PATTERN-BASED EXPECTATIONS: INTERNATIONAL EXPERIMENTAL EVIDENCE AND APPLICATIONS IN FINANCIAL ECONOMICS

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Abstract—We study how subjects extrapolate simple patterns in financial time series in order to develop a descriptive model of actual agent behavior. The laboratory experiment for this analysis was conducted in Germany and Japan. Statistical analyses indicate considerable similarity in expectations formation across cultures and document that agents' expectations are at variance with the notion of standard trend extrapolation. The paper then proposes a method for computing expectations for any economic time series based on the experimental data. Such pattern-based expectations are shown to explain stock prices and the dynamics of the forward discount on the foreign exchange market.

I. Introduction

THE concept of rational expectations has been the most important contribution to the modeling of expectations in the past fifty years. The idea that economic agents use the same analysis and data as econometricians do for their forecasts has to a large extent replaced the various notions of extrapolative expectations that had been popular before Muth's (1961) contribution. However, much empirical research investigating expectations elicited through survey techniques or through experiments casts doubt on the general empirical validity of the rational expectations hypothesis (Pesaran, 1987; Manski, 2004; & Rötheli, 2007). This paper proposes a model of extrapolative expectations based on the notion that agents—for example, investors—form expectations based on the visual pattern shown by the time series to be projected into the future.[1] *Pattern* in this context means a specific sequence of changes over the recent past of a time series. Psychological research documents the importance of relying on patterns (shapes, forms) and shows how networks of neurons can learn and generate fast responses to a vast number of patterns (Puccetti, 1974; Rumelhart, McClellan, and the PDP Research Group, 1986; Posner 1989; Lund, 2001). From the perspective of evolution, the ability of organisms to detect patterns (that is, recognize similarities in situations) and draw quick inferences has a high survival value (see Edelman & Reeke, 1990).

Several studies document that subjects rely on simple visual patterns when forming expectations. In particular, runs and zigzag movements in time series stand out as patterns on which subjects rely when forming one-step-ahead expectations (Feldman, 1963; Jones, 1971, Eggleton, 1982; Rötheli, 1998). These contributions study behavior when subjects face a binary series. Several researchers have already experimentally studied intuitive time series forecasting using financial data. While De Bondt (1993) shows subjects historical data, Bloomfield and Hales (2002) work with stylized financial data. We follow the latter approach but study subjects' responses over a broad spectrum of possible circumstances (that is, not just for a few interesting sequences in a time series). The ultimate goal of eliciting such an array of responses is the use of these data in a new method for calculating expected values for any economic time series. We would like to make it clear that it is a positive (that is, behavioral) model of expectations we propose, not a normative one. In an important theoretical contribution, Barberis, Shleifer, and Vishny (1998; BSV) propose a behavioral model of investors' expectations, or "investor sentiment," that is able to account for asset price regularities that are at variance with rational expectations. Their contribution is related to the approach presented here, and we will compare its implications with our experimental findings. In order to check the generality of pattern-based expectations, we compare expectations data collected in Japan and Germany.[2]

The paper is structured as follows. Section II describes the experiment. Section III presents the statistical analysis of the experimental data and shows striking similarities in

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[1] Pattern-based expectations of ordinary people should not be confused with the various methods applied in so-called technical analysis as used by some professional financial forecasters (see Brock, Lakonishok, & LeBaron, 1992).

[2] Such an East-West comparison is also interesting given the substantial literature addressing cultural differences in economic decision making (Hofstede, 1997; Usunier, 1998; Gannon and Newman, 2002; Mattock and Bannon, 2003). With respect to experimental investigations in the field of economics, authors like Cason, Saijo, and Yamato (2002) and Brandts, Saijo, and Schram (2004) have presented relevant international comparative studies. Brandts et al. (2004) conclude from their experiment on voluntary contribution mechanisms that there are only minor behavioral differences across Japan, the Netherlands, Spain, and the United States. In contrast, Cason et al. (2002) report significant differences in the behavior of Japanese subjects as compared to U.S. subjects. The main behavioral trait investigated in that study is spiteful behavior. Concerning cultural effects that influence the formation of expectations, there is, to my knowledge, no relevant evidence. With respect to forward-looking behavior Aggarwal and Mohanty (2000) compare Japanese and U.S. behavior. Their analysis of survey data of macroeconomic variables does not indicate important systematic East-West differences. As to financial behavior, at least one study indicates that cultural factors make a difference: Brown, Chua, and Mitchell (2002) document effects of Chinese culture (and superstition) on asset prices. A different (noncultural) national variation in asset pricing is documented in Garrett, Kamstra, and Kramer (2005): in a multicountry comparison, these authors document that variations in the length of the day across countries can explain some cross-national differences in the data.

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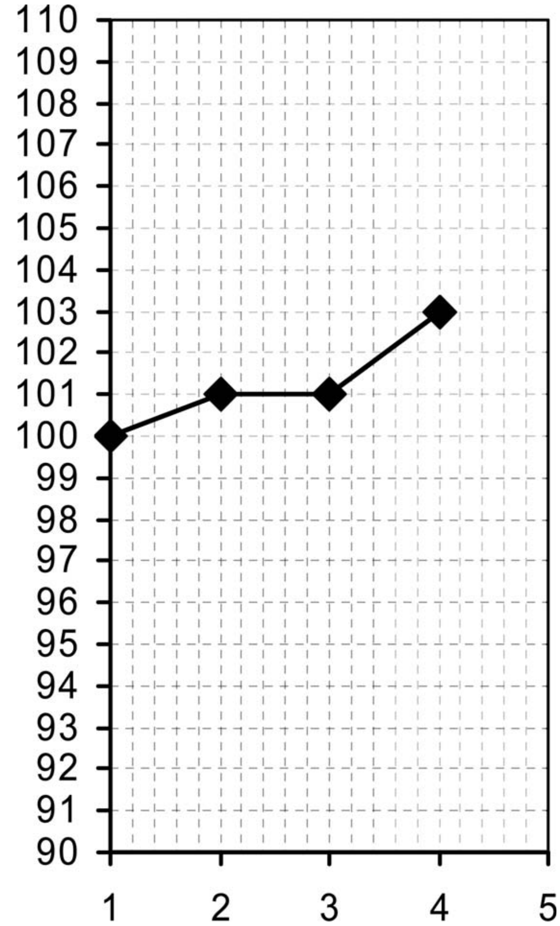
pattern-based extrapolation across German and Japanese subjects. Section IV documents that neither the linear model of trend extrapolation nor the BSV model of expectations adequately describes the expectations elicited experimentally. Section V shows how the experimental data can be used to compute historical expectations data for any time series. Section VI applies this approach to the econometric study of historical U.S. stock price data. Section VII documents that the pattern-based expectations help to explain the forward discount on the foreign exchange market. Section VIII concludes the article.

II. The Experimental Design

Our data are elicited using an experimental design introduced by Rötheli (2007).[3] Appendix A provides the instructions for the experiment. Subjects are shown short sequences in a time series they are told to consider to be a financial time series, such as a stock price or an exchange rate. They are informed that they are going to see different possible cases of how this financial series can evolve over the course of four periods and their task is to assess the likely continuation of this series. The experiment simplifies the possible course of the time series: the series can proceed only in steps of +2, +1, 0, −1, −2. With this restriction, there exist 125 cases of how a series can evolve over four periods. We limit the number of cases to 63 on the basis of tests presented in Rötheli (1998), indicating that the hypothesis of symmetry is not rejected for a majority of subjects—that is, agents' forecasts based on a sequence of changes for example, −1, +1 is typically the same as that based on the sequence +1, −1 multiplied by −1. In principle, subjects could be asked for responses to a wider array of possible patterns, including changes of steps of size three or larger. However, this widening of the set of observations would imply a vastly larger set of patterns and a longer experiment. The number of tasks (in our experiment, a total of 3 $\times$ 63 = 189) also depends on the length of the data window shown in each case. The length of our windows is chosen based on earlier research (Rötheli, 1998) indicating that in a similar task, few subjects rely systematically on information that reaches further back than the last three steps of a time series. Moreover, Carlson and Shu (2007), based on a variety of data, document that it typically takes people three observations to conclude that a series of outcomes forms a streak.

To be clear, subjects see 63 different four-period sequences and give their individual projection without receiving feedback on how the series continues into the fifth period. The instructions strictly use the term *case* and avoid the term *pattern*. A further point concerns the display of the time series. Given the evidence on the importance of visualization of information (see, Chaomei & Czerwinski, 2000),

Figure 1.—Visual Display of Case 16



we present subjects with graphs instead of numbers, and in accordance with earlier similar studies (like Bloomfield & Hales, 2002), we present the graphs in level form and not in the form of changes. Figure 1 presents one of the cases shown to subjects (case 16). In our terminology, this is pattern 16. We chose this particular pattern to illustrate that we elicit expectations for many types of sequences and not just for a few prominent patterns like straight trends (patterns 20 and 51) or zigzags (patterns 30 and 61). Each case is presented separately along with the tasks described below. Table 1 documents the full list of the 63 patterns shown in the experiment. The experimental data in the column with the heading Expected Change to Period 5 in this table will be explained in section IV.

Subjects in the experiment were given three tasks. Task a asks for an estimate of the likely change in the series from period 4 to period 5 expressed in probability values (in steps of 0.1) for the different possible steps (+2, +1, 0, −1, −2).[4] Task b asks for an estimate of the population mean (the average response over all subjects) of the expected

[3] This reference offers a detailed descriptive analysis of the German data in chapter 9 of Rötheli (2007).

[4] Manski (2004) has argued strongly for eliciting expectations in the form of probabilities.

TABLE 1.—AVERAGE EXPECTED CHANGES

Pattern Number	Periods 1	2	3	4	Expected Change to Period 5
1	100	100	100	100	0.005544
2	100	100	100	101	0.538867
3	100	100	100	102	0.686689
4	100	100	101	101	0.212963
5	100	100	101	102	0.727654
6	100	100	101	103	1.054439
7	100	100	101	100	−0.207800
8	100	100	101	99	−0.624428
9	100	100	102	102	0.456296
10	100	100	102	103	0.608878
11	100	100	102	104	1.149170
12	100	100	102	101	−0.138911
13	100	100	102	100	−0.347082
14	100	101	101	101	0.167783
15	100	101	101	102	0.525808
16	100	101	101	103	0.768911
17	100	101	101	100	−0.451128
18	100	101	101	99	−0.741481
19	100	101	102	102	0.318777
20	100	101	102	103	0.857794
21	100	101	102	104	1.132244
22	100	101	102	101	−0.251128
23	100	101	102	100	−0.601998
24	100	101	103	103	0.385578
25	100	101	103	104	0.980017
26	100	101	103	105	1.212244
27	100	101	103	102	−0.229983
28	100	101	103	101	−0.429888
29	100	101	100	100	0.090258
30	100	101	100	101	−0.030011
31	100	101	100	102	0.339077
32	100	101	100	99	−0.540022
33	100	101	100	98	−0.913339
34	100	101	99	99	0.070443
35	100	101	99	100	−0.161302
36	100	101	99	101	−0.002233
37	100	101	99	98	−0.415561
38	100	101	99	97	−0.976667
39	100	102	102	102	0.294141
40	100	102	102	103	0.531133
41	100	102	102	104	0.723344
42	100	102	102	101	−0.392598
43	100	102	102	100	−0.473934
44	100	102	103	103	0.191089
45	100	102	103	104	0.769871
46	100	102	103	105	1.015443
47	100	102	103	102	−0.539456
48	100	102	103	101	−0.585449
49	100	102	104	104	0.387323
50	100	102	104	105	0.692570
51	100	102	104	106	1.401223
52	100	102	104	103	−0.185561
53	100	102	104	102	−0.576436
54	100	102	101	101	0.132206
55	100	102	101	102	0.121106
56	100	102	101	103	0.238883
57	100	102	101	100	−0.485556
58	100	102	101	99	−0.763148
59	100	102	100	100	0.081111
60	100	102	100	101	0.184428
61	100	102	100	102	−0.551807
62	100	102	100	99	−0.395309
63	100	102	100	98	−0.835533

values given under task a. “All subjects” here means the 45 subjects present in one (national) location on the day of the experiment. Hence, this task asks for an estimate of the answers of the other subjects. Here, subjects were asked to provide a single value between +2 and −2 down to one decimal point. Finally, task c calls on subjects to express their confidence in their response to task b. Here, subjects bet between 0 and 10 euro cents (in Japan between 0 and 15 yen) on the proposition that their answer in task b differs by no more than 0.5 in absolute value from the actual mean of the expected values computed over all (national) subjects. The financial rewards in the two countries were adjusted to local hourly wages for student aides and for the exchange rate so as to make financial incentives as similar as possible across countries. In concrete terms, the show-up fee in Germany was 4 euros (in Japan, 650 yen). Moreover, subjects received 10 euro cents in Germany (15 yen in Japan) per case for completing tasks a and b. This (respective) per case amount is the maximum that can be entered as an answer in task c. The main purpose of task c is to offer a financial incentive for diligent processing of the tasks at hand given that there is no measure of forecast accuracy in task a.

An analysis of the individual responses (means and standard deviations) for each of the 90 subjects shows that the deviations of the individual responses regarding their own expectation (task a) and their assessment of the collective expectation (task b) differ frequently and substantially. Hence, nothing in the data indicates that subjects would not truthfully report their expectations. The 45 subjects participating in Germany were undergraduate students from the University of Erfurt who had completed at least one principal course of economics prior to the experiment. The 45 subjects participating in Japan were undergraduate students from the University of Osaka from the fields of economics and social science. Subjects (after 15 minutes of instructions) had to allocate a minimum of 60 minutes for completing the task. The maximum time allowed was 90 minutes. During this time, participants were allowed to make changes to any of their answers. The average German (Japanese) subject earned 11. 57 euros (1,756 yen).

III. Experimental Findings

Here we document in what respect pattern-based expectations of German and Japanese subjects are similar and how they differ. We start with the expected change as calculated from the probabilities given for task a and compare the means over the respective (national) population of subjects for each of the 63 patterns. Figure 2 shows the expected changes in the experimental series elicited from the Japanese subjects plotted against the mean of the German subjects. Specifically, a single point in the scatter plot shows the average of the Japanese expected values associated with one pattern against the average of the German expected values for the same pattern. The display indicates a strong similarity in answers across the two national pools of subjects (with a coefficient of correlation of 0.96). The statistical analysis documents that the slope of the regression line is not signifi-

FIGURE 2.—EXPECTED CHANGES IN JAPAN AGAINST EXPECTED CHANGES IN GERMANY

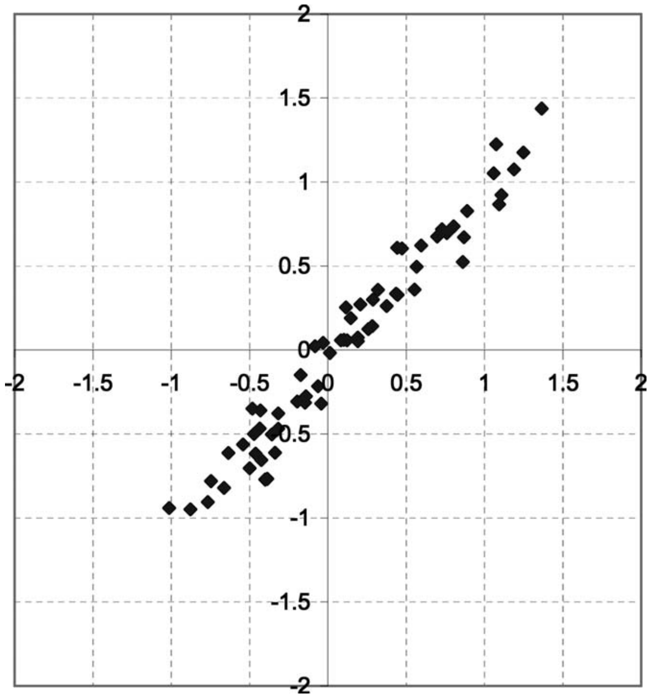
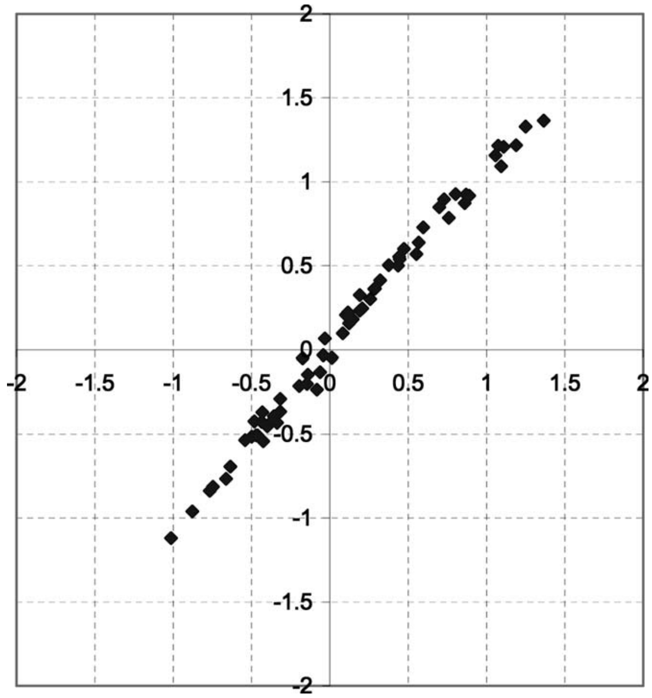


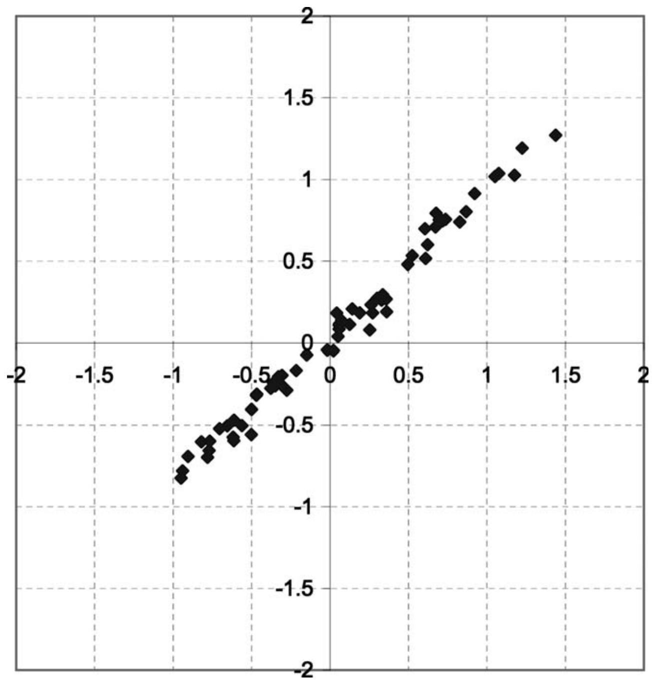
FIGURE 3.—ASSESSMENT OF COLLECTIVE EXPECTATIONS AGAINST ACTUAL COLLECTIVE EXPECTATION: THE GERMAN DATA



cantly different from 1 at the 1% level of significance. However, the intercept term of −0.075 (statistically different from 0 at the 1 % level) indicates a small difference in the expectations in the two subject pools. Hence, the predictive behavior in the two subjects' groups is very similar, although on average, Japanese subjects appear to predict the next value of the series to lie slightly lower than the value predicted by the German subjects.

Next, we turn to task b, which addresses the individual's estimate of collective assessments. Here, we observe interesting differences between the German and the Japanese subjects. For the German subjects, figure 3 shows the scatter plot of the assessment of the expected change as attributed to the collective—the average of the answers under task b for any of the 63 patterns—plotted against the actual expected change over all German subjects (the average of the mean under task a). Figure 4 shows the same display for Japan. In the German case, the statistical analysis indicates that the regression line goes through the origin (the intercept is not significantly different from 0) and has a slope of 1.086, which is different from 1 at the 1% level of significance. Hence, the German subjects assess themselves as being more extreme (in the sense of expecting a larger absolute change in the variable) than they actually are. Compare this to the Japanese subjects: the slope coefficient of the regression line (of 0.892) is significantly different from 1 and the positive intercept (of 0.038) is significantly different from 0 (both assessed at the 1% level). Hence, in the case of Japanese subjects, the collective self-assessment is (for most patterns) overly reserved.

FIGURE 4.—ASSESSMENT OF COLLECTIVE EXPECTATIONS AGAINST ACTUAL COLLECTIVE EXPECTATION: THE JAPANESE DATA



Summing up, we find that in terms of the personal assessments of the future course of a financial series, Japanese and German subjects make very similar predictions. Only when it comes to pondering collective behavior do we find interesting differences between the two national samples.

IV. Pattern-Based Expectations and Alternative Models

In this section we document that neither the model of linear trend extrapolation nor the BSV model adequately captures the subjects' time series extrapolations. Wald tests (statistics reported below) indicate that in the estimates shown below, there is no significant difference between the responses of German and Japanese subjects. Hence, we pool the data of all subjects. The endogenous variable in our analysis (denoted by ${X_{j,5}}^e - X_{j,4}$) is the expected change (that is, the probability weighted sum of possible changes) of the series from period 4 to period 5 as elicited in task a averaged over all 90 subjects. This variable is indexed by j, with j running from 1 to 63. Hence, we have 63 observations. Table 1 lists these expected changes for all 63 different patterns. The notion of linear trend extrapolation means that these elicited expectations should be explainable by a weighted average of the observed lagged changes in the experimental series where the weights of the lagged changes should be constant over all possible histories of a series. The symbols $X_{j,4}$, $X_{j,3}$, $X_{j,2}$, $X_{j,1}$ stand for the values of the series shown to the subjects in periods 4, 3, 2, and 1. These values are also indexed because each of the j patterns has a different history of Xs. In order to assess whether the experimentally elicited expectations can be captured by linear trend extrapolation, we estimate the following regression equation:

$$
\begin{aligned}
X^e_{j,5} - X_{j,4} = {} & \beta_0 + \beta_1\left(X_{j,4} - X_{j,3}\right) \\
& + \beta_2\left(X_{j,3} - X_{j,2}\right) \\
& + \beta_3\left(X_{j,2} - X_{j,1}\right) + \varepsilon_j.
\end{aligned} \tag{1}
$$

This is what linear trend extrapolation means: the weight attached to lagged changes in X varies only with the lag of the change (that is, β_1, β_2, β_3 can differ). In this regression, the first four terms make up the part of the experimental data that can be captured by the model of trend extrapolation, and ε_j denotes the part of the elicited expectation that cannot be explained by the linear extrapolation scheme. As hinted at earlier a Wald test indicates (with a p-value of 0.116 based on the chi-square statistic) that the β-coefficients for the German data do not statistically differ from the β-coefficients of the Japanese data. Hence, we report the estimates from the pooled data set. Similarly, a Wald test (with a p–value of 0.514) indicates that β_3 is not statistically different from 0. The regression result incorporating this parameter restriction is

$$
\begin{aligned}
X^e_{j,5} - X_{j,4} = {} & \underset{(0.028)}{0.041} + \underset{(0.021)}{0.356}\left(X_{j,4} - X_{j,3}\right) \\
& + \underset{(0.024),}{0.171}\left(X_{j,3} - X_{j,2}\right)
\end{aligned} \tag{2}
$$

$R^2 = 0.864$, $SEE = 0.225$.

As a first impression, trend extrapolation appears to be a reasonable explanation of the experimental data. However, we want to find out whether prominent types of patterns (that is, trends and zigzags) lead to expectations formation at odds with the generalization proposed in equations (1) and (2). For this purpose, we define two dummy variables: (a) D^{Trend} is 1 whenever X has changed in the same direction over the previous four periods, and 0 otherwise, and (b) D^{Zigzag} is 1 whenever the changes in X have a reversed sign at every step over the previous four periods, and 0 otherwise.[5] Equation (3) alters equation (1) by allowing the β-coefficients to differ across classes of patterns. Specifically, equation (3) shows the results when zigzag patterns and trend patterns and all other patterns are considered as three classes of patterns:

$$
\begin{aligned}
X^e_{j,5} - X_{j,4} = {} & \Big[\underset{(0.220)}{0.501} - \underset{(0.101)}{0.022}\left(X_{j,4} - X_{j,3}\right) \\
& + \underset{(0.101)}{0.300}\left(X_{j,3} - X_{j,2}\right)\Big] \times D^{Zigzag} \\
& + \Big[\underset{(0.220)}{0.268} + \underset{(0.101)}{0.365}\left(X_{j,4} - X_{j,3}\right) \\
& + \underset{(0.101)}{0.128}\left(X_{j,3} - X_{j,2}\right)\Big] \times D^{Trend} \\
& + \Big[\underset{(0.022)}{0.107} + \underset{(0.016)}{0.399}\left(X_{j,4} - X_{j,3}\right) \\
& + \underset{(0.016)}{0.090}\left(X_{j,3} - X_{j,2}\right)\Big] \\
& \times \left(1 - D^{Zigzag}\right) \times \left(1 - D^{Trend}\right)
\end{aligned} \tag{3}
$$

$R^2 = 0.950, SEE = 0.143.$

Here we have three different linear models—one for each class of patterns. Wald tests again indicate that the coefficient estimates for Germans and Japanese subjects do not differ significantly (p–value of 0.118) and an inclusion of $X_2 - X_1$ is not justified (p–value of 0.874). Furthermore, the hypothesis that the parameters of the three linear models are the same across the three different classes of patterns is rejected at the 1% level of significance. Moreover, when considering R^2 values computed from the three subregressions of equation (3), the model of linear trend extrapolation captures least well the subjects' responses after zigzag patterns ($R^2 = 0.331$). The linear model describes the expectations better for the case of trends ($R^2 = 0.781$) and the remaining circumstances ($R^2 = 0.953$). In summary, the estimates indicate that the standard model of

[5] Given that we show only 63 patterns (no patterns starting with a negative change), the condition for the trend dummy D^{Trend} to be 1 is $(X_4 - X_3) > 0$, $(X_3 - X_2) > 0$, $(X_2 - X_1) > 0$ and, likewise, the condition for the zigzag dummy D^{Zigzag} to be 1 is $(X_4 - X_3) > 0$, $(X_3 - X_2) < 0$, $(X_2 - X_1) > 0$.

linear extrapolation is not a satisfactory representation of agents' time series forecasts.

Next, we want to assess whether the expectations elicited by our experiments can be explained by the "investor sentiment" model of Barberis et al. (1998). The BSV model proposes that agents base their expectations on the notion that the series to be projected into the future (earnings in their analysis) is generated either by a regime (model 1) where changes are mean reverting or a regime (model 2) where they are trending, when in fact the series follows a random walk. The probabilities capturing the mean-reverting or the trending behavior are assumed by agents and interact with a further process where a Markov chain determines which of the two regimes rules. In this approach, agents update the probabilities pertaining to either of the two models in a Bayesian rational way. This idea of the sophisticated updating of probabilities sets the BSV model clearly apart from the notion of pattern-based extrapolation proposed here. Yet it is interesting to investigate whether the expectations elicited by our experiments are in accordance with the BSV model. For this purpose, a subset of patterns shown to subjects is of relevance. Consider pattern 2: 100, 100, 100, 101. The subjects' response to this pattern is an expected change of 0.538, that is, a rather strong increase. The BSV model in this case would not yet see a rationale for changing expectations compared to pattern 1 (100, 100, 100, 100): since changes (positive or negative) are announced as possible, a one-time change does not induce Bayesian learners to alter their probabilities whether mean reversion or trending is more likely to generate the next observation. The same argument holds for patterns 4 and 14. A revision of probabilities should occur only after a further change, that is, after pattern 5 (100, 100, 101, 102) or pattern 7 (100, 100, 101, 100). Further, consider our subjects' responses to one of these patterns (pattern 7). Here, the "investor sentiment" type of agent would calculate a probability of model 1 of above 0.5 and hence should project a likely increase (that is, a reversion of the latest decline). In contrast, the subjects' expectations in this case clearly point toward a decline. Finally, consider pattern 30: 100, 101, 100, 101. This sequence of ups and downs should lead to an assessment of increased probability of change reversion and thus an expected decrease for the immediate future. The experimental data instead show a decline very close to 0 (-0.030). Altogether these findings indicate that the expectations elicited by our experiments cannot be explained by the probability updating idea of the BSV model.[6]

The findings in this section document that the pattern-based expectations measured here are distinct from other notions of expectations formation. Hence, we proceed to show how these experimental expectations data can be transformed to a model of agents' time series expectations, which can then be used in the econometric analysis of financial data.

V. Applying the Experimental Data to Empirical Analysis

This section describes how the information in the experimental data can be used to compute a time series of expected values for any particular economic variable. More concretely, we use the expected changes (averaged over all subjects in our study) of the series as elicited in task a after the 63 different patterns reported in table 1 to compute a historical series reflecting how agents forecast under the assumption that, on average, agents function as our subjects do. The key problem to be solved here is to scale the stylized experimental patterns to actual economic time series. The process of scaling the experimental data to historical data consists of several steps. First, we split the historical time series into rolling data windows of four data points at a time. Second, focusing on one data window at a time, correlation analysis (to be explained in more detail) is used to determine the stylized pattern most similar to the given four data points in the historical series. For this step, the list of 63 patterns first has to be shortened to the 50 patterns that are (in terms of the changes) linearly independent. Consider table 1, and note that the changes over time in the thirteen pairs of patterns numbered 2-3, 4-9, 5-11, 7-13, 14-39, 15-41, 17-43, 19-49, 20-51, 22-53, 29-59, 30-61, and 32-63 are related to each other by a factor (denoted β) of two. That is, in terms of the changes of the experimental series, thirteen of our patterns show changes twice the size of the changes in exactly one other pattern. So, for example, in first differences, pattern 3 is twice pattern 2, and pattern 9 is twice pattern 4.

From the subjects' responses to the thirteen related pairs, we can derive an estimate of how subjects react when a pattern is altered in terms of the size of the steps (that is, to a scaling). In order to quantify this scaling relation, we regress the reported expected changes (as shown in table 1) after the thirteen patterns 3, 9, 11, 13, 39, 41, 43, 49, 51, 53, 59, 61, 63 to the expected changes after patterns 2, 4, 5, 7, 14, 15, 17, 19, 20, 22, 29, 30, 32. This yields a regression coefficient of 1.51 (with a standard error of 0.11). From this estimate (and rejecting at the 1% level the hypothesis that the regression coefficient is 2), it is clear that agents' responses do not simply vary in proportion to changes. Instead, based on the estimate, we use the scale transformation $\beta^{0.6}$, meaning that for values of β of 2, 3, 4 . . . , we will use factors of proportion 1.51, 1.93, 2.29, . . . Figure 5 shows this scaling relation. Next, we match the historical series (that is, four consecutive observations at a time) to the stylized pattern most similar to it. Here, we take the absolute level of the correlation coefficient between the logarithmical actual data and each of the fifty linearly independent patterns as the criterion. Finally, the thus selected

[6] Were it not for these contradictions, the experimental data would lend themselves to an empirical determination of the parameters of the BSV model. Responses to patterns 5, 7, 20, 22, 30, and 32 would suffice to calculate values for all fixed probability values of the BSV model.

FIGURE 5.—SCALING THE ELICITED EXPECTATIONS: EXPECTED CHANGE AS A FUNCTION OF β

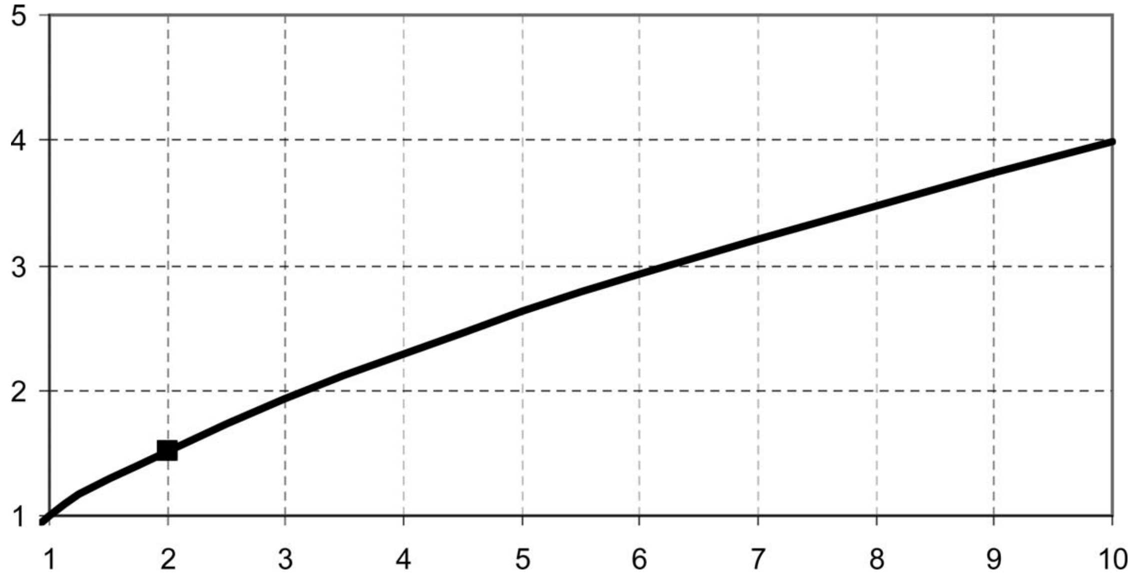
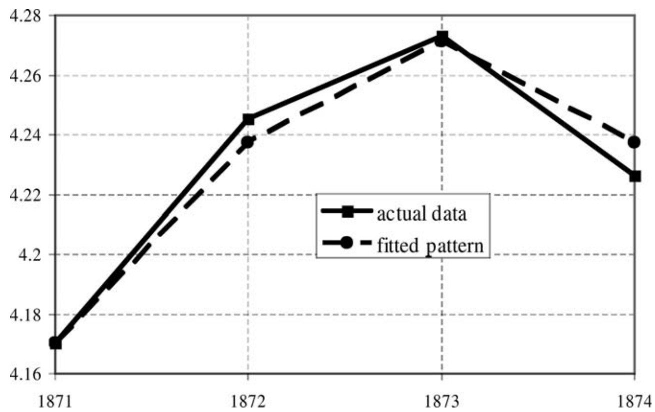


FIGURE 6.—ACTUAL ANNUAL DATA (NATURAL LOGS) AND FITTED PATTERN NUMBER 47 FOR STOCK PRICE



most similar stylized pattern is scaled to the historical data by linear regression.

To illustrate this procedure, we take the first four data points of the log of the deflated stock price series that Shiller (1981) used with the data points 4.170 in 1871, 4.245 in 1872, 4.273 in 1873, and 4.226 in 1874. The pattern with the highest (absolute) correlation with these historical data is pattern 47, that is, the sequence 1.00, 1.02, 1.03, 1.02. The average subject participating in our study in this case expects the experimental series to change by −0.539%. Minimizing the sum $(4.170 - \alpha - 1.00\beta)^2 + (4.245 - \alpha - 1.02\beta)^2 + (4.273 - \alpha - 1.03\beta)^2 + (4.226 - \alpha - 1.02\beta)^2$ while enforcing $4.170 = \alpha - 1.00\beta$ (making the actual and fitted pattern start from the same point) yields the coefficient estimates $\alpha = 0.806$ and $\beta = 3.363$. Figure 6 shows the fit of the appropriately scaled pattern 47 ($0.806 + 1.00 \times 3.363, 0.806 + 1.02 \times 3.363, 0.806 + 1.03 \times 3.363, 0.806 + 1.02 \times 3.363$) to the first four data points of the Shiller stock market data set. Based on this procedure, the computed expected stock price for the year 1875 is $\exp(4.226 - 0.00539 \times 3.363^{0.6}) = 67.683$. By a step-wise application of the described procedure, we reach the expected value one step ahead for the whole historical time series.

VI. Stock Price Expectations

In this section, we pursue the issue of stock price expectations and switch to the use of monthly data compiled and updated by Robert Shiller (2000). We first compute the series of the expected stock price starting in May 1871 up to December 2006. This series of (pattern-based) expected stock price values can now be used for further econometric analysis. Let us first subject the expectations data to a standard test of rationality. equation (4) shows the result when we regress the stock price on the pattern-based expected stock price (both in natural logs) and a constant:[7]

$$\begin{aligned} q_t &= 0.009 + 0.998 q_t^e, \\ &\quad (0.007)(0.001) \\ R^2 &= 0.997, SEE = 0.0399, DW = 1.871. \end{aligned} \tag{4}$$

[7] The standard errors reported are White heteroskedasticity-consistent estimates. They are not used in any of the tests reported.

A Wald test (with a p-value of 0.393) does not reject the hypothesis that the constant is 0 and the coefficient of q_t^e is 1. Hence, pattern-based expectations here do not violate a basic requirement of rationality. Appendix B documents various further efficiency tests applied to the pattern-based stock price expectations and documents that no clear rejection of efficiency emerges.

Next, consider an empirical version of the BSV model of investor sentiment as an alternative behavioral model of expectations formation. One way to compare that model with the pattern-based expectations model is to estimate its parameters using Shiller's monthly stock price data. The BSV model is based on four probability parameters and a parameter capturing the size of a typical shock to the series. Parameter π_L denotes the probability that the series trends in the mean-reversion regime (model 1), π_H denotes the probability that the series trends in the trending regime (model 2), λ is the probability that a switch occurs from model 1 to model 2 and vice versa.[8] The last parameter to be determined is y, which describes the (absolute size) of the shock to the series. Barberis et al. (1998) document formally how the expected change of the series to be projected depends on these parameters, and we estimate the relevant parameter values by grid-searching for the minimum of the sum of squared forecast errors. The parameters thus determined are $\lambda = 0.49$, $\pi_L = 0.50$, $\pi_H = 0.58$ and $y = 0.09$. The BSV model gives a sum of squared errors over the period from 1871 to 2006 of 2.644. In comparison, the pattern model (assessing $q_t - q_t^e$) gives a lower sum of squared forecast errors of 2.604. This is notable considering that for the former model, we vary four parameters to find the version that forecasts best (ex ante), and for the latter, we do not vary any parameter. The decisive point, however, is that the parameters of the BSV model estimated from actual stock prices are not in accordance with the story of Barberis et al. (1998). In order for their behavioral model to capture interesting asset price phenomena, investors should base their expectations, in particular, on lower values of λ and π_L. Hence, it becomes apparent that minimizing the sum of squared errors with historical realizations of stock prices does not yield parameter values that could be considered behaviorally sensible.

In a further step, we investigate whether pattern-based stock price expectations can shed light on issues of stock price determination. Specifically, we ask whether evidence for competing models of expectations can be found in present value models of stock prices (see Chow, 1989). The analysis starts with the equilibrium condition

$$Q_t = \delta\left(Q_{t+1}^e + D_t\right), \tag{5}$$

where Q_t denotes the price of a stock (or index) and D_t denotes the dividend paid to the owner of the stock. We start with the hypothesis of rational expectations. In order to derive a testable hypothesis in logs of observable variables, we formulate rational expectations as $Q_{t+1}^e = \eta_{t+1}Q_{t+1}$ with $\ln \eta_{t+1} = c + u_{t+1}$ where c is a constant and $E_t u_{t+1} = 0$. Using Campbell and Shiller's (1987) approximation $\rho q_{t+1} + (1-\rho)d_t + k$ for $\ln(Q_{t+1} + D_t)$, where $\rho = 1/[1 + \exp(\delta)]$, δ is the mean of $d_t - q_{t+1}$ and $k = \ln[1 + \exp(\delta)] - \delta \exp(\delta)/[1 + \exp(\delta)]$, the following relationship is implied:

$$\begin{aligned} q_t = {} & \rho^{-1}\left(\ln\delta^{-1} + c - k\right) + \rho^{-1}q_{t-1} \\ & - \rho^{-1}(1-\rho)d_{t-1} + \rho^{-1}u_t \end{aligned} \tag{6}$$

Estimating this linear relationship between q_t, on the one hand, and q_{t-1} and d_{t-1}, on the other hand, by ordinary least squares, we find

$$\begin{aligned} & q_t = 0.0013 + 1.002q_{t-1} - 0.005d_{t-1}, \\ & \qquad (0.0062)\ (0.005) \qquad (0.008) \\ & R^2 = 0.9971, SEE = 0.0410, DW = 1.455. \end{aligned} \tag{7}$$

According to a Wald test (with a p-value of 0.427) the estimated coefficients of equation (7) agree with the parameter restriction on the coefficients of equation (6).[9]

Next, consider the hypothesis of adaptive expectations formation. This is the preferred expectations hypothesis by Chow (1989). According to Chow's analysis with annual data, the notion of adaptive expectations is in accordance with the stock price, while the notion of rational expectations is not. The specification to be estimated under adaptive expectations (Chow, 1989) is

$$\begin{aligned} q_t = {} & \frac{\lambda\left(k - \ln\delta^{-1} + c\right)}{1-\lambda\rho} + \frac{1-\lambda}{1-\lambda\rho}q_{t-1} \\ & + \frac{\lambda(1-\rho)}{1-\lambda\rho}d_{t-1} + \frac{1}{1-\lambda\rho}\varepsilon_t, \end{aligned} \tag{8}$$

which in terms of the variables included is the same as equation (6). When comparing the estimated coefficients of equation (7) with the model parameters of equation (8), we find that the estimated coefficients are not consistent with the theoretically presumed parameter values for λ and ρ between 0 and 1. This does not lend credence to the hypothesis of adaptive expectations in this context.

Finally, consider the hypothesis of pattern-based expectations. Based on the Campbell and Shiller approximation scheme, the stock price equation in log form in this case becomes

$$q_t = k + \ln\delta + \rho q_{t+1}^e + (1-\rho)d_t + u_t, \tag{9}$$

[8] We simplify here by equalizing two probabilities, that is, by setting $\lambda_1 = \lambda_2 = \lambda$.

[9] Instrumenting the dividend variable as suggested by Chow (1989) makes no noticeable difference in the results on the level of monthly data used here.

where now q^e_{t+1} is the log of the pattern-based expectation of the stock price formed at t. Clearly, given that q^e_{t+1} builds on q_t it is correlated with u_t, and hence this variable needs to be instrumented. Using the estimate

$$\begin{aligned} q^e_{t+1} &= \underset{(0.009)}{0.010} + \underset{(0.029)}{1.422}q_{t-1} - \underset{(0.029)}{0.425}q_{t-2} \\ &\quad + \underset{(0.083)}{0.095}d_{t-1} - \underset{(0.082)}{0.090}d_{t-2}, \\ R^2 &= 0.9959, SEE = 0.0486, DW = 1.968, \end{aligned} \tag{10}$$

that does not include q_t as a regressor to compute the fitted variable $\hat{q}^e_{t+1}$, we proceed (after instrumenting for d_t along the same lines) to estimate

$$\begin{aligned} q_t &= \underset{(0.009)}{0.006} + \underset{(0.004)}{0.996}\hat{q}^e_{t+1} + \underset{(0.007),}{0.006}\hat{d}_t \\ R^2 &= 0.9972, SEE = 0.0399, DW = 2.233. \end{aligned} \tag{11}$$

A Wald test (with a p–value of 0.498) supports the restriction on the coefficients for $\hat{q}^e_{t+1}$ and d_{t-1} suggested by equation (9). Finally, a comparison with the previous estimate under rational expectations indicates that the hypothesis of pattern-based expectations fares better when assessed by the standard error of the estimate.

VII. Exchange Rate Expectations

This section takes up the analysis of the determinants of the forward discount on the foreign exchange market. To start, we apply the procedure introduced to compute the pattern-based expectations of the spot exchange rate of the pound sterling in relation to the U.S. dollar (denoted by s^e). The data used are monthly data (end-of-month daily numbers) provided by the Bank of England for the period 1979:01 to 2006:12. As a next step, the test for rationality of expectations suggested by Froot and Frankel (1989) is conducted with these data. This test avoids problems due to potential errors in the measurement of expectations. It consists of running a regression where the expectations error is regressed on a constant and the forward discount from the previous period. The forward discount expressed in percentage terms is the difference between the log of the forward rate (f_t) for delivery in the next period determined in a given period and the log of the spot rate (s_t) in that period. The result of running this regression is

$$\begin{aligned} s^e_t - s_t &= \underset{(0.002)}{0.001} + \underset{(0.927),}{0.857}(f_{t-1} - s_{t-1}) \\ R^2 &= 0.0033, SE = 0.0313, DW = 2.3105, \end{aligned} \tag{12}$$

where $s_t{}^e$ denotes the pattern-based expectation for time t formed in $t-1$. A Wald test (with a p–value of 0.612) indicates that we cannot reject unbiasedness and efficiency (the hypothesis that the two coefficients do not differ from 0). With this result, pattern-based expectations perform better than survey expectations of exchange rates (see Froot & Frankel, 1989; Cavaglia, Verschoor, & Wolff, 1994). When including on the right-hand side of equation (12) lagged expectations errors (with lags of one and two months), the assessment of the efficiency of the pattern-based expectations has to be qualified. In this specification, the Wald test rejects the hypothesis of efficiency at the 5% level of significance. Hence, when pattern-based exchange rate expectations are evaluated ex post, they appear not to be fully efficient. Moreover, when we compare our expectations data against random walk expectations (that is, static expectations), pattern expectations are not superior. The root mean square forecasting error of the pattern expectations over the whole sample is 0.0313, whereas static expectations yield a number of 0.0303. Hence, in terms of predictive power, the pattern-based expectations are not able to beat the random walk model for the exchange rate data studied here.

However, the key issue is not the predictive accuracy of pattern-based exchange rate expectations but the question of whether this expectations hypothesis helps to explain important characteristics of exchange rates. In this respect, one of the most hotly debated issues is the question of how expectations errors and the risk premium affect the forward discount (see Froot & Frankel, 1989). This is addressed here by estimating the following regression equation:

$$\begin{aligned} f_t - s_t &= \underset{(0.000)}{-0.000} + \underset{(0.025)}{0.943}(f_{t-1} - s_{t-1}) \\ &\quad + \underset{(0.004),}{0.014}\left(s^e_t - s_{t-1}\right) \\ R^2 &= 0.8567, SEE = 0.0008, DW = 1.8357. \end{aligned} \tag{13}$$

Note that for the second explanatory variable (the expected change of the exchange rate), we instrument s_t by s_{t-1} in order to avoid any possible endogeneity bias. When applying a Wald test (with a p–value of 0.249), we cannot reject the hypothesis that the constant in equation (13) is 0 and that the sum of the two remaining coefficient is 1. Equation (14) shows the outcome of estimating the regression with these restrictions:

$$\begin{aligned} f_t - s_t &= \underset{(0.005)}{0.981}(f_{t-1} - s_{t-1}) + \underset{(0.005)}{0.018}\left(s^e_t - s_{t-1}\right), \\ R^2 &= 0.8552, SEE = 0.0008, DW = 1.8759. \end{aligned} \tag{14}$$

Thus, the following picture emerges: the forward exchange bias is highly persistent and affected by the expectations term. In the long run, an expected change (the expectations error if we understand s_{t-1} to be an instrument for s_t) in the exchange rate is fully reflected in a corresponding change in the forward rate relative to the spot rate. In the short run, however, the forward rate adjusts slowly, giving the appearance of a time-varying risk premium. This interpretation is in accordance with results by Zietz (1995) and Lothian and Wu (2003).

VIII. Conclusion

This study elicits pattern-based expectations in a comprehensive way in order to develop a descriptive model of actual agent behavior. The experimental evidence indicates that pattern-based expectations are very similar in Germany and in Japan, suggesting that the way humans extrapolate time series is little affected by differences in culture and history. Econometric analysis indicates that the data on expectations collected cannot be adequately represented by either the model of linear trend extrapolation or the investor sentiment model of Barberis et al. (1998). For example, expectations of changes after zigzag movements cannot be understood as a weighted sum of past changes. The paper further introduces a method for using the expectations data gathered for computing expectations for any economic time series. Econometric analysis of historical data shows that thus derived stock price expectations and exchange rate expectations help to explain variations in stock prices and in the forward discount on the foreign exchange market. Hence, the experimentally informed model of pattern-based time series extrapolation shows promise as a model of expectations.

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APPENDIX A

Instructions for the Experiment (translated from German)

You are participating in an experiment investigating the formation of expectations on financial markets. Hence, think of the data shown to you in the experiment as stock prices or exchange rates. In what follows, you are presented with 63 cases of how the price of an asset (like a stock or a currency) can develop over four periods. The experiment is simplified inasmuch as only the four following steps are possible:

- An increase by 2 (that is, a change by +2)
- An increase by 1 (that is, a change by +1)
- No change (that is, a change by 0)
- A decrease by 1 (that is, a change by –1)
- A decrease by 2 (that is, a change by –2)

It is your task in this experiment to forecast the development for the fifth period for all 63 cases presented to you. This means (this is task a) that you have to assign probability values to the different possibilities of the continuation of the displayed path (+2, +1, 0, −1, −2). Please select probability values in steps of 0.1, and note that the sum of the probabilities must equal 1. By way of an example, here are three of many possible answers:

Example 1

+2	0.1
+1	0.2
0	0.4
−1	0.2
−2	0.1

Example 2

+2	0
+1	1.0
0	0
−1	0
−2	0

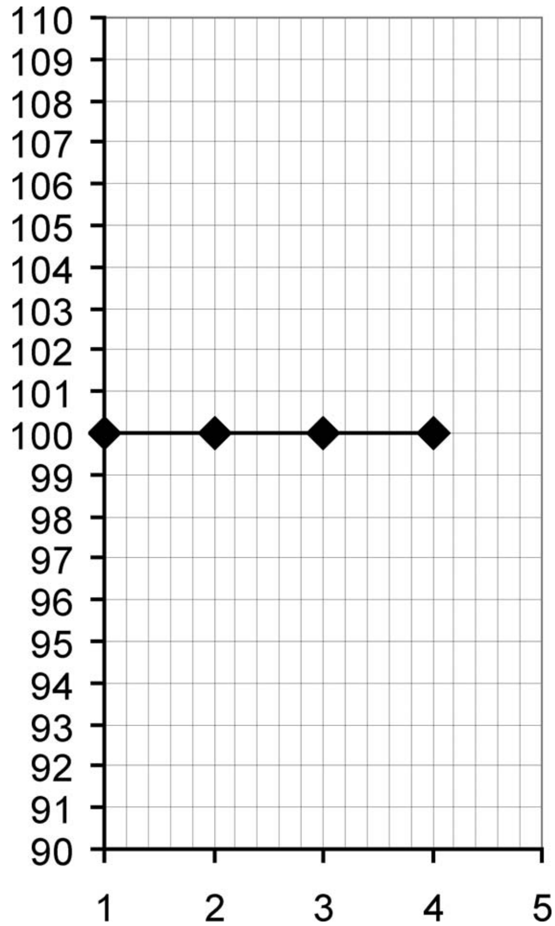
Example 3

+2	0
+1	0
0	0.5
−1	0.5
−2	0

In addition, we would like to obtain your estimation of the average of the forecasts of all test persons taking part in the experiment here today for each of the 63 cases (this is task b). This means that you are asked to estimate the forecasts of the other test persons as accurately as possible. Specifically, we are asking you for a single value between +2.0 and –2.0 down to one decimal point. The following will give you a hint of a possible procedure to solve this task: Start with your own expected value (your expected value of the change of the displayed variable is the sum of the possible changes weighted with their probability values as given by you) in task a. In the three examples above, this would be 0 (example 1), +1 (example 2), and –0.5 (example 3). Now predict the average of the expected values of all test persons in task a, and write down this value. If you, for example, in this position enter a value of 1.2 while your personal expected value is 0 (as in example 1), you judge the average of forecasts to be significantly above your personal forecast.

You will receive, as financial compensation, a basic fee of 4 euros. For answers to tasks a and b, you additionally earn 10 euro cents per case (for a maximum of 6.30 euros). Moreover, we would like to measure the degree of your confidence regarding your answer in task b. In this part of the experiment, you can gain or lose money. Specifically (this is task c), we want to know how much (between 0 and 10 cents) you bet on your assessment in task b being no more than 0.5 above or below the actual average of all subjects' expected values in task a. If your assessment is within a range of 0.5 of the average, you will gain the amount you enter in task c. However, if your assessment deviates by more than 0.5, you lose this amount. Thus, your final payoff consists of the participation fee of 4 euros and between 0 cents and 20 cents per case.

Case 1



a. Your personal probability forecast (down to one decimal point):

Change	Probability
+2	
+1	
0	
−1	
−2	

b. Your assessment of the average of the expected values of all test persons (down to one decimal point): _____

c. The amount you bet on your assessment of the average forecast of the test persons (figure without decimal points between 0 and 10): _____

[Here cases 2 to 63 follow].

APPENDIX B

Results for Efficiency Tests for Pattern-Based Monthly Stock Price Expectations

We start with an efficiency test over the full data set (from 1871:05 to 2006:12). Equation (A1) shows the result of regressing the expectations error on lagged terms of the expectations error:

$$\begin{aligned} q_t - q_t^e &= \underset{(0.001)}{0.001} + \underset{(0.037)}{0.071}\left(q_{t-1} - q_{t-1}^e\right) \\ &\quad - \underset{(0.040)}{0.132}\left(q_{t-2} - q_{t-2}^e\right), \end{aligned} \tag{A1}$$

$R^2 = 0.021, SEE = 0.0396, DW = 2.030.$

TABLE A1.—EFFICIENCY TESTS OVER SUBSAMPLES WITH DEFLATED STOCK PRICE DATA

Sample	Constant	$q_{t-1} - q_{t-1}^e$	$q_{t-2} - q_{t-2}^e$	Probability for Rejection of Efficiency
1871M07–1889M12	0.0005	0.1340	−0.2041	0.0332
1890M01–1909M12	0.0021	0.0372	−0.0952	0.6602
1910M01–1929M12	0.0021	0.0841	−0.2548	0.0092
1930M01–1949M12	−0.0015	0.1717	−0.1235	0.0619
1950M01–1969M12	0.0040	−0.0596	−0.1718	0.0462
1970M01–1989M12	0.0034	0.0656	−0.1571	0.0753
1990M01–2006M12	0.0042	−0.0363	−0.1254	0.2632

TABLE A2.—EFFICIENCY TESTS OVER SUBSAMPLES WITH NOMINAL STOCK PRICE DATA

Sample	Constant	$q_{t-1} - q_{t-1}^e$	$q_{t-2} - q_{t-2}^e$	Probability for Rejection of Efficiency
1871M07–1889M12	0.0015	0.0111	−0.1462	0.1379
1890M01–1909M12	0.0013	−0.0770	−0.0180	0.6298
1910M01–1929M12	0.0004	0.1145	−0.2661	0.0259
1930M01–1949M12	−0.0023	0.1569	−0.1343	0.0641
1950M01–1969M12	0.0029	−0.0393	−0.1816	0.0483
1970M01–1989M12	0.0001	0.0914	−0.1457	0.1374
1990M01–2006M12	0.0027	−0.0275	−0.1214	0.4093

TABLE A3.—COMPARISON OF ROOT MEAN SQUARE FORECAST ERROR OF PATTERN-BASED EXPECTATIONS AND STATIC EXPECTATIONS WITH DEFLATED STOCK PRICE DATA

Sample	Pattern-Based Expectations	Static Expectations	Ratio
1871M07–1889M12	0.0290	0.0296	0.9790
1890M01–1909M12	0.0350	0.0357	0.9826
1910M01–1929M12	0.0395	0.0410	0.9627
1930M01–1949M12	0.0636	0.0657	0.9685
1950M01–1969M12	0.0298	0.0303	0.9825
1970M01–1989M12	0.0381	0.0389	0.9787
1990M01–2006M12	0.0331	0.0335	0.9907

TABLE A4.—COMPARISON OF ROOT MEAN SQUARE FORECAST ERROR OF PATTERN-BASED EXPECTATIONS AND STATIC EXPECTATIONS WITH NOMINAL STOCK PRICE DATA

Sample	Pattern-Based Expectations	Static Expectations	Ratio
1871M07–1889M12	0.0275	0.0286	0.9596
1890M01–1909M12	0.0340	0.0355	0.9589
1910M01–1929M12	0.0381	0.0392	0.9734
1930M01–1949M12	0.0636	0.0660	0.9633
1950M01–1969M12	0.0296	0.0304	0.9729
1970M01–1989M12	0.0378	0.0385	0.9818
1990M01–2006M12	0.0325	0.0331	0.9820

A Wald test rejects the hypothesis that the coefficients in this regression are jointly 0 at the 1% level of significance. However, consider the results from similar tests using rolling data windows sequentially covering two decades starting with the sample 1871 to 1889 proceeding to 1890 to 1909, 1910 to 1929 up to the last sample 1990 to 2006. Table A1 presents the results of estimating equation (A1) for these shortened samples. The table presents the estimated coefficients and the probability level at which a Wald test rejects the null hypothesis that all coefficients are 0. From this sequential analysis, no clear rejection of the efficiency of the pattern-based stock price expectations emerges. Table A2 gives basically the same picture with estimates based on nominal stock price data instead of deflated data.

Finally, the forecast performance of pattern-based expectations is compared with static expectations. Table A3 presents root mean square errors over the seven subsamples used above. The numbers indicate that in all subsamples, random walk expectations were inferior to pattern-based expectations. The results in table A4 document that this conclusion is also warranted when analyzing nominal stock price variables.