Why do Brazilian investors allocate so little of their investments to stocks? An experimental study on the investment allocation behavior of the Brazilian investor.[1]

Martin Casals Iglesias
Bank Boston
mciglesias@bkb.com.br

Jolanda E. Ygosse Battisti
Escola de Economia São Paulo e
Escola de Administração de Empresas São Paulo
Fundação Getulio Vargas
jolanda_ygosse@fgvsp.br
Tel.: (+55-11) – 3281.7765

Julia von Maltzan Pacheco
Escola de Economia São Paulo e
Escola de Administração de Empresas São Paulo
Fundação Getulio Vargas
juliapacheco@fgvsp.br
Tel.: (+55-11) – 3281.7765

Version June 2006

[1] We would like to thank Karl Czymoch for his dedication in developing software for the laboratory experiment. We would also like to thank Daniel Kahneman and Richard Gonzalez for their comments on our research. Any error is ours.

Abstract

The objective of this study is to analyze investment allocation behavior in the Brazilian capital market. In Brazil only around 3% of investments are allocated to the stock market, versus 97% to fixed income. In the USA, for example, this distribution is almost 50-50.

The high rate of return on stocks in the long run led Mehra and Prescott in 1985 to describe this as an Equity Premium Puzzle. Since then various authors have tried to explain this apparent puzzle for developed countries. In 1995 Benartzi e Thaler presented an explanation based on Kahneman and Tversky's prospect theory. They concluded that the investment allocation in the USA can be explained by the phenomenon that investors suffer from myopic loss aversion. In this paper we use the approach suggested by Benartzi and Thaler to analyze investment allocation behavior in Brazil.

When using this approach, the question arises whether we can actually speak of an equity premium and of an equity premium puzzle in Brazil. Various studies found an ex post equity premium in Brazil varying between 10 and 14 percentage points. However, these studies report such a large variance that it cannot be confirmed that an actual premium exists.

Although we cannot confirm the existence of a true equity premium in the Brazilian capital market, this does not impede the use of Benartzi and Thaler's approach to try to explain the observed allocation behavior of the Brazilian investor. In their study, Benartzi and Thaler use the parameters for risk and loss aversion that Kahneman and Tversky obtain in an experimental study of 1992. In that study the subjects were graduate students from Berkeley and Stanford University. We did an exact replica of the experimental study by Kahneman and Tversky, with Brazilian subjects. Our experimental results are similar to those found by Kahneman and Tversky. This leads to the conclusion that there are no behavioral differences between Brazilian subjects and the subjects in the original study.

Using the parameters of our experimental study for risk and loss aversion, we calculate the utility maximizing investment allocation for evaluation horizons between 1 and 24 months, following Benartzi and Thaler's approach. We find that the most plausible investment evaluation horizon in Brazil lies between 9 and 17 months, and that the optimal, utility maximizing, investment allocation decision for these optimal horizons is to hold only between 0% to 8% of investments in stocks. Given the actual allocation observed in Brazil, of approximately 3%, we conclude that observed investors' behavior is compatible with prospect theory and the phenomenon that investors suffer from myopic loss aversion. Moreover, we conclude that the difference between investment allocations to stocks in Brazil compared to, for example, in the USA, is due to the extraordinary high return of relatively low-risk government bonds, and not due to behavioral differences.

Keywords: Prospect Theory, Utility Theory, Value Function, Probability Weighting Function, Behavioral Finance, Equity Premium, Laboratory Experiment, Myopic Loss Aversion, Risk Aversion, Asset Allocation.

JEL code: C91, B14, G11

Why do Brazilian investors allocate so little of their investments to stocks? An experimental study on the investment allocation behavior of the Brazilian investor

1. Introduction

The objective of this study is to analyze investment allocation behavior in the Brazilian capital market. In Brazil only around 3% of investments are allocated to the stock market, versus 94% to fixed income. In the USA, for example, this distribution is almost 50-50 (Benartzi and Thaler (1995)).

The high rate of return on stocks in the long run led Mehra and Prescott in 1985 to describe the actual investment allocation in the USA as an Equity Premium Puzzle. Since then various authors explained this apparent puzzle for developed countries. In 1995 Benartzi and Thaler presented an explanation based on Kahneman and Tversky's prospect theory. They concluded that the investment allocation in the USA can be explained by the phenomenon that investors suffer from myopic loss aversion. In this paper we use this approach suggested by Benartzi and Thaler to analyze investment allocation behavior in Brazil.

The main objective of our study is to analyze investment allocation in the Brazilian stock market. In section 2 we describe the current asset allocation in Brazil. We also present the literature on the equity premium in the United States, Brazil and other countries. In section 3, we present the prospect theory explaining individual's behavior when submitted to risky situations. In section 4, we show the set up and results of our laboratory experiment that was undertaken in order to obtain parameters of the prospect theory. Finally, in section 5, we use our results in order to verify whether Brazil's investment allocation is in accordance with prospect theory and the return level of Brazilian assets.

2. The Brazilian Investment and the Equity Premium

The Brazilian stock market is the most important stock market in Latin America. 66% of Latin America's trading volume is undertaken at the BOVESPA (Bolsa de Valores de São Paulo). The second largest trading volume, with 24%, is in Mexico and the third largest, with 7%, in Chile. The Brazilian stock market capitalization more than doubled over the last two years, attaining US$ 341 billions.

Today market capitalization represents 56% of the Brazilian GDP, pointing out a significant growth potential. Mature markets such as the United States attain a market capitalization of 140% of GDP.

The focus of our paper is on investment behavior of private investors[2] in Brazilian financial markets. Our study aims to determine and explain the current asset allocation in Brazil. One specific characteristic of the Brazilian stock market is that private investors use commonly mutual funds to access the Brazilian stock and financial market.

In Table 1 we present a list of mutual funds acquired by Brazilian private investors from financial institutions (retail banks, investment banks and independent asset managers). We obtained for each fund the net asset value (NAV), evaluated on October, 30th 2005, from ANBID's Information System (SI-ANBID). In Table 1, we also included the referring values of saving accounts and CDBs acquired by private investors.

Table 1 Investment Allocation for Private Investors in Brazil

Investment	AUM (R$ MM)	%
Equity Funds	14.271	2,68%
Foreign Exchange Funds	1.125	0,21%
Hedge Funds	25.674	4,82%
Money Market Funds	94.561	17,77%
Inflation Related Funds	407	0,08%
Asset Allocation Funds	1.427	0,27%
Fixed Income Funds	104.970	19,72%
Other	91	0,02%
Total - Mutual funds	242.526	45,56%
CD	54.885	10,31%
Saving accounts	162.777	30,58%
Retirement plans	72.100	13,55%
Total	532.288	100,00%

Source: SI-ANBID, Central banking e ANAPP (October of 2005)

In Table 1, we discern a significantly lower percentage of mutual funds investing in stocks than investing in fixed income. According to Table 1, asset allocation into stocks represents around 2,7% of total private investment in Brazil.

[2] The term "private investor" is here applied in the sense of the investor being a physical or natural person. The term thus excludes all entities that represent juridical persons or institutional investors.

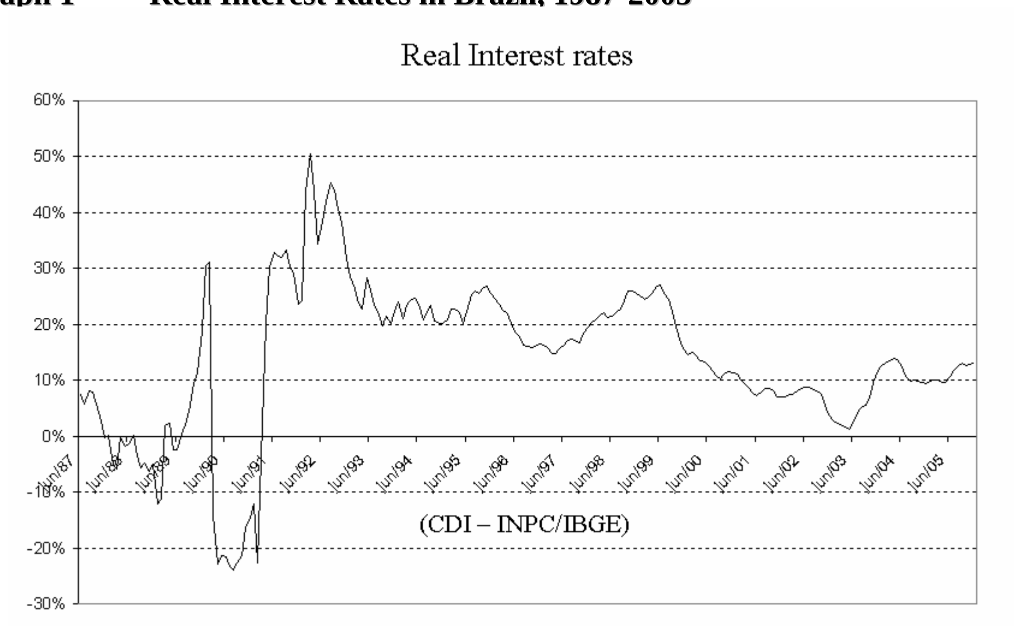
The asset allocation into stocks raises to around 6,0%, when including stock market investment by institutional investors and juridical persons (Iglesias (2006)).

The main question of this study is whether this asset allocation of 3-97 can be considered small or not. However, in order to achieve an adequate evaluation, it is essential to discuss factors that can affect investment allocations such as risk return relation, equity premium, risk aversion and evaluation periods.

2.1 Equity Premium in Various Countries

Over the last three decades Brazilian financial markets have suffered from several repeated external and internal crises, such as the hyperinflation period in the Eighties and early Nineties, the emerging market's crises (Asian, Mexican, Russian, Brazilian, and Argentinean etc.), a self-fabricated energy shortness in 2001 and a capital flight crisis induced by an to-be-elected left-wing president in 2002. These economic turbulences combined with high fiscal deficits and debt levels implied more than a decade of exceptionally high real interest rates in Brazil, as shown in Graph 1. During the same period the Brazilian stock market, BOVESPA, presented a quite volatile performance, offering at time exceptionally high returns, in other times considerable losses, as shown in Graph 2.

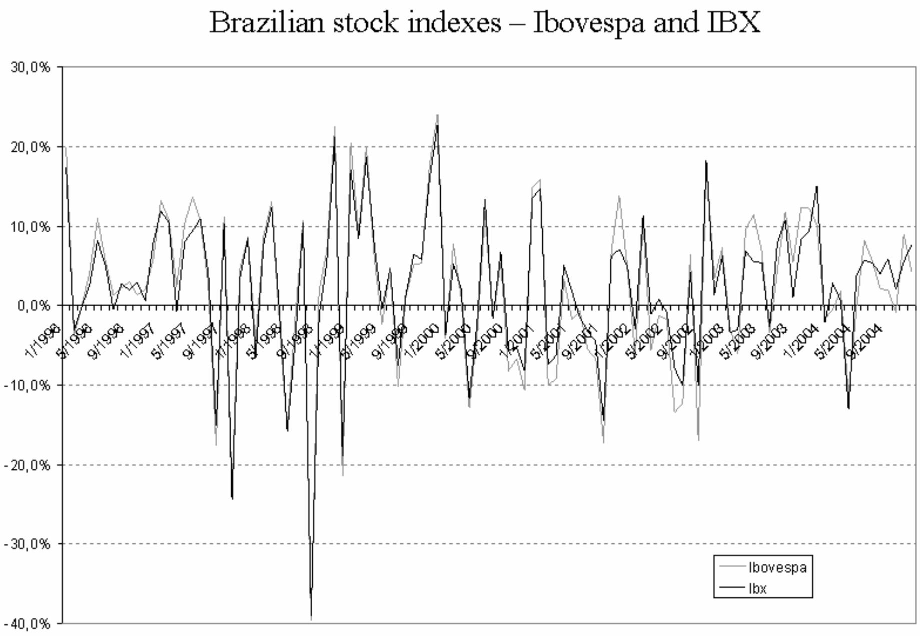
Graph 1 Real Interest Rates in Brazil, 1987-2005



12-months moving average

Source: Own calculation. Banco Central and Sistema Smart Investor (2005).

Graph 2 Bovespa and IBX – Brazilian Stock Market Indexes, 1995-2005



Source: Own calculation. Banco Central and Sistema Smart Investor (2005).

Having this stock market volatility in mind, it is evident that in order to invest partly in stocks, investors must believe that stocks have something to offer that will compensate for higher risks taken by those, who invested in stocks instead of bonds. In more general terms, what take investors into account when deciding between investing into risky assets, such as stocks, or to concentrate their investment into less risky assets, such as government bonds?

Studies using US data show that the returns on stocks consistently outperform the returns on bonds, implying the existence of an equity premium. According to Shiller (2000), an equity premium is the additional return required in order to compensate for the higher investment risk in the stock market. In their paper of 2003 Mehra and Prescott summarize these studies confirming a considerable equity premium that varies between 4.1 and 8.4 percentage points, as shown in Table 2.

Table 2 Equity Premium in U.S.A.

	Real Return		
Data	Stock Market	Risk free Asset	Equity premium
1802 - 1988 (Siegel)	7,0	2,9	4,1
1871 - 1999 (Shiller)	7,0	1,7	5,3
1889 - 2000 (Mehra and Pı	8,1	1,1	6,9
1926 - 2000 (Ibotson)	8,8	0,4	8,4

Source: Mehra and Prescott (2003).

The equity premium is not an exclusive phenomenon only for the United States, but it can also be observed in other countries, such as the UK, Japan, Germany and France. In 2003, Mehra and Prescott undertook a study on equity premiums in these countries. For the UK, Japan, Germany and France, the authors estimate annual ex post equity premiums, similar in magnitude as to equity premiums found in the United States, varying between 3.3 – 6.6 percentage points, as presented in Table 3.

Table 3 Ex-Post Equity Premiums for Diverse Countries

	Real Return		
Country	Stock Market	Risk free Asset	Equity premium
UK (1947 - 1999)	5,7	1,1	4,6
Japan (1970 - 1999)	4,7	1,4	3,3
Germany (1978 - 1997)	9,8	3,2	6,6
France (1973 - 1998)	9,0	2,7	6,3

Source: Mehra and Prescott (2003).

In Brazil, measuring equity premiums is not a simple task, because it is difficult to find long time series. While in the United States data exists since 1802, and for other developed countries data is available for up to 6 decades, studies on the Brazilian equity premium are based on time series of a maximum length of two decades. Furthermore, as mentioned above, there have been a couple of economic breaks and ruptures that affected the fixed income market in the late eighties and early nineties. When studying the Brazilian financial markets three principal difficulties should be mentioned: (1) the IBOVESPA index only exists since 1968, (2) the need to correct for inflation during the hyperinflation period in the late eighties and early nineties, and, (3) the need to correct for restructuring of Brazil's internal debt (*tablitas* and default).

Nevertheless, various studies estimated the Brazilian equity premium. Schor, Bonomo and Valls (1998) estimate an ex post equity premium of 10 percentage points per the year, using data during the period between 1987 and 1997. However, estimated standard deviation is 20 percentage points per month. Given this high variability the existence of an equity premium in Brazil becomes uncertain. Sampaio (2002) presents an ex post equity premium of 11.2 percentage points, based on data from 1980 until 1998. Bonomo and Domingues (2002) measure a premium of 10.25 percentage points per year, using data from 1986 until 1990. Finally, Cysne (2005) based on data between 1992 and 2004, finds a premium of 14.3 percentage points per year (3.4 percentage points per quarter).

In conclusion, empirical research on the Brazilian equity premium shows an equity premium of 10 to 15 percentage points per year. However, we cannot confirm the existence of a Brazilian equity premium because of the observed high standard deviation.

3. Prospect Theory and its Application to Brazilian Data

In order to deduce whether Brazilian allocation into risky assets is small, we need to understand what investors take into account when submitted to situations that involve risk. Another question is what factors influence the investors' decision whether to take or to avoid risks. Utility theory offers models that conciliate asset returns with investors' behavior allow us to verify whether Brazil's asset allocation is or is not in accordance with the theory.

3.1 The Prospect Theory

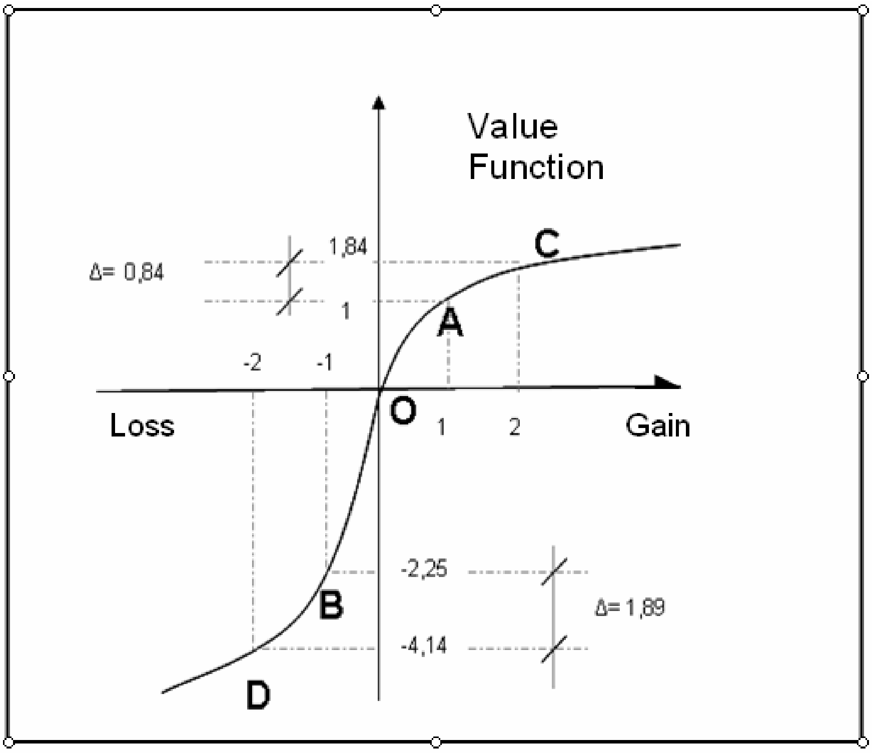
In 1944 von Neumann and Morgenstern described in their book *Theory of Games and Economic Behavior* how individuals react when exposed to risk and what utility is generated from assets in which they invest. Basic assumptions of their model are that individuals are risk averse and their utility depends on states of wealth resulting from investments. In 1979, Kahneman and Tversky propose a different utility function, in which individuals are loss averse and not necessarily risk averse: prospect theory. They present a utility function that does not depend on the final state of wealth, but on obtained results (profits or losses). The expected utility of an investment results from the interaction of two functions: the value function (VF) and the probability weighting function (PWF). The value function calculates the subjective value associated to each profit or loss. The probability weighting function measures how individuals mentally evaluate probabilities. In general, individuals overestimate small probabilities, while they underestimate large probabilities. The expected utility of investment is the sum of values associated to potential results (VF) weighed by the perceived probability of each potential result (PWF). The calculation of expected utility allows us to verify which investment offers more gains to the investor and which portfolio maximizes his gains. We are also able to test whether current investment allocation in Brazil is in accordance with prospect theory.

The value function has two main characteristics: (i) loss aversion, implying that a value destroyed by a loss is greater than a value created by a profit of equal sum, and (ii) marginal values for both gains and losses are decreasing. The function is thus concave in the domain of profits, convex in the domain of losses and defined as follows:

$$(1) \qquad V(x) = \begin{cases} x^{a} & \text{if } x \geq 0 \\ -\lambda(-x)^{\beta} & \text{if } x < 0 \end{cases}$$

where λ is defined as the coefficient of loss aversion. In 1992, Tversky and Kahneman estimated a value of 2.25 for λ in laboratory tests. In the same work they estimated a value of 0.88 for α and β. The value function has a characteristic S-shape, as presented in Graph 3.

Graph 3 Value Function



Source: Own elaboration

The value function reveals the investor's subjective values that are generated by profits or losses resulting from his risky decisions. The expected utility is the value created by each profit or loss weighted for its probability to occur. However, this probability is not simply a mathematical probability *(p)*, but a behavioral transformation of *p* called probability weighting function (PWF) and is defined as follows:

(2)
$$W+(p)=\frac{p^{\gamma}}{\left(p^{\gamma}+(1-\rho)^{\gamma}\right)^{1/\gamma}}$$
$$W-(p)=\frac{p^{\delta}}{\left(p^{\delta}+(1-\rho)^{\delta}\right)^{1/\delta}}$$

where *W+(p)* is the PWF for profits, *W-(p)* is the PWF for losses, *p* is the mathematical probability. Tversky and Kahneman (1992) also estimated the parameters γ and δ, with values of 0.61 and 0.69 respectively.

If the individual's preferences are described by the prospect theory (with loss aversion and mental balance of probabilities), we can calculate the expected utility of an investment, as described in Figure 1 and in the following instructions:

i) Assuming that returns follow a normal distribution, one can calculate the return, x_i, associated to its probability p_i, with i varying from 0% up to 100%, for any given return mean and any given return standard deviation of one asset or a portfolio of assets.

ii) To each value of x_i, we can associate its subjective value by applying the value function, *V(x)*.

iii) For each p_i exits a mental probability of occurrence $W(p_i)$, calculated by the PWF.

iv) Finally, the expected utility (EU) of an asset or a portfolio of assets is given by:

$$EU = V(X_1) * p_1 + V(X_2) * p_2 + \ldots + V(X_{100}) * p_{100}$$

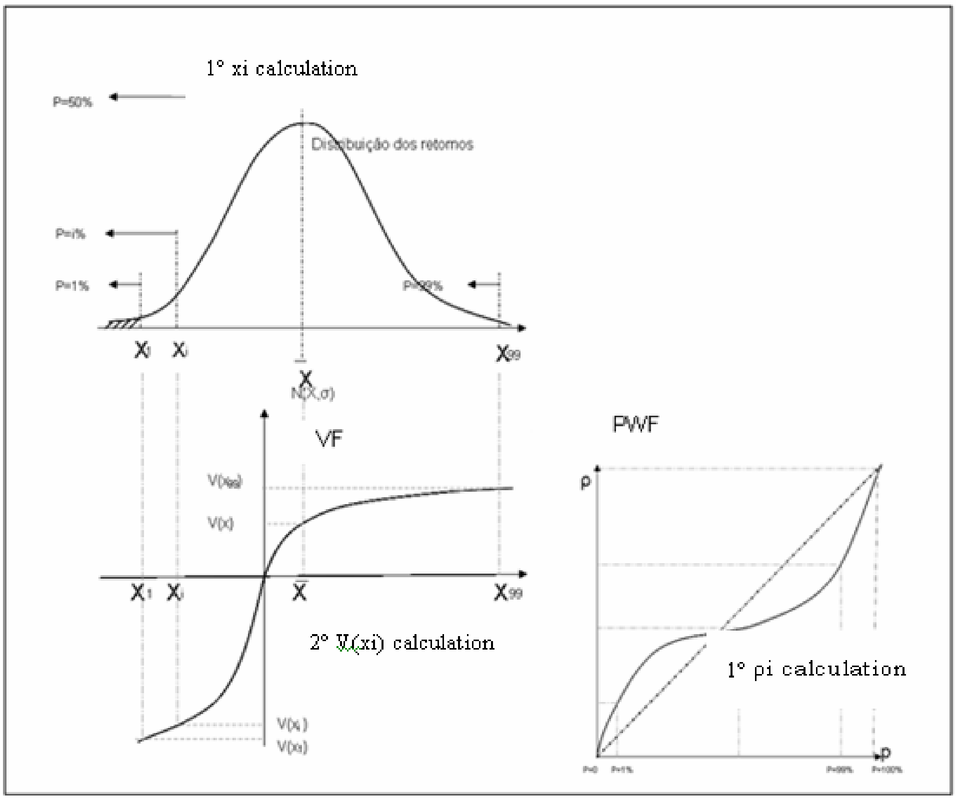
$$or$$

(3) $$EU = \sum V(X_i) * p_i$$

$$where$$

$$p_i = W(p_i) - W(p_{i-1})$$

Figure 1 Utility Calculation: Distribution of Returns, Value Function and Probability Weighting Function.



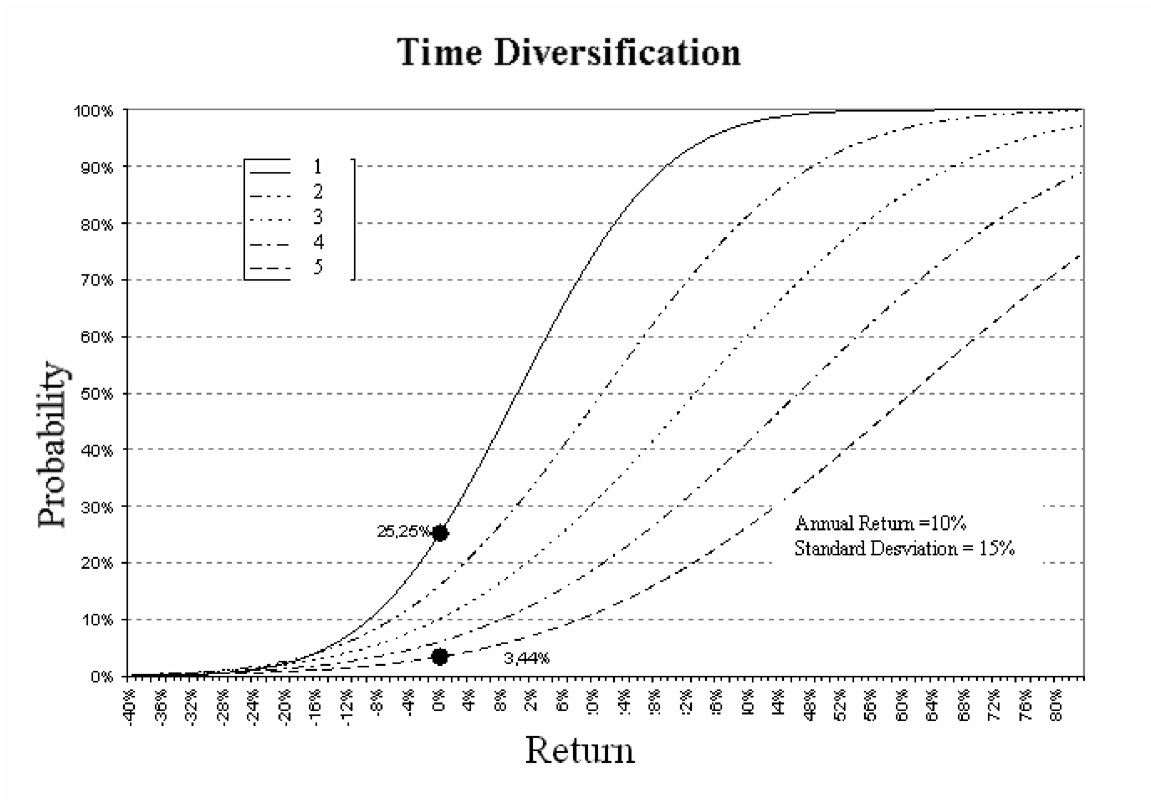
Source: Own elaboration.

3.2 Time Diversification

Time diversification is related to portfolio diversification. Investors benefit from risk reduction, when diversifying their portfolio into assets that are not perfectly correlated. When diversifying over time, investors benefit from the ownership of a single asset over a long time period. Time diversification can be defined as the benefit from risk reduction created by the ownership of volatile assets over long investment horizons.

Return means and return standard deviations of an investment depend on the time horizon over which it is analyzed. Returns, when kept constant over long time period, grow according to an exponential function $(1+i)^t$, while standard deviations grow multiplied by the factor $(\sigma * t^{1/2})$. We can then define the variation coefficient as the relation of the mean to the standard deviation. The coefficient declines over time, implying an increasing utility of risk assets, in the measure where if it increases the evaluation stated period. This dilution of risk over the time is also called time diversification.

Graph 4 Effect of Time Diversification over Probability to Realize Negative Returns



Source: Own elaboration

Assuming an annual expected return of 10% and an annual expected volatility of 15%, we present in Graph 4 the probabilities of an asset suffering losses or gains. The graph shows, for instance, when the asset is kept for one year, the probability of realizing a return equal or worse than zero is 25.5%. If the asset is kept for five years, the probability of realizing a return equal or worse than zero falls to 3.44%.

3.3 Evaluation Period

Two different concepts have a strong impact on asset allocations and their performance: the investment horizon and the evaluation period. The investment horizon is the period during which an investment is carried through. For example, an investor, who saves resources for a trip in one year from now, has an investment horizon of twelve months; a person that saves for his retirement has an investment horizon of twenty years. The objective of the evaluation period or horizon is a different one. The evaluation period is the period, over which the investor evaluates his profits or losses. In fact, for the investor, in order to create value from his investments, the evaluation period is a far more important concept than the investment horizon. Let's assume two investors, who keep resources invested with the objective to buy a house in five years. One of them evaluates his investment results on a monthly basis, while the other one evaluates his investments quarterly. In accordance to prospect theory, both investors will attain different levels of utility from their investments. Furthermore, it is possible to say that their optimal portfolio that maximizes each utility of them will be composed differently. For instance, what would be the investment horizon of an investor, who keeps savings for emergencies? Such an investor does not have a defined investment horizon. However, he certainly uses an evaluation horizon in order to evaluate his investments regularly.

According to Benartzi and Thaler (1995), the average evaluation horizon of a north-american investor is around twelve months. In their study, they also conclude that utility of stock investments is inferior to utility of investment in fixed income over short evaluation horizons. However, utility of stock investments increases over longer evaluation horizons and becomes larger than utility generated by fixed income investment. This effect occurs due to the high volatility of stocks, provoking negative results with great frequency over short evaluation periods. The latter diminishes the investor's utility because of his loss aversion. When expanding the evaluation horizon, two effects occur: the frequency of negative returns diminishes (time diversification) and equity premiums emerge.

Benartzi and Thaler found a point where the utility of both the investments (bonds and stocks) is equal and at this moment investor would be indifferent to have a portfolio composed exclusively for stocks or fixed income. Before this point, the investor not it would have stocks, given that the utility generated for the assets of fixed income it is superior, or either, the fixed income dominates stocks in shorter periods. Before the meeting point must not have action market, therefore nobody would desire to buy them.

The critique is that for any period it is always possible to find a portfolio composed for stocks and fixed income that generates more utility that any one of the assets taken individually, or either, there is a place for two assets in any investment horizon and not only in the period where the same utility occurs (where two curves have the same utility). This occurs due to the "portfolio effect" of traditional finance because assets are not perfectly correlated.

Portfolio theory of Markovitz (1952) evidences the existence of an efficient frontier that nothing more is than a portfolio that maximizes the return for each level of risk. The problem is that investor nor always defines a priori its level of risk tolerance, especially if measured as being the standard deviation of the returns. In the vision of the present work, it has to be maximized not the level of return for each risk, but the utility generated for the investor, in each evaluation horizon. The great advantage of this kind of maximization is that the investor has a much clearer notion of its horizons (either of investments or of evaluation) than it has on the risk level (standard deviation of the returns).

3.4 Utility of Products of investment and indifference curves

Calculating the expected utility of investments allows us to compare various investments, even if undertaken in different asset classes. If we calculate the investor's utility that each investment product generates, we can deduct the necessary additional return so that the investor changes his current investment into one with a higher risk level.

In Table 4 we show investment products generating the same utility for the investor. The investor chooses whether he allocates his investment into a risk free mutual fund (Money Market or fixed income) or a risky mutual fund. We assume that the risk free mutual fund charges investors with an annual fee of 0.50%. Furthermore, the portfolio of the risk free fund is exclusively composed of government bonds or other bonds classified as low credit risk. We also assume that the risk free mutual fund (CDI) offers an annual return of 19 % or 13%, generating a utility of 0.24 or 0.16 respectively. As a result, the investor can realize an annual return of 18.5% (19 % minus 0.50%), when investing exclusively into the risk free mutual fund. As for alternative investment products, we chose various relevant mutual funds from the Brazilian financial market and estimate approximates for their respective volatilities. We report only one index for each fund representing applications into Money Market or Fixed Income, because these two fund types present low risk level. We divided multi-markets into funds with low, medium and high volatilities, with sub-divisions for each volatility level. For stock funds we applied a similar division method.

In Table 4 we report the returns that are necessary in order to compensate the investor for his additional risk exposure when investing into risky mutual funds. We can see, for example, that, at an annual interest rate level of 19%, investors would only opt to sell all their investment of a Money Market fund and invest in a Hedge fund with volatility of 3.0% per year, if the Hedge fund offers an expected return of 103.3% of CDI. If the investor desires to move his resources to a Equity fund with an annualized volatility of 22%, for example, he would demand at least a 33.4% return or 176.0% of CDI. However, Table 4 does not include portfolio diversification. For instance, the investor could move only part of its resources into an asset of higher volatility and compound a portfolio that generates higher benefits from diversification.

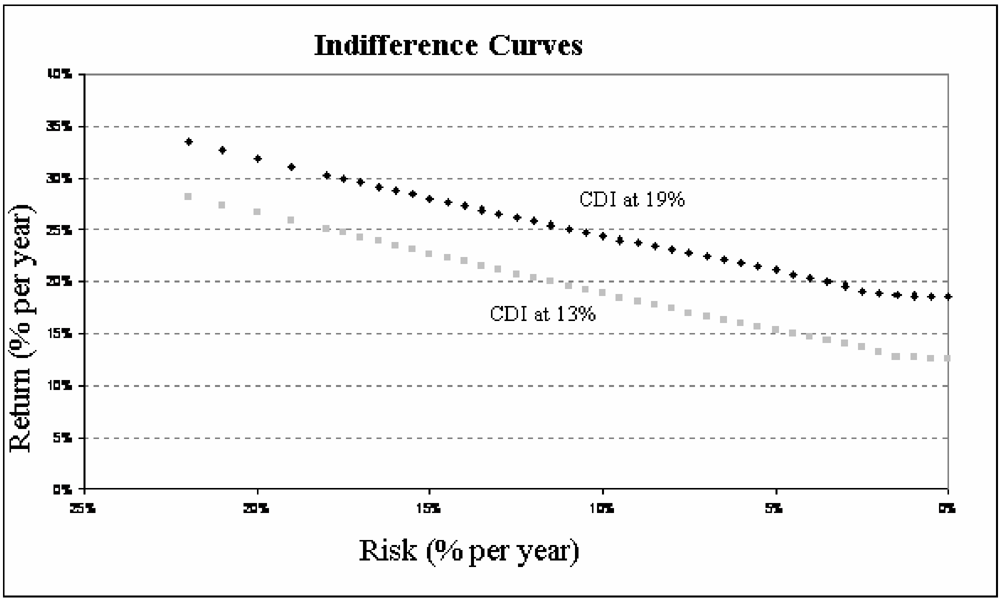
Table 4 Investment Products, Evaluation Horizon of 12 Months and Constant Utility of 0.24 or 0.16.

		CDI at 19% per year			CDI at 13% per year		
Fund class	Risk	Return	% of CDI	Utility	Return	% of CDI	Utility
Money Market Funds	0,01%	18,5%	97,4%	0,24	12,5%	96,2%	0,16
Fixed Income Funds	0,5%	18,6%	97,7%	0,24	12,6%	96,6%	0,16
Low Volatility Hedge Funds (1)	1,0%	18,6%	98,1%	0,24	12,7%	97,3%	0,16
Low Volatility Hedge Funds (2)	1,5%	18,7%	98,5%	0,24	12,8%	98,2%	0,16
Intermidiate Volatility Hedge Funds (1)	2,0%	18,9%	99,5%	0,24	13,2%	101,8%	0,16
Intermidiate Volatility Hedge Funds (2)	2,5%	19,1%	100,5%	0,24	13,6%	104,8%	0,16
Intermidiate Volatility Hedge Funds (3)	3,0%	19,6%	103,3%	0,24	14,0%	107,6%	0,16
High Volatility Hedge Funds (1)	5,0%	21,2%	111,6%	0,24	15,4%	118,1%	0,16
High Volatility Hedge Funds (2)	7,0%	22,5%	118,2%	0,24	16,7%	128,1%	0,16
Equity funds (1)	19,0%	31,1%	163,7%	0,24	25,8%	198,6%	0,16
Equity funds (2)	20,0%	31,9%	167,8%	0,24	26,6%	204,7%	0,16
Equity funds (3)	21,0%	32,7%	171,9%	0,24	27,4%	210,6%	0,16
Equity funds (4)	22,0%	33,4%	176,0%	0,24	28,2%	216,7%	0,16

Source: Own elaboration

In Graph 5 we present in detail the risk-return pairs that generate same utility for the investor. As in any curve of indifference, the graph was plotted with increasing order of benefits, implying that the variable of the X-axe (the asset's risk or volatility) is plotted in decreasing order, since less risk brings more benefits. In Graph 5 as well as in Table 4, which is a summary of the graph, we use annualized data for our calculations. This implies that our analysis is only valid, if investor use an evaluation period of one year.

Graph 5 Indifference Curves - Evaluation Period of 12 Months



Source: Own elaboration

4. Experiment to Determine Parameters of the Prospect Theory

A condition of calculating a portfolio's utility is the knowledge of investors' levels of risk and loss aversion, or in other words, knowing the values of all parameters in the functions VF and PWF. Although experimental studies exist that provide estimated values of these parameters, undertaken with subjects recruited among students at universities in the United States, we chose to estimate these parameters in an experiment in which we used Brazilian subjects, at the São Paulo School for Business and Economics of the Vargas Foundation (FGV-SP) in São Paulo, Brazil. In this way we are able to capture any behavioral differences, which may or may not exist between subjects in the US study and Brazilian subjects.

4.1 Description of the Experiment

Our experiment follows basically the experiment undertaken by Tversky and Kahneman in 1992, with some alterations taken from the experiment of González (1999). For our experiment we recruited 23 students (12 men and 11 women) from the undergraduate course[3] in business and public administration. Each student participated in three sessions over three separate days in the experimental laboratory of FGV-SP, LIJIA[4]. Each subject received a fixed amount of R$ 30 for his participation in the experiment. The payment was made at the end of the 3rd and last session. Thus, remuneration did not depend on subjects' decisions. Kahneman and Tversky (1992) confirm that there are not significant differences in answers of subjects remunerated with predetermined, fixed fees and answers of subjects whose remuneration depended on their decisions.[5]

The experiment was conducted on computers, where subjects had to answer a total of 128 questions. Our questions are identical to those that Tversky and Kahneman applied in their experiment in 1992, allowing the comparison of our results with Tversky and Kahneman's results. The laboratory has partitioned units, so subjects were isolated of each other while answering the questions, giving us independent observations.

In each question in the experiment subjects have the option to choose between a game and a certain event: in the game their profits or losses depend on probabilities, while the certain alternative would be the receipt or payment of a fixed value and the exit of the game. A typical question for the game, for example, would be the following:

"*Let's assume a game with a probability of 90% of you gaining zero and a probability of 10% of you gaining R$ 50*".

The subject sees on his screen six values linearly distributed between minimum, "zero" and maximum "R$ 50". For each of these six values, the subject is requested to indicate whether he prefers the game over the certain value or vice versa. We provided subjects in each round with additional information on the expected value of the game. This information would also show up in the screen.

[3] According to various studies (Davis (1992, p.16f); Dyer et al. (1989); Smith et al. (1988), Mestelman and Feeny (1988); De Jong et al. (1988)), the behavior of undergraduate or graduate students does not significantly differ from behavior of decision takers in the real economy.
[4] Laboratório de Investigação em Jogos Interdisciplinares Aplicados (LIJIA).
[5] Camerer and Hogarth (1999) put forward that ethic committees in many universities reject remunerations that could impose losses on the participating subjects. The financial loss could provoke a psychological discomfort of the subject, which many universities are not willing to assume.

Let assume that in this example a participant preferred the game over any certain value inferior to R$ 10 and preferred to give up the game when receiving any certain amount equal or superior to R$ 20, as shown in Figure 2. Once the subject marked his answers in the screen, the program generates a new screen, as shown in Figure 3, showing the same question, but now with new six values, linearly distributed between new maximum (R$ 20) and minimum (R$ 10) values. This additional question aims at increasing the precision of the subject's answer. The average between the maximum rejected value and the minimum preferred value is then the final result of the game composed by these two questions (in Figure 3 the result would be 11, which is the average between 10 and 12). In this manner the 128 questions are summarized in 64.

Figure 2 Screen of Laboratory Test. Question 1

Let's assume a game in which you have 90% of possibility to gain zero and 10% of possibility to gain R$50				
I prefer to win R$ 0 and give up the game			I prefer the game	x
I prefer to win R$ 10 and give up the game			I prefer the game	x
I prefer to win R$ 20 and give up the game	x		I prefer the game	
I prefer to win R$ 30 and give up the game	x		I prefer the game	
I prefer to win R$ 40 and give up the game	x		I prefer the game	
I prefer to win R$ 50 and give up the game	x		I prefer the game	

Source: Own elaboration

Figure 3 Screen of Laboratory Test. Question 2

Let's assume a game in which you have 90% of possibility to gain zero and 10% of possibility to gain R$50				
I prefer to win R$ 10 and give up the game			I prefer the game	x
I prefer to win R$ 12 and give up the game	x		I prefer the game	
I prefer to win R$ 14 and give up the game	x		I prefer the game	
I prefer to win R$ 16 and give up the game	x		I prefer the game	
I prefer to win R$ 18 and give up the game	x		I prefer the game	
I prefer to win R$ 20 and give up the game	x		I prefer the game	

Source: Own elaboration

4.2 Data Treatment

Before analyzing the results of our experiments, we run several consistency tests on our data set, following Gonzalez (1999). Gonzalez undertook consistency tests verifying monotonicity violations and as a result, he excluded all answers of one of his eleven subjects. We run two types of consistency tests.

In the first consistency test, we regrouped all questions with the exactly same underlying probability for gains. We then verified whether subjects answered with increasing order of payment values. For instance, assume the following two types of games: (1) Game A offers a gain of R$ 100 with a probability of 10% and gaining nothing with a probability of 90%. (2) Game B offers a gain of R$ 50 with a probability of 10% and gaining nothing with a probability of 90%. In this example, the monotonicity condition requires that subjects prefer game A over game B, or in other words, the cash equivalent[6] of the first game must be larger than of the second. In cases, where this condition was not fulfilled, we assumed the monotonicity condition as being violated. However, it is important to emphasize that questions has not been asked in increasing order of preference in order to avoid biased answers.

In the second consistency test, we regrouped questions that had the same payment values. We then verified whether subjects preferred games with higher probability over games with lower probability, if outcomes would have been positive. If outcomes would have been negative, we verified whether subjects preferred games with lower probability over games with higher probability. For example: (1) Game A offers a gain of R$ 200 with a probability of 10% and gaining nothing with a probability of 90%. (2) Game B offers a gain of R$ 200 with a probability of 50% and gaining nothing with a probability of 50%. In this case, the monotonicity condition requires that participants prefer game B over game A. In cases, where this condition was not fulfilled, we assumed the monotonicity condition as being violated.

As a result of our consistency analysis, all interviewed subjects presented violations of the monotonicity condition. In order to preserve data quality, we decided to exclude those subjects with elevated number of violations, leaving us with 10

[6] The cash equivalent is the certain value that the participating subject is willing to receive (or to pay) instead of playing the game that involves uncertainty.

interviewed subjects[7]. However, even within this smaller sample of subjects, we still had to exclude individual answers, when diverging significantly from other subjects' answers.

4.3 Our Results

The experiments' questions always obeyed the same model: participants had to choose between a certain value and a game involving two possible results. In Table 5 we present a summary of all answers to our experiment's questions, comparing them directly to Tversky and Kahneman's results of 1992.

For instance, take the pair (9,13) in Table 5. The pair (9,13) is combined with a probability of 10% (or 0.1 in the table) and an outcome pair of (0,50). The experiment question behind this pair is the following: choose between a payment of a certain value, the cash equivalent *c*, or a game, which involves a gain of R$ 50 (or *x*) with a probability of 10% or a zero gain with a probability of 90%. In the experiment the participating subject has then to determine his cash equivalent in such a way that he is indifferent between the game and the cash equivalent. In our example, Tversky and Kahneman (1992) obtained for the above presented question a cash equivalent of 9, while in the Brazilian experiment the value was of R$ 13.

In Graph 6 and in Graph 7, we present our results by using the relation *c/x*. The relation *c/x* is the relation between the cash equivalent *c* and the value *x* that would be received or lost in the game. In Graph 6 we show the relation *c/x* for games with positive outcomes and in Graph 7 for games with negative outcomes. As you can observe in both graphs, we can divide the relation *c/x* into two different behavior regions.

In Graph 6, the first region, defined by probabilities of below 40%, represents risk seeking behavior, implying that the attributed value is higher than the expected value. In other words, the relation *c/x* is greater than the mathematical profit probability would imply. This type of behavior can be observed in lotteries, where the value of one lottery ticket is substantially higher than the expected value of the prize. For probabilities above of 40%, the propensity behavior towards risks inverts and participants became risk averse, attributing values to outcomes below the

[7] We opted for 10 participants, because 10 subjects participated in Gonzalez' experiment (1999).

mathematical expected value. In this region, the relation *c/x* is inferior to the mathematic probability.

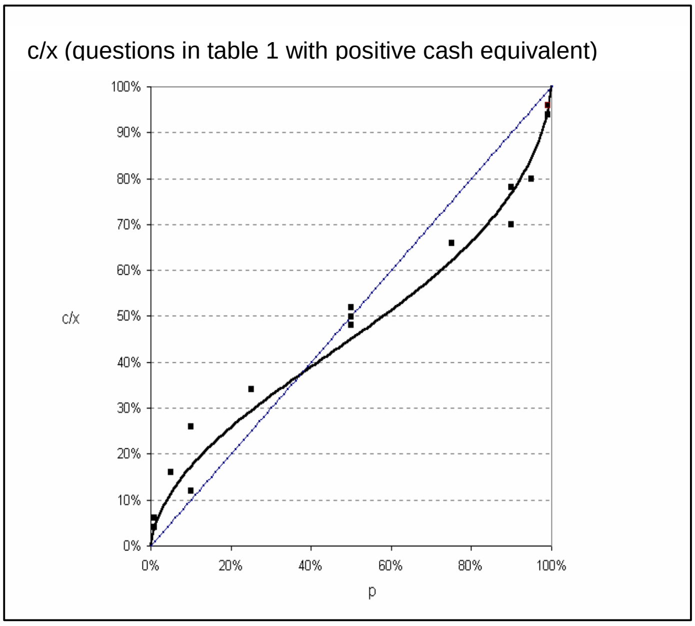
Table 5 Comparison of Experiment Answers in Brazil and United States

	Probability								
Outcomes	0,01	0,05	0,1	0,25	0,5	0,75	0,9	0,95	0,99
(0,50)			(9,13)		(21,26)		(37,39)		
(0,-50)			(-8,-14)		(-21,-20)		(-39,-35)		
(0,100)		(14,16)		(25,34)	(36,50)	(52,66)		(78,80)	
(0,-100)		(-8,-22)		(-23,5,-28)	(-42,-46)	(-63,-66)		(-84,-90)	
(0,200)	(10,12)		(20,24)		(76,96)		(131,140)		(188,188)
(0,-200)	(-3,-8)		(-23,-44)		(-89,-92)		(-155,-148)		(-190,-188)
(0,400)	(12,16)								(377,384)
(0,-400)	(-14,-8)								(-380,-376)
(50,100)			(59,63)		(71,76)		(83,87)		
(-50,-100)			(-59,-30)		(-71,-76)		(-85,-85)		
(50,150)		(64,66)		(72,5,76)	(86,100)	(102,104)		(128,126)	
(-50,-150)		(-60,-72)		(-71,-72)	(-92,-96)	(-113,-104)		(-132,-128)	
(100,200)		(118,122)		(130,146)	(141,150)	(162,166)		(178,180)	
(-100,-200)		(-112,-116)		(-121,-142)	(-142,-146)	(-158,-152)		(-179,-178)	

Source: Tversky e Kahneman (1992) and brazilian experiment
Notation (Tversky and Kahneman, Author)
Note; For example value (9,13) represents Tversky - Kahneman median value (9) and Author median value (13) in a gamble with 0% of chance of win zero and 10% of win 50.

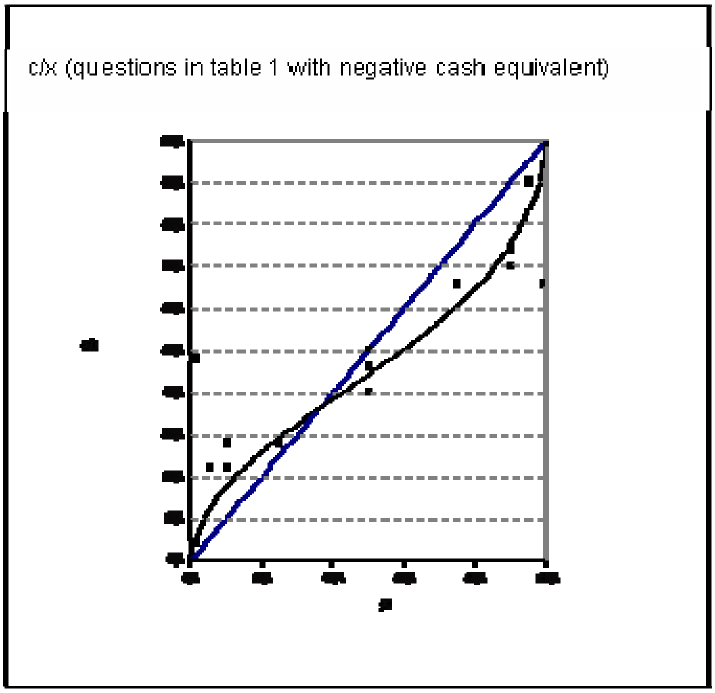
In Graph 7 we can observe that participants invert their behavior in games with negative outcomes. Subjects become risk averse in games with low probabilities and risk seeking in games with medium or high probabilities. The behavior of risk aversion explains very well that insurance premiums are generally higher than its expected loss.

Graph 6 C/X Relation for Games with Profits



Source: Own elaboration

Graph 7 C/X Relation for Games with Losses



Source: Own elaboration

The results of our experiment illustrate a pattern, the Fourfold pattern, first mentioned by Tversky and Kahneman (1992). It is one of prospect theory's pillars and is characterized by: a) Risk seeking in situations with low probability of profits or high probability of losses; b) Risk aversion in situations with low probability of losses or with high probability of profits.

4.4 Parameter Calculation

In order to estimate the parameters of prospect theory, $\alpha, \beta, \lambda, \gamma$ and δ, we calculate the median of all answer for each question.[8] As a result, we obtain 64 equations with five unknown variables that are $\alpha, \beta, \lambda, \gamma$ and δ. The solution of this system it is not that trivial. Each one of the answers is the result of the interaction between the PWF and VF. They are nonlinear functions and with different behaviors

[8] We use only the answers of those 10 participants that have passed our consistency tests.

for profits and losses. We opt to solving this system through a nonlinear regression. This regression aims to minimize following sum:[9]

$$\sum\left(Y_0 - Y^e\right)^2 \qquad (4)$$

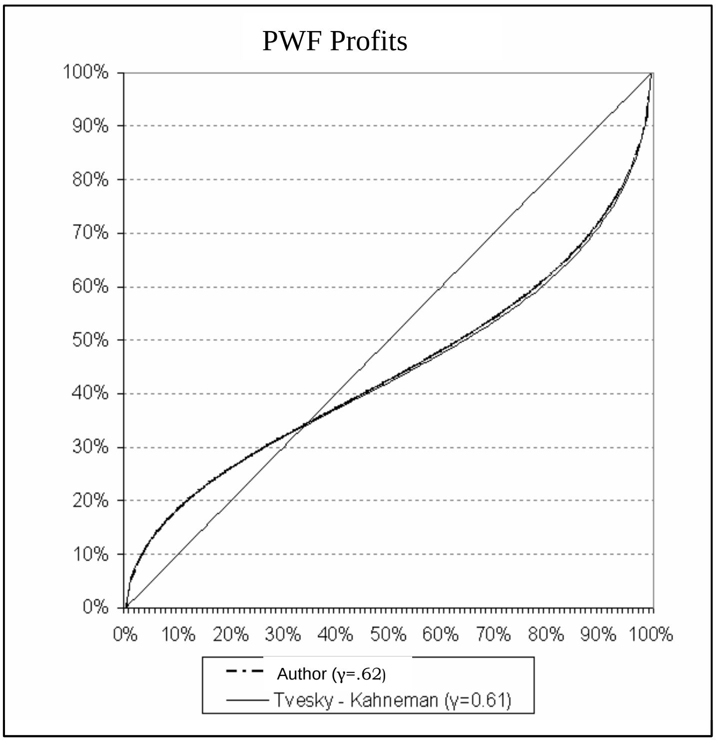
where Y_0 is the observed value (participant's answer) and Y^e is the expected value. The expected value is the result of the interaction between the VF and the PWF, depending on the unknown variables α, β, λ, γ and δ. The results of this minimizing problem are summarized in Table 6, compared to values obtained by Tversky and Kahneman (1992).

Table 6 Parameters from Experiments in Brazil and in United States

	Results	
	Tversky and Kahnemar	Brazilian Data
α	0,88	0,95
β	0,88	0,98
γ	0,61	0,62
δ	0,69	0,66
λ	2,25	2,21

Source: Tversky e Kahneman (1992) and experiment of the author

Graph 8 PWF for Profits

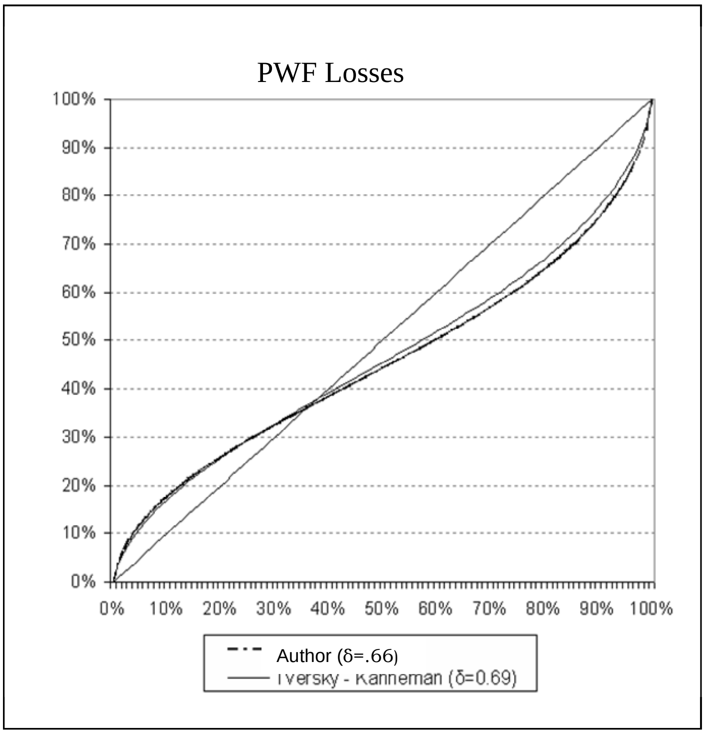


Source: Own elaboration

[9] We use the interactive program Solver, offered by Microsoft Excel.

Graph 8 and Graph 9 compare the PWF's curves obtained in our experiment with those obtained by Tversky and Kahneman (1992): the shape of both curves shows a close fit. Only in Graph 9, we can observe a slight difference in the PWF region of losses (probabilities greater than 40%). It appears that there is higher risk seeking in the case of the Brazilian experiment.

Graph 9 PWF for Losses



Source: Own elaboration

Summarizing, we can state that the results obtained in our experiment confirm the basic properties as laid out in prospect theory. Furthermore, the coefficients that we obtained are very similar to the ones obtained by Tversky and Kahneman (1992).

5. Expected asset allocation in Brazil

With parameters of the prospect theory estimated using Brazilian subjects, we can now calculate the Brazilian asset allocation that would be in accordance with prospect theory. In order to calculate the expected utility, we apply a similar procedure as Benartzi and Thaler (1995).

For our calculation of Brazil's asset allocation, we use two types of data, stock market data and fixed income data. In order to measure the Brazilian stock market, we employ two different indices of nominal returns, IBX and Ibovespa. IBX data are available since 1996, while Ibovespa data are available since 1995. As reference of the fixed income market, we used the CDI index, also measured in nominal returns and available since 1986. Because of missing information on fees for administrating funds, we estimate our model twice, once with a proxy of an annual administration fee and once without this proxy. In the case of stock funds we assume an annual fee of 3,5% and for fixed income funds a fee of 1%.

We calculate the portfolio that maximizes utility for specific evaluation periods that can vary from 1 month up to 24 months. For each evaluation period we calculate nominal average returns and standard deviations for the stock market and the fixed income market, using the above mentioned indices. As one month has 21 working days, we use mobile windows of multiplies of 21 days in order to calculate the average return and its standard deviation. For instance, when calculating our portfolio with an evaluation period of 4 months, we apply a mobile window of 84 days.

Once we have determined the average return and its standard deviation of each series for each time horizon, we calculate returns and standard deviations for all portfolio options. First, we calculate a portfolio, in which 100% of its assets are allocated into stocks. Then we calculate a portfolio, in which 99% of its assets are allocated into stocks and 1% into the fixed income. We continue varying the ratio by one percentile point, until we calculate the portfolio, in which 100% of its assets are allocated into fixed income.

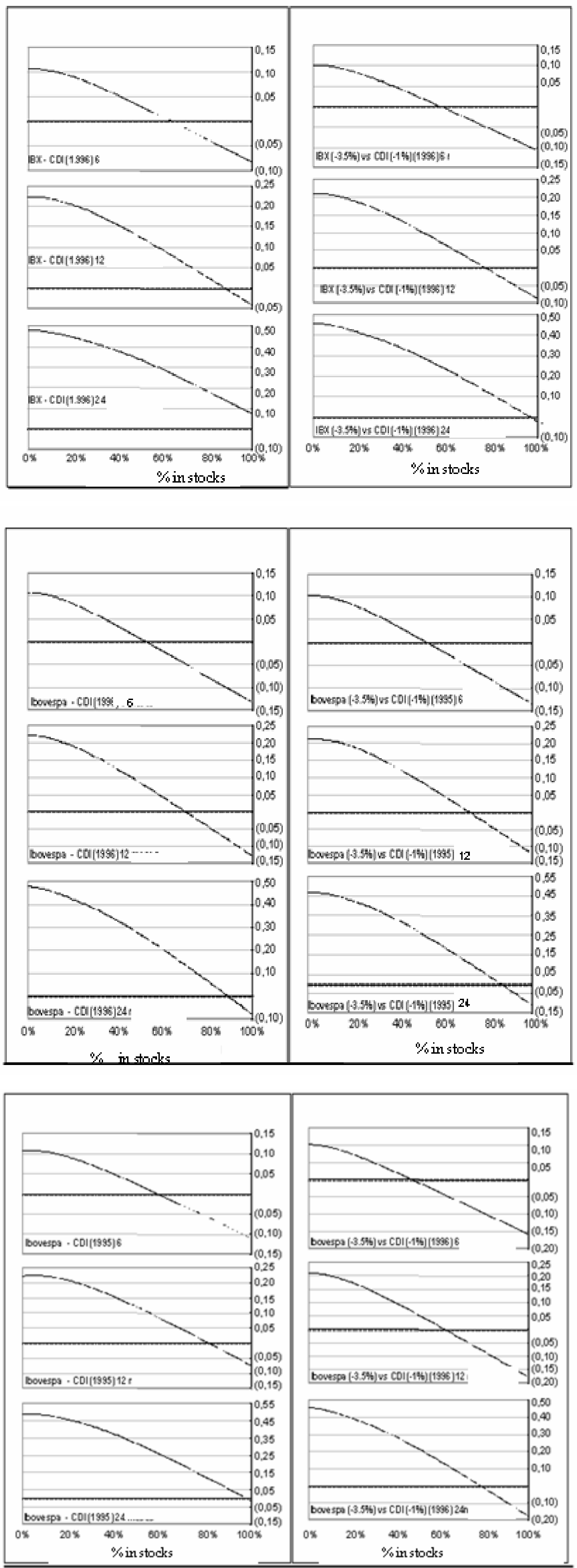
In order to calculate the utility of each portfolio, we use the PWF and VF functions and we assume that returns follow a normal distribution. We calculate the returns with 0% of probability of occurrence up to 100% of occurrence probability, with steps of one percentile point. Then we calculate the expected utility of the investment, as shown in section 3.

In Graph 10 we show how utility varies in function of the portfolio's allocation into stocks. Utility curves were calculated for evaluation periods of 6, 12 and 24 months. All curves show the same general picture. Utility maximizes with portfolios that exhibit low participation of stocks in its composition. Actually, we cannot detect a single maximum, but utility is maximized in an entire region. In this region, the utility function is practically parallel with the x-axis, indicating that utility is maximized with an allocation into stocks anywhere between 0% and 8%. Our calculations with Brazilian data show that, for any index and for any evaluation period, utility is maximized with less than 8% of investment into shares. Comparing this result with Brazilian actual investment share into stocks of around 3%, we can conclude that the Brazilian market is compatible with results predicted by prospect theory and the theory.

One question that remains is the determination of the average evaluation period for Brazil. In Table 7 we present the stock market participation that maximizes utility for all evaluation periods (values that have been already presented in Graph 10). We can observe in the table that in no circumstance utility is maximized with more than 5% of investment into the stock market (see for instance the composed portfolio for Ibovespa and CDI calculated since 1995 and with evaluation periods between 12 and 17 months). Finally, in the last column of Table 7 we present the average allocation that maximizes utility calculated with different indices. Here we can identify portfolios that are similar to the one observed in Brazilian market, pointing at an average evaluation period between 9 to 17 months.

Our analysis leads us to conclude that according to prospect theory Brazilian portfolios that maximize utility allocate up to 8% of its assets into stocks. Furthermore, the estimated average evaluation period in the Brazilian market lies between 9 and 17 months. Our estimates of the evaluation period are similar to Benartzi and Thaler’s results in 1995. However, Brazilian portfolios that maximize utility exhibit a much smaller participation of stocks (between 0% and 8%) than portfolios that maximize utility in the United States (between 30% and 55%).

Graph 10 Utility of the portfolios



Source: Own elaboration

Table 7 Allocations that they maximize the utility

	Data since 1996				Data since 1995		
Horizon	IBX vs CDI	Ibov. vs CDI	IBX vs CDI (1)	Ibov. vs CDI (1)	Ibov. vs CDI (1)	Ibov. vs CDI (1)	Median
1	0%	0%	0%	0%	1%	0%	0%
2	1%	0%	0%	0%	1%	1%	1%
3	1%	1%	1%	0%	2%	1%	1%
4	1%	1%	1%	0%	2%	2%	1%
5	2%	1%	1%	1%	3%	2%	2%
6	2%	1%	1%	1%	3%	2%	2%
7	2%	1%	1%	1%	4%	2%	2%
8	2%	1%	2%	1%	4%	3%	2%
9	3%	1%	2%	1%	4%	3%	3%
10	3%	1%	2%	1%	4%	3%	3%
11	3%	2%	2%	1%	4%	3%	3%
12	3%	2%	2%	1%	5%	3%	3%
13	3%	2%	2%	1%	5%	3%	3%
14	3%	2%	2%	1%	5%	3%	3%
15	3%	2%	2%	1%	5%	3%	3%
16	3%	2%	2%	1%	5%	3%	3%
17	3%	1%	2%	0%	5%	3%	3%
18	3%	1%	1%	0%	4%	3%	2%
19	2%	1%	0%	0%	4%	3%	2%
20	2%	0%	0%	0%	4%	2%	1%
21	2%	0%	0%	0%	4%	2%	1%
22	1%	0%	0%	0%	3%	2%	1%
23	1%	0%	0%	0%	3%	1%	1%
24	1%	0%	0%	0%	3%	1%	1%

(1)(Stocks: -3,5%. Money Market -1%)

Source: For series historical: System Smart Investor. For coefficients of the theory of prospect: experiment of the author

6. Conclusions

In this paper we analyzed the Brazilian investment allocation in the stock market. Only approximately 3% of the investments are destined to this market, while the remaining 97% are destined to the less risky fixed income assets. In the United States, for example, this distribution is almost 50-50. Taking into account that stocks market returns are greater in the long run, Mehra and Prescott pointed out the existence of an Equity Premium Puzzle in their pioneering work of 1985. Several authors attempted to explain this puzzle for other developed countries. Benartzi and Thaler obtained good results through a behavioral finance approach, based on the prospect theory of Kahneman and Tversky. Benartzi and Thaler's conclusion was that the investment distribution in the United States can be explained by investor's behavior: investors suffer from myopic loss aversion. In this study we use the same approach to analyze the Brazilian asset allocation.

Using Benartzi and Thaler approach, the first question that needs to be answered is whether there exits an Equity Premium in Brazil or not. Several authors found premiums varying from 10 to 14 percentile points. At the same time, these studies point out a great standard deviation, so that we can not confirm the existence of a Brazilian equity premium. However, even without knowning whether there is or not an Equity Premium, or even an Equity Premium Puzzle, Benartzi and Thaler's model can be applied to evaluate the Brazilian asset allocation.

The methodology of Benartzi and Thaler utilizes in the calculation parameters of risk aversion and loss aversion. Benartzi and Thaler use values for these parameters obtained by an experimental study, undertaken by Kahneman and Tversky in 1992 with graduate students of the Universities of Berkeley and Standford. In the present work, we replicate this study with Brazilian participants. We opted for the replication in order to allow the comparison of our results with those obtained by Kahneman and Tversky and by Benartzi and Thaler. Our estimated parameters are very similar to those found in the experiment in the United States. While Kahneman and Tversky find for the parameters $\alpha, \beta, \lambda, \gamma$ and δ the values of 0,88, 0,88, 0,61, 0,69 and 2,25, respectively, we find for the same parameters the following values: 0,95, 0,98, 0,62, 0,66 and 2,21. One important result of our study is that parameters for risk and losses aversion, found for Brazilian subjects, are very similar to those parameters, found by the original study in the USA. Brazilian subjects seem not to show a significantly different risk behavior than the subjects in the experiments run in the US.

Using these parameters, we calculate portfolios that maximize utility for evaluation periods between 1 and 24 months. While Benartzi e Thaler find an evaluation period of 12 months in the United States, with an optimal portfolio investing between 35 and 50% in stocks, our calculations show that the Brazilian evaluation horizon can vary between 9 to 17 months, implying an optimal portfolio that invests between 0% and 8% into stocks. As the observed portfolio allocation in Brazil is approximately 2.7% (for private investors) into stocks, our study leads us to conclude that observed investment in the Brazilian stock market is in accordance with prospect theory and the phenomenon that investors are myopic loss averse.

Finally, given the similarity of the Brazilian and Kahneman & Tversky's parameters for risk and loss aversion, we can conclude that the difference in asset allocation between USA and Brazil is not motivated by behavioral issues. Our conclusion is that the significant difference between the stock market allocations in Brazil and the United States is caused by the risk–return performance of assets. This would suggest that if the Brazilian interest rates (the basic rate – Selic) drop, and the extraordinary high return on relative save fixed income drops, investors can be expected to allocate a larger proportion of their investments to the Brazilian stock market.

7. References

Abel. B., Mankiw, N.G., Summers L.H. and Zeckhauser R.J. (1989). "Assessing Dynamic Efficiency: Theory and Evidence ". The Review of Economic Studies 56(1), 1-19.

Auerbach. J. and Kotlikoff, L.J. (1988). "Fiscal Dynamic Policy ". The Economic Journal. 98(392), 873-874.

Baucells, M. and Heukamp, F.H. (2002), "Reevaluation of the results of Levy and Levy ". Organizational Behavior and Human Decision Process, 94 15-21.

Benartzi, S. and Thaler, R. (1995). "Myopic Loss Aversion and the Equity Premium Puzzle. "Quarterly Journal of Economics, 110(1), 73-92.

Bonomo, Landmark and Gabriela Bertol D. (2002) "the Puzzles Inverted in the Market Brazilian of Action ". In Bonomo Landmark (organizador)"Finances Applied to Brazil "(pp. 105 - 117). Rio De Janeiro, Brazil. Publishing company FGV.

Brock, W. (1979). "An Integration of Stochastic Growth Theory and the Theory of Finance, Part 1: The Growth Model."In J.Green and J. Scheinkman (eds.), General Equilibrium, Growth & Trade New York, U.S.A.: Publishing company Academic Press.

Constantinides, G.M., & Duffie, D. (1996)."Asset Pricing with Heterogeneous Consumers ". Journal of Political Economy, 104(2), 219-240.

Cox, J., Ingersoll, J.E. and Ross, S.. (1985). "the Theory of the Term Structure of Interests Rates." Econometrica, 53(2), 385-407.

Cysne, Rubens P. (2005) "Equity-Premium Puzzle: Evidence From Brazilian Dates ". Work Presented to 23º National Meeting of Economy, Christmas, 2005. Not published

Davis, Douglas D. and Charles. Holt, Experimental Economics. Princeton, U.S.A.. Publishing company: Princeton University Press, 1992.

DeJong, Douglas D., Robert Forsythe and Wilfred C. Uecker (1988). " The Use notices on of Businessmen the Subjects in Sealed Offer Markets ", Journal of Economic Behavior and Organization, 9, 240 - 259.

Donaldson, J.B., and Mehra, R. (1984). "Comparative Dynamics of an Equilibrium Intertemporal Asset Pricing Model ". Review of Economic Studies 51(3), 491-508.

Dyer, Douglas, John Kagel and Dan Levin (1989). "the Comparison of Naive and Experienced Bidders in Common Value Offer Auctions: The Laboratory Analysis ". Economic Journal, 99, 108-115.

Gonzalez, R. and Wu, G. (1999). "On the Shape of the Probability Weighting Function ". Cognitive Psychology, 38 129-166.

Iglesias, M.(2006). “O comportamento do investidor brasileiro na alocação de ativos”. Dissertação de mestrado. Fundação Getulio Vargas – São Paulo.

Kahneman, D., and Tversky. (1979). "Prospect Theory: Analysis of Decision to under Risk.” Ecometrica, 47(2), pag 263-292.

Kahneman, D. and Tversky. (1984). "Choices, Values and Frames". In Kahneman, D., & Tversky. (eds.), "Choices, Values and Frames", (pp. 67-92). New York, U.S.A.: Publishing company: Russell Sage Foundation, Cambridge University Press.

Fisher, K. and Statman, M (1999). "the Behavioral Framework will be Teams Diversification ", Financial Analysts Journal 55(3), 88-97.

Levy, H. and Levy M. (2002). "Experimental test of the prospect theory: The Stocastic dominance approach ". Behavior organization and Human decision Process, 89, 1058-1081.

Markovitz, Harry M. (1952). "Portfolio selection". The Journal of Finance, 7, 77-91.

McGrattan, E.R. and Prescott, E.C. (2000). "Is the Market Overvalued?”. Federal of Minneapolis Quarterly Review Reserves Bank, 24, 20-40.

Mehra, R. and Prescott E.C. (1985). "The Equity Premium: Puzzle.” Journal of Monetary Economics, 15, 145-161.

Mehra, R. and Prescott, E.C. (2003)."The Equity Premium in Retrospect ". Handbook of the economics of finance Capitulo 14, 888-935.

Mestelman, Stuart and D.H. Feeny (1988). "You donate Ideology Matter?" Experimental Anecdotal Evidence on the Voluntary Provision of Public Goods ", Public Choice, 57, 281 - 286.

Morgenstern, Oscar. and von Neumann, John.. Theory of Games and Economic Behavior. Princeton, U.S.A.. Publishing company: Princeton University Press, 1980. 648 p.

Prelec, D. (2000) "Compound Invariant Weighting Functions in Prospect Theory”. In Kahneman, D., and Tversky. (eds.), Choices, Values and Frames, (pp 67-92). New York, U.S.A.: Publishing company: Russell Sage Foundation, Cambridge University Press.

Sampaio, Frederico S. (2002) "Exists Equity Premium Puzzle in Brazil?”. In Bonomo Landmark (organizador)"Finances Applied to Brazil" (pp. 86 - 104). Rio de Janeiro, Brasil. Editora FGV.

Schor, Adriana, Bonomo, Marco A. and Valls, Pedro L. P. (1998) “Arbitrage Pricing Theory (APT) e Variáveis Macroeconomicas. Um estudo empírico sobre o mercado acionário brasileiro. Não publicado.

Shiller, Robert J. Irrational Exuberance. Princeton, EUA. Editora: Princeton University Press, 2001. 312p.

Smith, Vernon L., Gerry L. Suchanek and Arlington W. Williams (1988). “Bubbles, Crashes, and Endogenous Expectations in Experimental Spot Asset Markets”. Econometrica, 56, 1119-1151.

Tversky, A and Kahneman, D. (1992). “Advances In prospect Theory: Cumulative Representation of Uncertainty”, Journal of Risk and Uncertainty , 5(4), 297-323.

Thaler, R., Tversky A., Kahneman, D. and Schwartz, A. (1997). “The effect of myopia and loss aversion on risk taking: An experimental test”, Quarterly Journal of Economics, 112(2), 647-661.

Wakker, P. (2003), "The Data of Levy and Levy (2002), "Prospect Theory: Much Ado about Nothing?". Support Prospect Theory" Management Science Volume 49(7), 979-981.