is continuous.*

Proof Write $\mathbb{P}^\sigma(h'|h)$ for the probability of reaching h' from h, *e.g.*,

$$\mathbb{P}^\sigma(a^1, a^2|\varnothing) = \left(\prod_{j \in I} \sigma_j(a_j^1|\varnothing)\right)\left(\prod_{j \in I} \sigma_j(a_j^2|a^1)\right).$$

Define α_i^σ as $\alpha_i^\sigma(z|h) = \mathbb{P}^\sigma(z|h)$ for all $i \in I$, $h \in H$, and $z \in Z$. Define β^σ as $\beta_i^\sigma(\cdot|h) = \alpha_i^\sigma(\cdot|h) \times \delta_{\alpha_{-i}^\sigma}$ for all $i \in I$, $h \in H$. It can be checked that (1) $\beta^\sigma \in \Delta_i^2$ for each $i \in I$, (2) (σ, β^σ) is a consistent assessment, and (3) if $\beta \neq \beta^\sigma$, then either (a) or (b) of the definition of consistency is violated. It is also apparent from the construction that the map $\sigma \mapsto \beta^\sigma$ is continuous, because $\sigma \mapsto \alpha^\sigma$ is obviously continuous, and the Dirac-measure map $\alpha_{-i} \mapsto \delta_{\alpha_{-i}}$ is continuous. ■

Lemma 4 *The set of consistent assessments is compact.*

Proof Lemma 2 implies that $\times_{i \in I}(\Sigma_i \times \Delta_i^2)$ is a compact metrizable space that contains the set of consistent assessments. Therefore, it is enough to show that the latter is closed. Let $(\sigma^n, \beta^n)_{n \in \mathbb{N}}$ be a converging sequence of consistent assessments with limit $(\sigma^\infty, \beta^\infty)$. For each $i \in I$, let α_i^n be the first-order belief derived from β_i^n ($n \in \mathbb{N} \cup \{\infty\}$), that is,

$$\alpha_i^n(Y|h) = \beta_i^n(Y \times \Delta_{-i}^1|h)$$

for all $h \in H$ and $Y \subseteq Z(h)$. By consistency, for all $n \in \mathbb{N}$, $i \in I$, $h \in H$, $a \in A(h)$, it holds that

- (a.n) $\alpha_i^n(a|h) = \beta_i^n\left(Z(h,a) \times \Delta_{-i}^1|h\right) = \prod_{j \in I} \sigma_j^n(a_j|h)$,

- (b.n) $\mathrm{marg}_{\Delta^1_{-i}}\beta_i^n(\cdot|h) = \delta_{\alpha^n_{-i}}$, where each α^n_j is determined as in (a.n).

Then,

$$\alpha_i^\infty(a|h) = \beta_i^\infty\left(Z(h,a)\times\Delta^1_{-i}|h\right) = \prod_{j\in I}\sigma_j^\infty(a_j|h)$$

for all $i\in I$, $h\in H$, $a\in A(h)$. Furthermore, $\mathrm{marg}_{\Delta^1_{-i}}\beta_i^\infty(\cdot|h) = \delta_{\alpha^\infty_{-i}}$ for all $i\in I$ and $h\in H$, because $\alpha^n_{-i}\to\alpha^\infty_{-i}$ and the marginalization and Dirac maps $\beta_i \mapsto \mathrm{marg}_{\Delta^1_{-i}}\beta_i$ and $\alpha_{-i}\mapsto\delta_{\alpha_{-i}}$ are continuous. ■

A.2 Proof of Remark 2

Fix $i\in I$ arbitrarily. First-order belief α_i is derived from β_i and, by consistency, gives the behavioral strategy profile σ. Therefore, by assumption each $h'\preceq h$ has probability one under α_i, which implies that $\mathbb{E}[\pi_i|h';\alpha_i] = \mathbb{E}[\pi_i;\alpha_i]$, hence $\mathrm{F}_i(h';\alpha_i) = 0$. Since blame is capped by frustration, $u_i(h',a_i';\beta_i) = \mathbb{E}[\pi_i|h';\alpha_i]$. Therefore, sequential rationality of the equilibrium assessment implies that $\mathrm{Supp}\sigma_i(\cdot|h')\subseteq\arg\max_{a_i'\in A_i(h')}\mathbb{E}[\pi_i|h';\alpha_i]$. If there is randomization only in the last stage (or none at all), then players maximize locally their expected material payoff on the equilibrium path. Hence, the second claim follows by inspection of the definitions of agent form of the material-payoff game and Nash equilibrium. ■

A.3 Proof of Proposition 1

Let $(\bar\sigma,\bar\beta) = \left(\bar\sigma_i,\bar\beta_i\right)_{i\in I}$ be the SE of the material payoff game, which is in pure strategies by the perfect information assumption. Fix decision utility functions $u_i(h,a_i;\cdot)$ of the ABI, or ABB kind, and a sequence of real numbers $(\varepsilon_n)_{n\in\mathbb{N}}$, with $\varepsilon_n\to 0$ and $0<\varepsilon_n<\frac{1}{\max_{i\in I,h\in H}|A_i(h)|}$ for all $n\in\mathbb{N}$. Consider the constrained psychological game where players can choose mixed actions in the following sets:

$$\Sigma_i^n(h) = \{\sigma_i(\cdot|h)\in\Delta(A_i(h)) : \|\sigma_i(\cdot|h)-\bar\sigma_i(\cdot|h)\|\leq\varepsilon_n\}$$

if h is on the $\bar\sigma$-path, and

$$\Sigma_i^n(h) = \{\sigma_i(\cdot|h)\in\Delta(A_i(h)) : \forall a_i\in A_i(h), \sigma_i(a_i|h)\geq\varepsilon_n\}$$

if h is off the $\bar\sigma$-path. By construction, these sets are non-empty, convex, and compact. Since the decision utility functions are continuous, and the consistent assessment map $\sigma\longmapsto\beta^\sigma$ is continuous (Lemma 3), correspondence

$$\sigma\longmapsto\times_{h\in H}\times_{i\in I}\arg\max_{\sigma_i'(\cdot|h)\in\Sigma_i^n(h)}\sum_{a_i\in A_i(h)}\sigma_i'(a_i|h)u_i(h,a_i;\beta_i^\sigma)$$

is upper-hemicontinuous, non-empty, convex, and compact valued; therefore (by Kakutani's theorem), it has a fixed point σ^n. By Lemma 4, the sequence of consistent assessments $\left(\sigma^n, \beta^{\sigma^n}\right)_{n=1}^{\infty}$ has a limit point (σ^*, β^*), which is consistent too. By construction, $\bar{\sigma}(\cdot|h) = \sigma^*(\cdot|h)$ for h on the $\bar{\sigma}$-path, therefore $(\bar{\sigma}, \bar{\beta})$ and (σ^*, β^*) are realization-equivalent. We let $\bar{\alpha}_i$ (respectively, α_i^*) denote the first-order beliefs of i implied by $(\bar{\sigma}, \bar{\beta})$ (respectively, (σ^*, β^*)).

We claim that the consistent assessment (σ^*, β^*) is a SE of the psychological game with decision utility functions $u_i(h, a_i; \cdot)$. We must show that (σ^*, β^*) satisfies sequential rationality. If h is off the $\bar{\sigma}$-path, sequential rationality is satisfied by construction. Since $\bar{\sigma}$ is deterministic and there are no chance moves, if h is on the $\bar{\sigma}$-path (*i.e.*, on the σ^*-path) it must have unconditional probability 1 according to each player's beliefs and there cannot be any frustration; hence, $u_i(h, a_i; \beta_i^*) = \mathbb{E}[\pi_i|h, a_i; \alpha_i^*]$ $(i \in I)$ where α_i^* is determined by σ^*. If, furthermore, it is the second stage ($h = \bar{a}^1$, with $\bar{\sigma}(\bar{a}^1|\varnothing) = 1$), then —by construction— $\mathbb{E}[\pi_i|h, a_i; \alpha_i^*] = \mathbb{E}[\pi_i|h, a_i; \bar{\alpha}_i]$, where $\bar{\alpha}_i$ is determined by $\bar{\sigma}$. Since $\bar{\sigma}$ is a SE of the material-payoff game, sequential rationality is satisfied at h. Finally, we claim that (σ^*, β^*) satisfies sequential rationality also at the root $h = \varnothing$. Let $\iota(h)$ denote the active player at h. Since $\iota(\varnothing)$ cannot be frustrated at $\varnothing$, we must show that action $\bar{a}^1$ with $\bar{\sigma}(\bar{a}^1|\varnothing) = 1$ maximizes his expected material payoff given belief $\alpha_{\iota(\varnothing)}$. According to ABB and ABI, player $\iota(a^1)$ can only blame the first mover $\iota(\varnothing)$ and possibly hurt him, if he is frustrated. Therefore, in assessment (σ^*, β^*) at node a^1, either $\iota(a^1)$ plans to choose his (unique) payoff maximizing action, or he blames $\iota(\varnothing)$ strongly enough to give up some material payoff in order to bring down the payoff of $\iota(\varnothing)$. Hence, $\mathbb{E}[\pi_{\iota(\varnothing)}|a^1; \alpha^*_{\iota(a^1)}] \leq \mathbb{E}[\pi_{\iota(\varnothing)}|a^1; \bar{\alpha}_{\iota(a^1)}]$ (anger). By consistency of (σ^*, β^*) and $(\bar{\sigma}, \bar{\beta})$, $\alpha^*_{\iota(a^1)} = \alpha^*_{\iota(\varnothing)}$ and $\bar{\alpha}_{\iota(a^1)} = \bar{\alpha}_{\iota(\varnothing)}$ (cons.). Since (σ^*, β^*) is realization-equivalent to $(\bar{\sigma}, \bar{\beta})$ (r.e.), which is the material-payoff equilibrium (m.eq.), for each $a^1 \in A(\varnothing)$,

$$
\begin{aligned}
&\mathbb{E}[\pi_{\iota(\varnothing)}|\bar{a}^1; \alpha^*_{\iota(\varnothing)}] \overset{\text{(r.e.)}}{=} \mathbb{E}[\pi_{\iota(\varnothing)}|\bar{a}^1; \bar{\alpha}_{\iota(\varnothing)}] \overset{\text{(m.eq.)}}{\geq} \\
&\mathbb{E}[\pi_{\iota(\varnothing)}|a^1; \bar{\alpha}_{\iota(\varnothing)}] \overset{\text{(cons.)}}{=} \mathbb{E}[\pi_{\iota(\varnothing)}|a^1; \bar{\alpha}_{\iota(a^1)}] \overset{\text{(anger)}}{\geq} \\
&\mathbb{E}[\pi_{\iota(\varnothing)}|a^1; \alpha^*_{\iota(a^1)}] \overset{\text{(cons.)}}{=} \mathbb{E}[\pi_{\iota(\varnothing)}|a^1; \alpha^*_{\iota(\varnothing)}].
\end{aligned}
$$

This completes the proof for the ABB and ABI cases. If there are only two players, then we have a leader-follower game and SA is equivalent to ABB (Remark 1 of "Frustration and Anger in Games"), so (σ^*, β^*) is a SE in this case too. ■

A.4 Proof of Remark 3

Fix $h \in H$. We consider the following simple extension of could-have-been blame in multistage games under fast play:

$$\mathrm{B}_{ij}(h;\alpha_i) = \min\left\{\left[\max_{h' \prec h, a'_j \in A_j(h')} \mathbb{E}\left[\pi_i|(h',a'_j);\alpha_i\right] - \mathbb{E}[\pi_i|h;\alpha_i]\right]^+, \mathrm{F}_i(h;\alpha_i)\right\}. \tag{19}$$

We must show that $\mathrm{B}_{ij}(h;\alpha_i) = 0$ if j is not active at any $h' \prec h$, and $\mathrm{B}_{ij}(h;\alpha_i) = \mathrm{F}_i(h;\alpha_i)$ if j is the only active player at each $h' \prec h$.

First note that if j was never active before, then $A_j(h')$ is a singleton for each $h' \prec h$, hence the term in brackets of (14) is zero. Next suppose that i is frustrated at h and j was the only active player in the past. Then there must be some $\bar{h} \prec h$ such that j deviated from i's expectations $\alpha_i(\cdot|\bar{h})$ for the first time, that is, $\bar{h}$ is the shortest predecessor $h' \prec h$ such that $\alpha_j(a'_j|h') < 1$ for $(h',a'_j) \preceq h$. Such $\bar{h}$ must have probability one according to the initial belief $\alpha_i(\cdot|\varnothing)$, thus $\mathbb{E}[\pi_i|\bar{h};\alpha_i] = \mathbb{E}[\pi_i;\alpha_i]$. Since $\max_{a'_j \in A_j(\bar{h})} \mathbb{E}\left[\pi_i|(\bar{h},a'_j);\alpha_i\right] \geq \mathbb{E}[\pi_i|\bar{h};\alpha_i]$, we have $\max_{a'_j \in A_j(\bar{h})} \mathbb{E}\left[\pi_i|(\bar{h},a'_j);\alpha_i\right] \geq \mathbb{E}[\pi_i;\alpha_i]$. Therefore

$$\begin{aligned}
&\max_{h' \prec h, a'_j \in A_j(h')} \mathbb{E}\left[\pi_i|(h',a'_j);\alpha_i\right] - \mathbb{E}[\pi_i|h;\alpha_i] \\
&\geq \max_{a'_j \in A_j(\bar{h})} \mathbb{E}\left[\pi_i|(\bar{h},a'_j);\alpha_i\right] - \mathbb{E}[\pi_i|h;\alpha_i] \\
&\geq \mathbb{E}[\pi_i;\alpha_i] - \mathbb{E}[\pi_i|h;\alpha_i] \\
&\geq \mathbb{E}[\pi_i;\alpha_i] - \max_{a_i \in A_i(h)} \mathbb{E}[\pi_i|(h,a_i);\alpha_i] = \mathrm{F}_i(h;\alpha_i),
\end{aligned}$$

which implies $\mathrm{B}_{ij}(h;\alpha_i) = \mathrm{F}_i(h;\alpha_i)$ according to (19). ■

References

[1] Akerlof, R. J. (2016): “Anger and Enforcement,” *Journal of Economic Behavior & Organization,* 126, 110-124.

[2] Alicke, M. D. (2000): “Culpable Control and the Psychology of Blame,” *Psychological Bulletin,* 126, 556.

[3] Aliprantis, C. R. and K. C. Border (2006): *Infinite Dimensional Analysis.* Berlin: Springer-Verlag.

[4] Anderson, E. and D. Simester (2010): “Price Stickiness and Customer Antagonism,” *Quarterly Journal of Economics,* 125, 729–765.

[5] Arnold, M. B. (1960): *Emotions and Personality.* New York: Columbia University Press.

[6] Attanasi, G., P. Battigalli, and E. Manzoni (2016): “Incomplete Information Models of Guilt Aversion in the Trust Game,” *Management Science*, 62, 648 - 667.

[7] Averill, J. R. (1983): “Studies on Anger and Aggression: Implications for Theories of Emotion,” *American Psychologist,* 38, 1145–1160.

[8] Bartling, B. and U. Fischbacher (2012): “Shifting the Blame: On Delegation and Responsibility,” *Review of Economic Studies*, 79, 67–87.

[9] Battigalli, P., G. Charness, and M. Dufwenberg (2013): “Deception: The Role of Guilt,” *Journal of Economic Behavior & Organization*, 93, 227–232.

[10] Battigalli, P., A. Di Tillio, and D. Samet (2013): “Strategies and Interactive Beliefs in Dynamic Games,” in D. Acemoglu, M. Arellano, E. Dekel (eds.) *Advances in Economics and Econometrics: Theory and Applications*, Tenth World Congress, Vol. 1 Economic Theory (Econometric Society Monographs No. 49), Cambridge: Cambridge University Press, 391–422.

[11] Battigalli, P. and M. Dufwenberg (2007): “Guilt in Games,” *American Economic Review, Papers and Proceedings,* 97, 170–176.

[12] Battigalli, P. and M. Dufwenberg (2009): “Dynamic Psychological Games,” *Journal of Economic Theory,* 144, 1–35.

[13] Battigalli, P. and M. Siniscalchi (1999): “Hierarchies of Conditional Beliefs and Interactive Epistemology in Dynamic Games,” *Journal of Economic Theory,* 88, 188–230.

[14] Baumeister, R. F. and B. F. Bushman (2007): “Angry Emotions and Aggressive Behaviors,” G. Steffgen and M. Gollwitzer (Eds.) *Emotions and Aggressive Behavior*, Cambridge, MA: Hogrefe, 61–75.

[15] Baumeister, R. F., L. Smart, and J. M. Boden (1996): “Relation of Threatened Egotism to Violence and Aggression: The Dark Side of High Self-Esteem,” *Psychological Review,* 103, 5–33.

[16] Bell, D. (1985): “Disappointment in Decision Making under Uncertainty,” *Operations Research*, 33, 1–27.

[17] Berkowitz, L. (1989): "Frustration-Aggression Hypothesis: Examination and Reformulation," *Psychological Bulletin*, 106, 59–73.

[18] Bosman, R., M. Sutter, and F. van Winden (2005): "The Impact of Real Effort and Emotions in the Power-to-take Game," *Journal of Economic Psychology,* 26, 407–429.

[19] Bosman, R. and F. van Winden (2002): "Emotional Hazard in a Power-to-take Experiment," *Economic Journal*, 112, 147–169.

[20] Brandts, J., and C. Solá (2001): "Reference Points and Negative Reciprocity in Simple Sequential Games," *Games and Economic Behavior*, 36, 138-157.

[21] Brams, S. (2011): *Game Theory and the Humanities: Bridging Two Worlds*, Cambridge, MA: MIT Press.

[22] Card, D. and G. Dahl (2011): "Family Violence and Football: The Effect of Unexpected Emotional Cues on Violent Behavior," *Quarterly Journal of Economics,* 126, 103–143.

[23] Carpenter, J. and P. Matthews (2012): "Norm Enforcement: Anger, Indignation or Reciprocity," *Journal of the European Economic Association,* 10, 555–572.

[24] Celen, B., A. Schotter, and M. Blanco (2014): "On Blame and Reciprocity: An Experimental Study," typescript.

[25] Chang, L., A. Smith, M. Dufwenberg, and A. Sanfey (2011): "Triangulating the Neural, Psychological, and Economic Bases of Guilt Aversion," *Neuron,* 70, 560–572.

[26] Chockler, H., and J.Y. Halpern (2004): "Responsibility and Blame: A Structural-Model Approach, " *Journal of Artificial Intelligence Research*, 22, 93-115.

[27] Dollard, J., L. Doob, N. Miller, O. Mowrer, and R. Sears (1939): *Frustration and Aggression.* New Haven, NJ: Yale University Press.

[28] Dufwenberg, M. and G. Kirchsteiger (2004): "A Theory of Sequential Reciprocity," *Games and Economic Behavior*, 47, 268–298.

[29] Dufwenberg, M., A. Smith, and M. Van Essen (2013): "Hold-up: With a Vengeance," *Economic Inquiry*, 51, 896–908.

[30] Elster, J. (1998): "Emotions and Economic Theory, " *Journal of Economic Literature*, 36, 47-74.

[31] Falk, A. and U. Fischbacher (2006): “A Theory of Reciprocity,” *Games and Economic Behavior*, 54, 293–315.

[32] Falk, A., E. Fehr, and U. Fischbacher (2003): “On the Nature of Fair Behavior,” *Economic Inquiry,* 41, 20-26.

[33] Falk, A., E. Fehr, and U. Fischbacher (2008): “Testing Theories of Fairness—Intentions Matter,” *Games and Economic Behavior,* 62(1), 287-303.

[34] Fehr, E. and S. Gächter (2002): “Altruistic Punishment in Humans,” *Nature*, 415, 137–140.

[35] Frank, R. H. (1988): *Passions Within Reason: The Strategic Role of the Emotions.* Chicago: W.W. Norton & Co.

[36] Frijda, N. H. (1986): *The Emotions.* Cambridge: Cambridge University Press.

[37] Frijda, N. H. (1993): “The Place of Appraisal in Emotion,” *Cognition and Emotion*, 7, 357–387.

[38] Geanakoplos, J., D. Pearce, and E. Stacchetti (1989): “Psychological Games and Sequential Rationality,” *Games and Economic Behavior*, 1, 60–79.

[39] Gneezy, U. and A. Imas (2014): “Materazzi Effect and the Strategic Use of Anger in Competitive Interactions,” *Proceedings of the National Academy of Sciences,* 111, 1334–1337.

[40] Grimm, V. and F. Mengel (2011): “Let Me Sleep on It: Delay Reduces Rejection Rates in Ultimatum Games,” *Economics Letters*, 111, 113–115.

[41] Güth, W. and H. Kliemt (1998): “The Indirect Evolutionary Approach: Bridging between Rationality and Adaptation,” *Rationality and Society*, 10, 377–399.

[42] Gurdal, M., J. Miller, and A. Rustichini (2014): “Why Blame?” *Journal of Political Economy,* 121, 1205–1246.

[43] Harmon-Jones, E. and J. Sigelman (2001): “State Anger and Prefrontal Brain Activity: Evidence that Insult-Related Relative Left-Prefrontal Activation is Associated with Experienced Anger and Aggression,” *Journal of Personality and Social Psychology*, 80, 797–803.

[44] Hirshleifer, J. (1987): "On the Emotions as Guarantors of Threats and Promises," John Dupre (Ed.), *The Latest on the Best: Essays on Evolution and Optimality*, Cambridge: MIT Press, Chapter 14, 307–26.

[45] Kőszegi, B. and M. Rabin (2006): "A Model of Reference-Dependent Preferences," *Quarterly Journal of Economics,* 121, 1133–1166.

[46] Kőszegi, B. and M. Rabin (2007): "Reference-Dependent Risk Attitudes," *American Economic Review,* 97, 1047–1073.

[47] Kreps, D. and R. Wilson (1982): "Sequential Equilibria," *Econometrica*, 50, 863–894.

[48] Loomes, G. and R. Sugden (1986): "Disappointment and Dynamic Consistency in Choice under Uncertainty," *Review of Economic Studies*, 53, 271–282.

[49] Munyo, I., and M.A. Rossi (2013): "Frustration, Euphoria, and Violent Crime," *Journal of Economic Behavior & Organization*, 89, 136-142.

[50] Passarelli, F. and G. Tabellini (2016): "Emotions and Political Unrest," *Journal of Political Economy*, forthcoming.

[51] Pillutla, M. and K. Murningham (1996): "Unfairness, Anger, and Spite: Emotional Rejections of Ultimatum Offers," *Organizational Behavior and Human Decision Processes*, 68, 208–224.

[52] Potegal, M., C. Spielberger, and G. Stemmler (eds.) (2010): *International Handbook of Anger: Constituent and Concomitant Biological, Psychological, and Social Processes*. New York: Springer-Verlag.

[53] Rabin, M. (1993): "Incorporating Fairness into Game Theory and Economics," *American Economic Review*, 83, 1281–1302.

[54] Rauh, M. and G. Seccia (2006): "Anxiety and Performance: A Learning-by-Doing Model," *International Economic Review*, 47, 583–609.

[55] Reuben, E. and F. van Winden (2008): "Social Ties and Coordination on Negative Reciprocity: The Role of Affect," *Journal of Public Economics,* 92, 34–53.

[56] Rotemberg, J. (2005) "Customer Anger at Price Increases, Changes in the Frequency of Price Adjustment and Monetary Policy," *Journal of Monetary Economics,* 52, 829–852.

[57] Rotemberg, J. (2008): "Minimally Acceptable Altruism and the Ultimatum Game," *Journal of Economic Behavior and Organization,* 66, 457–476.

[58] Rotemberg, J. (2011): "Fair Pricing," *Journal of the European Economic Association,* 9, 952–981.

[59] Sanfey, A. (2009): "Expectations and Social Decision-Making: Biasing Effects of Prior Knowledge on Ultimatum Responses," *Mind and Society* 8, 93–107.

[60] Scherer, K. R. (1999): "Appraisal Theory," *Handbook of Cognition and Emotion*, 637–663.

[61] Schotter, A. and B. Sopher (2007): "Advice and Behavior in Intergenerational Ultimatum Games: An Experimental Approach," *Games and Economic Behavior*, 58, 365–393.

[62] Sebald, A. (2010): "Procedural Concerns and Reciprocity," *Games and Economic Behavior*, 68, 339–352.

[63] Selten, R. (1975): "Re-examination of the Perfectness Concept for Equilibrium Points in Extensive Games," *International Journal of Game Theory*, 4, 25–55.

[64] Silfver, M. (2007): "Coping with Guilt and Shame: A Narrative Approach," *Journal of Moral Education,* 36, 169–183.

[65] Smith, A. (2009): "Belief-Dependent Anger in Games," typescript, University of Arizona.

[66] Winter, E. (2014): *Feeling Smart: Why Our Emotions Are More Rational Than We Think.* New York: PublicAffairs.

[67] Winter, E., I. Garcia-Jurado, and L. Mendez Naya (2016): "Mental Equilibrium and Rational Emotions, "*Management Science*, forthcoming.

[68] Xiang, T., T. Lohrenz, and R. Montague (2013): "Computational Substrates of Norms and Their Violations during Social Exchange," *Journal of Neuroscience*, 33, 1099–1108.