Great expectations: Prospect theory with a consistent reference point

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Abstract

I apply a prospect theory model of risk preferences with an endogenously determined reference point to propose an alternative objective of maximizing expected outcome rather than maximizing expected utility. I show that an agent can always form a consistent expected outcome for any binary gamble and derive a parametric formula, which can then be used to examine the effects of loss aversion, risk aversion, and probability weighting on behavior. To illustrate the applicability of the results, I use this model to consider the incentives of an agent purchasing insurance against the possibility of a loss and show that it is optimal for him to either purchase full insurance or purchase no insurance.

JEL Classification D03, D81, D84

Key words: Prospect theory, Endogenous reference point, Consistent expectations, Insurance, Loss aversion

1. Introduction

Prospect theory (Kahneman and Tversky, 1979; Tversky and Kahneman, 1992) applies psychological principles to incorporate several important and frequently observed behavioral tendencies into the neoclassical expected utility model of preferences. This formulation helps resolve several apparent paradoxes and provides a useful descriptive model of choice under risk and uncertainty (Camerer, 2000). The model seeks to accommodate five important behavioral concerns that have been observed empirically:

- **Reference Dependence:** Outcomes are evaluated as deviations relative to some reference point rather than absolute levels. This reference point is therefore critical to determining how the agent assesses an outcome. Reference points are flexible and could be based on any number of judgments, such as the status quo (Kahneman and Tversky, 1979), an aspiration level (Siegel, 1957; Tversky and Kahneman, 1991), or past observations (Baucells, Weber, and Welfens, 2011). A prospect theory model with a flexible reference point has been used to help explain behavior by real-world agents facing risky decisions involving large monetary outcomes (see e.g. Post et al., 2008).

  Koszegi and Rabin (2006, 2007) argue that expectations about the future form the most natural reference point for valuing realized outcomes. According to this model, an agent makes two related judgements when facing a risky prospect: He determines a reference point that will allow him to evaluate the realized outcome. He also assesses his prospects relative to this reference point and forms an expectation about his outcome. For an agent who uses expectations as his reference point, this implies that reference point formation is an inherently endogenous process. Koszegi (2010) argues that, at least in a ex ante sense, the agent should then seek to maximize this anticipated outcome. Abeler et al. (2011) find support for the hypothesis that subjects use expectations as a reference point in labor supply decisions, and Crawford and Meng (2011) use this model to explain New York City cabdrivers' labor supply behavior.

- **Loss Aversion:** Outcomes that fall below the reference point ("losses") are felt more intensely than equivalent outcomes above the reference point ("gains"). This means that the value function is steeper for losses than for gains, which can be parameterized by applying a scaling factor $\lambda \geq 1$ to the portion of the value function below the reference point. The larger λ is, the more averse the agent is to (and the more heavily he negatively weights) outcomes which fall below his expectation.

- **Risk Aversion in Gains, Risk Seeking in Losses:** While the agent displays a general preference for an assured moderate-sized outcome over 50-50 chance of a large gain or zero gain, he prefers a 50-50 chance of taking a large loss or avoiding the loss altogether over an assured moderate-sized loss. This so-called reflection hypothesis is modeled by a concave value function in the domain above the reference point and a convex value function in the domain below the reference point and has been debated empirically. Budescu and Weiss (1987) and Baucells and Villasis (2010) find support for the reflection effect while Hershey and Schoemaker (1980) conduct experiments that provide evidence to the contrary.

- **Diminishing Sensitivity to Gains and Losses:** The marginal effect of changes in the outcome are smaller as the agent moves further away from the reference point. For example, if the reference point is \$0, the agent will experience a stronger effect when moving from \$100 to \$200 than when moving from \$1,100 to \$1,200. An intuitive, simple, and frequently utilized value function that captures this effect is a power function (Wakker, 2008) with parameters $\alpha \in (0,1)$ and $\beta \in (0,1)$ in the domain of gains and losses, respectively.

- **Probability Weighting:** Agents tend to overweight small probabilities and underweight probabilities close to 1. This effect can be modeled with an S-shaped probability weighting function, such as Goldstein and Einhorn's (1987) specification $w(p) = \frac{\delta p^\gamma}{\delta p^\gamma + (1-p)^\gamma}$, where δ and $\gamma \in (0,1)$ parameterize the shape of the probability weighting. Modern prospect theory (Tversky and Kahneman, 1992) allows for separate weighting functions in the domain of gains and losses while imposing additional restrictions to ensure that the total weights sum to 1.

This paper applies Shalev (2000) and Koszegi and Rabin's (2006, 2007, 2010) model of expectations as an endogenously determined reference point to propose a tractable objective of maximizing expected outcome EO (as an alternative to the classical objective of maximizing expected utility) under a prospect theory model of risk preferences. I show that an agent can always form this unique consistent expectation about his prospects and derive its functional form for binary prospects. Consequently, the model provides a parametric method to analyze incentives in the presence of risk and can be used to predict how changes in different risk attitudes affect behavior.

The rest of the paper is organized as follows: Section 2 presents the model and the main results. Section 3 demonstrates the applicability of this model by considering an agent making an insurance decision. Section 4 concludes and the appendix contains proofs of all results.

2. The Model

I consider an agent who faces a risky future prospect with n possible outcomes $x_1 \leq \cdots \leq x_n$, which could each occur with respective probabilities $p_1, \ldots, p_n$, with $\sum_{i=1}^{n} p_i = 1$. Before the uncertainty is resolved, the agent forms an expectation about his outcome X, and this expected outcome, which I will denote as EO, serves as a reference point for him to evaluate the outcome that he ultimately receives. When his outcome exceeds this expectation, he feels an additional gain equal to $(X - EO)^{\alpha}$. When his outcome falls below his expectation, he perceives this as an added loss equal to $-\lambda(EO - X)^{\beta}$.

Numerous studies attempt to estimate the parameters λ, α, and β for the general population by analyzing subjects' choices over risky gambles. Tversky and Kahneman (1992) estimate $\lambda = 2.25$, $\alpha = \beta = 0.88$, Tu (2005) finds $\lambda = 3.18$, $\alpha = 0.68$, and $\beta = 0.74$, and Abdellaoui, Bleichrodt, and Paraschiv (2007) find $\lambda = 2.54$, $\alpha = 0.72$, and $\beta = 0.73$. Booij et al. (2010) estimate $\lambda = 1.58$, $\alpha = 0.86$, and $\beta = 0.83$, and cannot reject that α and β are equal. Based on these results, I assume that $\alpha = \beta$, which simplifies the model and allows for a broader spectrum of definitions of the loss aversion parameter λ (see e.g. Kobberling and Wakker, 2005).

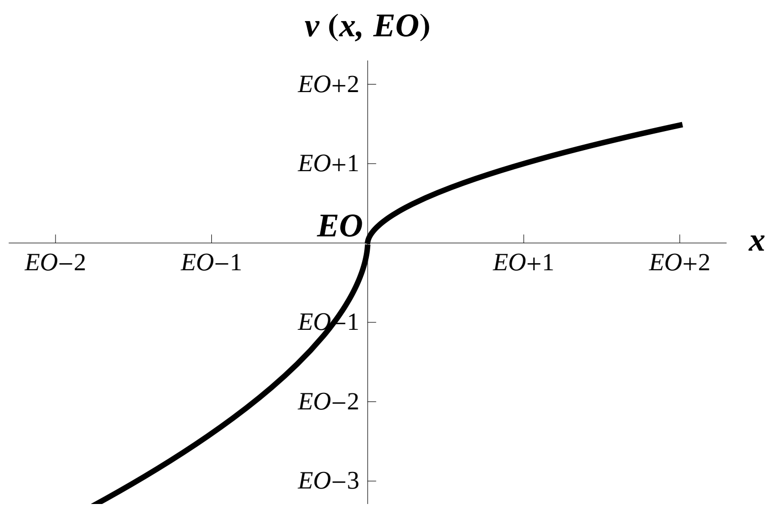

Figure 1: **A prospect theory type reference-dependent utility function $v(x, EO)$ displaying loss aversion with coefficient $\lambda > 1$, risk aversion in gains and risk seeking in losses.**

Specifically, I use a prospect theory type generalization of Shalev's (2000) utility function which incorporates all of the key features of cumulative prospect theory discussed earlier:

$$v(x, EO) = \begin{cases} EO - \lambda(EO - x)^{\alpha} & \text{if } x < EO \\ EO + (x - EO)^{\alpha} & \text{if } x \geq EO, \end{cases}$$

where $\lambda \geq 1$ represents the coefficient of loss aversion as before, and $\alpha \in (0, 1]$ determines the curvature of the utility function. Figure 1 depicts the shape of this utility function. For a fixed reference point EO, $v(x, EO)$ displays constant Arrow-Pratt relative risk aversion (CRRA) of $1 - \alpha$ in gains and $-1-\alpha$ (risk seeking) in losses. The power family of utility functions exhibits a diminishing sensitivity to absolute gains and losses, retains its properties under positive linear transformation of the payoffs, and is widely used for estimating risk attitudes from real-world data (Wakker, 2008). If $\alpha = \lambda = 1$ this formulation will revert back to a risk-neutral utility function equal to expected payoff.

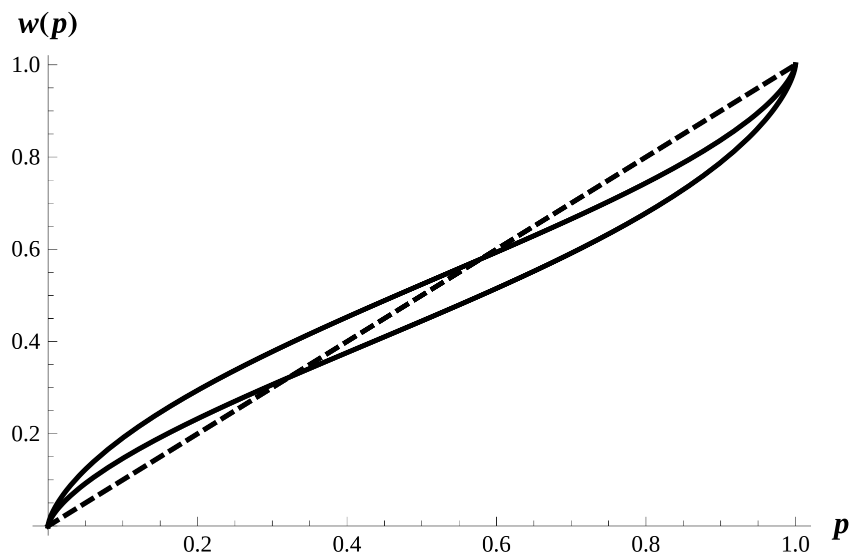

Figure 2: **Probability weighting functions $w^{-}(p)$ (the upper solid curve) and $w^{+}(p)$ (the lower solid curve) versus the true probability p (the dashed line) for $\delta^{+} = 0.8$, $\gamma^{+} = 0.7$, $\delta^{-} = 1.1$ and $\gamma^{-} = 0.7$.**

I also assume that the expert applies probability weighting functions $w^{+}(p)$ and $w^{-}(p)$ for scores above and below EO (positive and negative events), respectively. $w^+(\cdot)$ and $w^-(\cdot)$ are assumed to be strictly increasing and satisfy $w^+(0) = w^-(0) = 0$ and $w^+(1) = w^-(1) = 1$. For $w(p) = \frac{\delta p^\gamma}{\delta p^\gamma + (1-p)^\gamma}$, Abdellaoui (2000) estimates $\delta^+ = 0.65$, $\gamma^+ = 0.60$, $\delta^- = 0.84$ and $\gamma^- = 0.65$, while Abdellaoui, Vossmann, and Weber (2005) find $\delta^+ = 0.98$, $\gamma^+ = 0.83$, $\delta^- = 1.35$ and $\gamma^- = 0.84$ and Booij et al. (2010) estimate $\delta^+ = 0.77$, $\gamma^+ = 0.62$, $\delta^- = 1.02$ and $\gamma^- = 0.59$. Figure 2 displays the shape of the weighting function for a rough average of these parameter estimates.

The agent's expected-valuation function when he uses EO as a reference point for this prospect is

$$V(EO) \equiv \sum_{i=1}^{n} \pi_i(EO) v(x_i, EO)$$

where decision weights are given by

$$\pi_i(EO) \equiv w^+(\sum_{j=i}^{n} p_j) - w^+(\sum_{j=i+1}^{n} p_j) \text{ for all } i \text{ such that } x_i > EO,$$

$$\text{and } \pi_i(EO) \equiv w^-(\sum_{k=1}^{i} p_k) - w^-(\sum_{k=1}^{i-1} p_k) \text{ for all } i \text{ such that } x_i \leq EO,$$

as in cumulative prospect theory (Tversky and Kahneman, 1992). Of particular interest is $EO \in [x_1, x_n)$, in which case $\pi_1(EO) = w^-(p_1)$ and $\pi_n(EO) = w^+(p_n)$.

Definition 1 (Consistent Expectations). *Following the definition of personal equilibrium presented in Koszegi (2010), I say that the reference point EO is* **consistent** *with the agent's expectations if the expected-valuation of the prospect is equal to the reference point he is using. Mathematically, this requires that $V(EO) = EO$.*

A reference point that satisfies this consistency equation is the natural expectation formed by an agent who carefully deliberates over his future prospects. If he initially forms a reference point R that is higher than EO, then upon further introspection he will find that his potential future losses $-\lambda(R-x)^\alpha$ outweigh his potential future gains $(x-R)^\alpha$. As a result, he will adjust his expectation about the future downwards, in anticipation of these future feelings, which in expectation are negative. Likewise, if he initially forms a reference point that is lower than R, then upon contemplating his

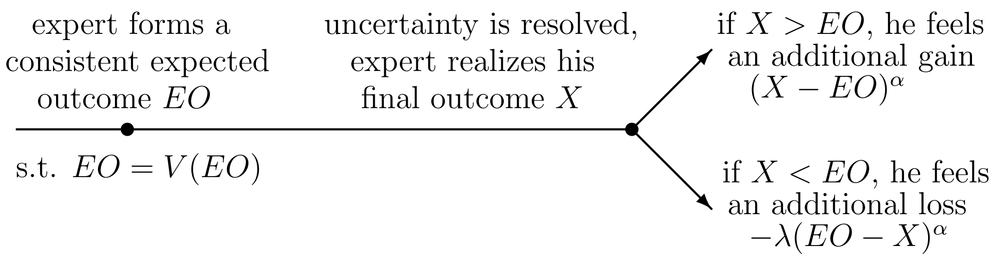

Figure 3: Timeline of the agent's *ex ante* reference point formation and *ex post* evaluation of his realized outcome.

prospects he will find that his expected future gains outweigh his expected future losses and adjust his expectation upwards. A thoughtful agent will thus continue to adjust his expectation closer and closer towards EO. A reference point EO that satisfies the consistency equation is stable because, in the agent's *ex ante* assessment, his potential future losses $\lambda(EO - x)^\alpha$ are perfectly balanced with his potential future gains $(x - EO)^\alpha$.

An agent choosing between multiple risky prospects should select the one that provides the highest *ex ante* expected outcome EO. In particular, the consistent expected outcome associated with a certain payoff of x, which is equivalent to a risky prospect that pays x with probability 1 and 0 with probability 0, is simply $EO = x$. As a corollary, the agent will then be indifferent between taking a gamble with expected outcome EO and receiving a payment of EO with certainty. EO is therefore exactly analogous to a certainty equivalent in classical expected utility theory, and as such, can be associated with a corresponding risk premium.

Definition 2 (Consistent Risk Premium)**.** *The risk premium RP_{EO} is the difference between the mathematical expected payoff of the gamble and the consistent expected outcome, $RP_{EO} = \sum_{i=1}^{n} p_i x_i - EO$.*

In other words, RP_{EO} is the most the agent would be willing to pay to remove the risk from the gamble and receive a certain payment equal to the average payoff of the gamble instead. RP_{EO} is typically positive and largely driven by the agent's degree of loss aversion λ, but can be negative as well. For example, when p is very small and λ is not significantly greater than 1, the overweighting of small probabilities can lead the agent to value the gamble higher than its average payoff.

2.1. Binary Prospects

The mechanics and implications of this model become much clearer in the case of a binary prospect. Consider an example of consistent reference point formation for an agent evaluating a risky prospect that pays \$0 with probability p and \$100 with probability $1-p$. Suppose that $w(p) = p$, $\alpha = 1$, and $\lambda = 2$. The agent's consistent expected outcome will be the reference point $EO \in [0, 100]$ such that $EO = p \cdot (EO - 2(EO - 0)^1) + (1-p) \cdot (EO + (100 - EO)^1)$. Then $EO(p) = \frac{100(1-p)}{1+p}$, as shown in Figure 4. Note that when $p = 0$, the agent's consistent expected outcome is \$100, since this is the amount that he will receive with certainty. As p increases, the consistent expected outcome gradually falls to \$0, which is the amount he will receive with certainty when $p = 1$. Also note that the units of EO are always the same as the units of the outcomes of the risky prospect, which in this case are dollars.

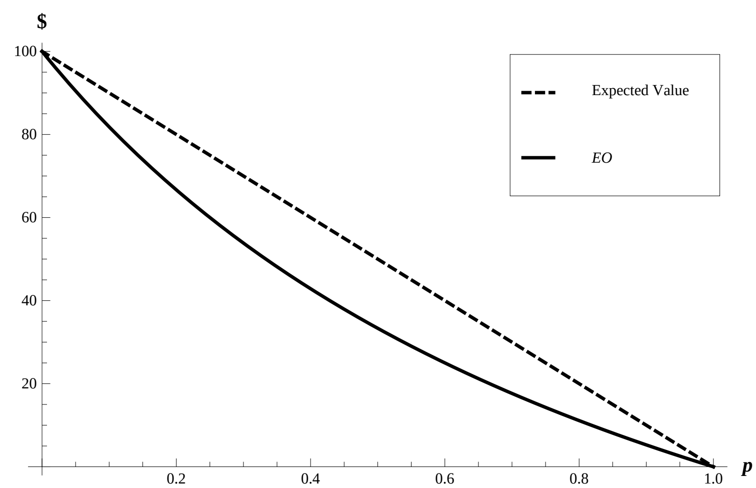

Figure 4: **The consistent expected outcome EO corresponding to a risky prospect that pays \$0 with probability p and \$100 with probability $1-p$ for various values of p when $w(p) = p$, $\alpha = 1$, and $\lambda = 2$.**

The consistent risk premium for this prospect is $RP_{EO} = 0 \cdot p + 100 \cdot (1-p) - EO = \frac{100(1-p)p}{1+p}$, which is the difference between the dashed line and the solid curve in Figure 4.

Lemma 1. *For any risky prospect that yields a payoff of A with probability* *p and B with probability $1-p$, where $A \leq B$, there exists a unique consistent expected outcome $EO \in [A, B]$ such that $V(EO) = EO$.*

Lemma 1 ensures that given any fixed risky prospect, the agent can always form this consistent *ex ante* expectation about her prospect. For prospects with more than two possible outcomes, EO must be solved for numerically and does not have a simple parametric form. However, for binary prospects, EO can be expressed as a closed-form parametric function.

Definition 3 (Coherent Probability Weighting)**.** *The weighting functions are said to be* **coherent** *if $w^-(p) + w^+(1-p) = 1$ for all $p \in [0, 1]$.*

Coherence holds trivially for the unweighted case $w(p) = p$ and approximately (with the sum generally between 0.96 and 1.00 for all values of p) for estimates of S-shaped weighting functions that overweight low probabilities and underweight high probabilities. While coherence is not critical to the results that follow, it is required to obtain closed-form parametric solutions rather than nonlinear equations that must be solved numerically. Coherence is therefore quite important for analyzing the solutions to the agent's insurance problem, and I will assume that it holds for binary prospects from here forward.

Proposition 1. *If the expert's weighting functions are coherent and he faces a binary prospect that yields a payoff of A with probability p and B with probability $1-p$, where $A \leq B$, then $EO = \frac{B + (\frac{\lambda w^-(p)}{w^+(1-p)})^{\frac{1}{\alpha}} A}{1 + (\frac{\lambda w^-(p)}{w^+(1-p)})^{\frac{1}{\alpha}}}$.*

Note that preferences are preserved under positive linear transformation of the payoffs. This means, for example, that if the payoffs were converted to a different currency, or if a constant amount were added to each payoff, the expression for EO would be adjusted according to the same linear transformation. In addition, in the cases where there is no risk, $EO = A$ for $p = 1$ and $EO = B$ for $p = 0$ as expected. In these situations, the agent knows for sure what his final outcome will be and his risk attitudes are not relevant. For risky cases $p \in (0, 1)$, a sensitivity analysis can easily be performed on EO by taking partial derivatives with respect to each of the parameters and with respect to A, B, and p. For example, for any fixed gamble, weighting functions, and risk attitude α, $EO \to A$ as $\lambda \to \infty$. In other words, as the agent becomes more and more loss-averse, he tends to focus only on the worst possible outcome A.

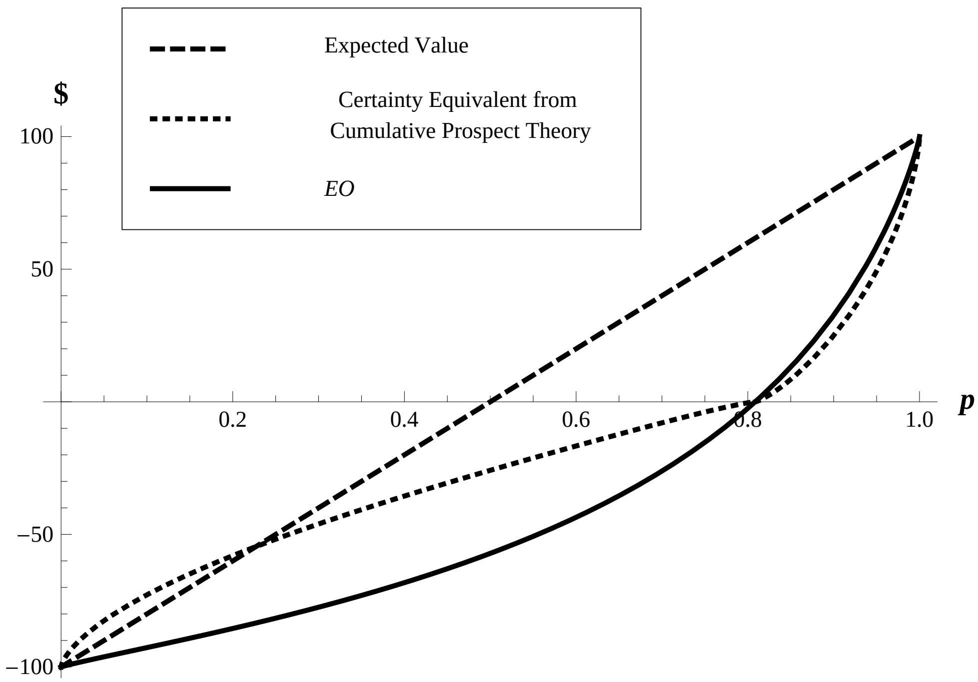

Figure 5: **The consistent expected outcome EO and certainty equivalents corresponding to a risky prospect that pays \$100 with probability p and $-\$100$ with probability $1-p$ for various values of p when $\lambda = 2.4$, $\alpha = 0.8$, $\delta^{+} = 0.8$, $\gamma^{+} = 0.7$, $\delta^{-} = 1.1$ and $\gamma^{-} = 0.7$.**

As another example, consider an agent evaluating a risky prospect in which he wins \$100 with probability p and loses \$100 with probability $1-p$, whose risk attitudes are characterized by parameters equal to the population average estimates of $\lambda = 2.4$, $\alpha = 0.8$, $\delta^+ = 0.8$, $\gamma^+ = 0.7$, $\delta^- = 1.1$ and $\gamma^- = 0.7$. Figure 5 displays the shape of his consistent expected outcome versus the risk-neutral expected value of the gamble for different values of $p \in [0,1]$. Observe that his *ex ante* assessment EO of this gamble remains negative until the positive event (winning \$100) becomes more than four times as likely as the negative event (losing \$100). This seemingly extreme aversion to risk is in fact driven by his high level of loss aversion. For agents with a smaller loss aversion parameter λ, EO follows more closely to the expected value of the gamble.

Figure 5 also shows a graph of the certainty equivalents of the gamble under the classical value function of Cumulative Prospect Theory (Tversky and Kahneman, 1992) with a reference point of 0. These certainty equivalents, which were calculated by finding the payoff x such that the value function of the gamble yielded the same level as the value function applied to a gamble yielding x with certainty, map the agent's evaluation of the gamble back to the payoff space and provide a way to make direct comparisons between EO and the predictions of classical Cumulative Prospect Theory. EO and the certainty equivalent coincide at three points: when the gamble yields either \$100 or $-$\$100 with certainty, in which case both EO and the certainty equivalent equal \$100 and $-$\$100, respectively, and when $p = 0.809$, when both EO and the certainty equivalent of the gamble equal \$0. For gambles that are viewed positively ($0.809 < p \leq 1$), the graphs of EO and the certainty equivalent track fairly closely to one another. However, the graph of EO remains much closer to the worst-case outcome of $-$\$100 for small values of p, while the certainty equivalents rise more quickly towards \$0. For example, for $p = 0.15$, the certainty equivalent of the classical Cumulative Prospect Theory value function equals $-$\$64.88, higher than even the expected value of the gamble $-$\$70, and much higher than the consistent expected outcome $EO = -\$89.15$.

3. Example: An Application To Insurance

Consider an agent with initial wealth W who faces a loss of size $L \in (0, W)$ which could occur with probability p. Before this uncertainty is resolved, he has the option to buy up to L units of insurance at a price of $q \in [p, 1]$ per

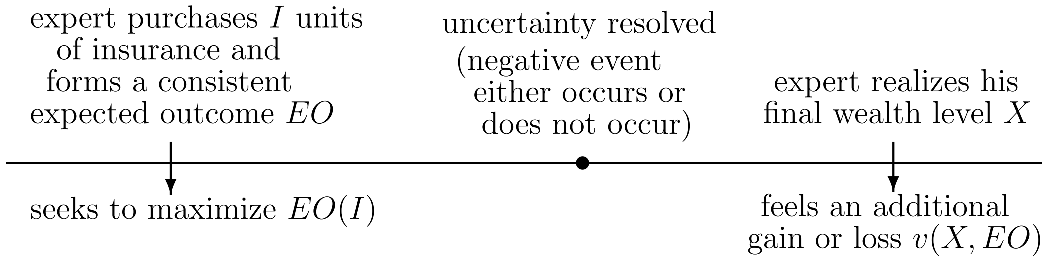

Figure 6: Timeline of the agent's insurance level decision, reference point formation, and ex post evaluation of the event.

unit. Let $I \in [0, L-D]$ denote the number of units of insurance that the agents decides to purchase, where $D \in [0, L)$ is the deductible amount. In the event that the loss occurs, each unit of insurance pays back $\rho \in [0,1]$ units of reimbursement to the agent, so that his final wealth level is $W - qI$ with probability $1-p$ (if the negative event does not occur) and his ultimate wealth level is $W - qI - L + \rho I$ with probability p (if the negative event does occur). The expert seeks to maximize his expected outcome $EO(I)$ over $I \in [0, L-D]$.

Proposition 2. *For a given probability p of the negative event occurring, it is optimal to not purchase any insurance ($I = 0$) if $\frac{\rho}{q} \leq 1 + (\frac{\lambda w^-(p)}{w^+(1-p)})^{\frac{1}{\alpha}}$, in which case $EO = W - \frac{L}{1+(\frac{\lambda w^-(p)}{w^+(1-p)})^{\frac{1}{\alpha}}}$. If $\frac{\rho}{q} \geq 1 + (\frac{\lambda w^-(p)}{w^+(1-p)})^{\frac{1}{\alpha}}$, the agent will find it optimal to fully insure ($I = L - D$) and $EO = W - \frac{L}{1+(\frac{\lambda w^-(p)}{w^+(1-p)})^{\frac{1}{\alpha}}} + \left(\frac{\rho}{1+(\frac{\lambda w^-(p)}{w^+(1-p)})^{\frac{1}{\alpha}}} - q\right)(L-D)$.*

This result has two key implications. First, observe that the insurance decision is independent of the initial wealth level W. This happens because that initial wealth level factors into the expected outcome regardless of the agent's insurance decision, which is made predominantly with the goal of balancing a pre-event expectation against the possible post-event value adjustments. Second, for a fixed price q and payout rate ρ, there exists a sufficiently low level of risk p such that the agent will want to choose an insurance level of $I = 0$. The agent's behavior in response to the risk he faces is categorical; either he finds it worthwhile to insure fully against the risk or he finds the unit cost of insurance to be too high and prefers to forgo insurance entirely.

An intriguing extension of this simple model would be to assume that the agent can also control to some extent the probability p of the negative event by choosing a risk-avoidance effort level. If the agent finds these preventative measures to be costly, it would be interesting to see how much effort he puts into avoiding risk ex ante (known as "self-insurance" in the literature) both when outside insurance is and is not available.

4. Discussion

The primary goal of this paper is to propose an alternative formulation of prospect theory that specifies a consistent mechanism for how an agent's reference point should be formed. The model preserves the underlying behavioral phenomena that prospect theory seeks to characterize, but simultaneously addresses the questions of what should serve as a reference point and what the agent seeks to maximize before the uncertainty is resolved. The model can then be used to develop normative implications for how an agent with these preferences should and will behave.

One advantage of the model proposed here is that it provides a closed-form expression for how the agent should evaluate a binary prospect *ex ante*. This expression is easy to use, and can be adapted to reflect an agent's individual risk preferences by simply plugging in his parameters λ, α, γ^+, δ^+, γ^-, and δ^- (the last four can all be set equal to 1 if probability weighting is not an important consideration). As a result, the formulation for EO derived in Proposition 1 can be readily used to incorporate the main insights of prospect theory into decision modeling, as the insurance example of Section 3 demonstrates. While several assumptions have to be made to derive this result, I feel that the probability weighting functions and parameters λ and α allow for a sufficiently rich set of risk attitudes, and setting $\alpha = \beta$ is necessary to derive explicit parametric solutions to the expert's evaluation of a binary prospect. This same methodology could be applied for $\alpha \neq \beta$ or other more general functional forms but the results would be similar and would have to be solved numerically.

Removing flexibility in specifying the reference point comes with disadvantages as well. If the agent anchors his evaluation of outcomes on some other value, then the results derived in this paper that rely on consistency will not hold. There are likely many situations where expectations may not be the appropriate reference point to use, and this possibility should be thoughtfully considered when selecting a model. The value function $V(\cdot)$ can in fact accommodate many reference points other than expectations. For example, Tversky and Kahneman's (1992) model of cumulative prospect theory, which implicitly assumes a reference point of 0, can be recovered by selecting the value function $V(0)$.

Finally, the proposed model could and should be tested experimentally to validate its conclusions. In particular, it would be useful to examine how subjects form an expectation about their outcome in the presence of risk and whether this expectation in fact serves as a reference point, and to compare the predictions of this model to actual decision making to see if it can accurately predict behavior. One immediate prediction of the model is that the agent should be indifferent between the prospect and receiving the fixed payment EO. An experimenter could elicit EO by varying the fixed payment amount and observing subjects' choices between the risky prospect and the certain payment. However, testing the model predictions about EO would require careful elicitation and estimation of individual risk attitude parameters beforehand.

5. Appendix: Proofs of Lemma 1 and Propositions 1 - 2

Proof of Lemma 1: Define the function f by $f(EO) \equiv V(EO) - EO = w^-(p)\big(EO - \lambda(EO-A)^\alpha\big) + w^+(1-p)\big(EO + (B-EO)^\alpha\big) - EO$. First suppose $p \in (0,1)$: If $A < B$, then $f(A) = w^-(p)A + w^+(1-p)\big(A + (B-A)^\alpha\big) - A = w^+(1-p)(B-A)^\alpha > 0$ and $f(B) = w^-(p)\big(B - \lambda(B-A)^\alpha\big) + w^+(1-p)B - B = -w^-(p)\lambda(B-A)^\alpha < 0$. f is continuous in EO, so by the Intermediate Value Theorem, there exists $\tilde{EO} \in [A,B]$ such that $f(\tilde{EO}) = 0$, or $V(\tilde{EO}) = \tilde{EO}$. Furthermore, $f'(EO) = -w^-(p)\lambda\alpha(EO-A)^{\alpha-1} - w^+(1-p)\alpha(B-EO)^{\alpha-1} < 0$ for all $EO \in (A,B)$, so f is strictly decreasing in (A,B) and the root $\tilde{EO}$ is unique. If $A = B$ then $\tilde{EO} = A = B$, $f(\tilde{EO}) = 0$ and the root is (trivially) unique. Second, suppose $p = 0$: Then $f(EO) = \big(EO + (b - EO)^\alpha\big) - EO = (b - EO)^\alpha$, so the only root is $\tilde{EO} = b$. Finally, suppose $p = 1$: Then $f(EO) = \big(EO - \lambda(EO-A)^\alpha\big) - EO = -\lambda(EO-A)^\alpha$ and the only root is $\tilde{EO} = A$. $\square$

Proof of Proposition 1: $A \leq EO \leq B$ so

$$v(X, EO) = \begin{cases} EO - \lambda(EO - A)^\alpha & \text{if } A \text{ is realized} \\ EO + (B - EO)^\alpha & \text{if } B \text{ is realized.} \end{cases}$$

Consistency requires that $w^-(p)\big(EO - \lambda(EO - A)^\alpha\big) + w^+(1-p)\big(EO + (B - EO)^\alpha\big) = EO$. By coherence of the weighting functions, $w^-(p)\big(-\lambda(EO - A)^\alpha\big) + w^+(1-p)\big((B - EO)^\alpha\big) = 0$, or $w^+(1-p)(B - EO)^\alpha = w^-(p)\lambda(EO - A)^\alpha$. Then $(B - EO) = (\frac{\lambda w^-(p)}{w^+(1-p)})^{\frac{1}{\alpha}}(EO - A)$, or $B + (\frac{\lambda w^-(p)}{w^+(1-p)})^{\frac{1}{\alpha}}A = (1 + (\frac{\lambda w^-(p)}{w^+(1-p)})^{\frac{1}{\alpha}})EO$. So EO $= \frac{B + (\frac{\lambda w^-(p)}{w^+(1-p)})^{\frac{1}{\alpha}}A}{1 + (\frac{\lambda w^-(p)}{w^+(1-p)})^{\frac{1}{\alpha}}}$. □

Proof of Proposition 2: $W - qI - L + \rho I \leq EO \leq W - qI$ so

$$v(X, EO) = \begin{cases} EO - \lambda(EO - W - qI - L + \rho I)^\alpha & \text{if the negative event occurs} \\ EO + (W - qI - EO)^\alpha & \text{if the negative event does not occur.} \end{cases}$$

Consistency requires that $w^-(p)\big(EO - \lambda(EO - W - qI - L + \rho I)^\alpha\big) + w^+(1-p)\big(EO + (W - qI - EO)^\alpha\big) = EO$. Then $EO(I) = W - qI + \frac{1}{1 + (\frac{\lambda w^-(p)}{w^+(1-p)})^{\frac{1}{\alpha}}}(\rho I - L) = W - \frac{L}{1 + (\frac{\lambda w^-(p)}{w^+(1-p)})^{\frac{1}{\alpha}}} + \big(\frac{\rho}{1 + (\frac{\lambda w^-(p)}{w^+(1-p)})^{\frac{1}{\alpha}}} - q\big)I$. EO is a linear function of I over the compact interval $[0, L - D]$, so its maximum value occurs at either $I = 0$ or $I = L - D$, depending on the sign of $\frac{\rho}{1 + (\frac{\lambda w^-(p)}{w^+(1-p)})^{\frac{1}{\alpha}}} - q$. If $\frac{\rho}{1 + (\frac{\lambda w^-(p)}{w^+(1-p)})^{\frac{1}{\alpha}}} - q \leq 0$, or equivalently, $\frac{\rho}{q} \leq 1 + (\frac{\lambda w^-(p)}{w^+(1-p)})^{\frac{1}{\alpha}}$, then $I = 0$ is optimal. If $\frac{\rho}{q} \geq 1 + (\frac{\lambda w^-(p)}{w^+(1-p)})^{\frac{1}{\alpha}}$, then $I = L - D$ is optimal. □

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References

Abdellaoui, M. (2000). Parameter-free elicitation of utilities and probability weighting functions. *Management Science* **46(11),** 1497–1512.

Abdellaoui, M., Bleichrodt, H., Paraschiv, C. (2007). Loss aversion under prospect theory: A parameter-free measurement. *Management Science* **53(10),** 1659–1674.

Abdellaoui, M., Vossmann, F., Weber, M. (2005). Choice-based elicitation and decomposition of decision weights for gains and losses under uncertainty. *Management Science* **51,** 1384–1399.

Abeler, J., Falk, A., Gotte, L., Huffman, D. (2011). Reference points and effort provision. *The American Economic Review* **101(2),** 470–492.

Baucells, M., Villasis, A. (2010). Stability of risk preferences and the reflection effect of prospect theory. *Theory and Decision* **68,** 193–211.

Booij, A. S., van Praag, B. M. S., van de Kuilen, G. (2010). A parametric analysis of prospect theory's functionals for the general population. *Theory and Decision* **68,** 115–148.

Budescu, D. V., Weiss, W. (1987). Reflection of transitive and intransitive preferences: A test of prospect theory. *Organizational Behavior and Human Decision Processes* **39,** 184–202.

Camerer, C. F. (2000). Prospect theory in the wild: Evidence from the field. In C.F. Camerer, G. Loewenstein, & M. Rabin (Eds.), *Advances in Behavioral Economics,* Princeton, NJ: Princeton University Press.

Crawford, V. P., Meng, J. (2011). New York City cabdrivers' labor supply revisited: Reference-dependent preferences with rational-expectations targets for hours and income. *The American Economic Review.* **101(5),** 1912–1932.

Goldstein, W. M., Einhorn, H. J. (1987). Expression theory and the preference reversal phenomena. *Psychological Review* **94,** 236–254.

Hershey, J. C., Schoemaker, P. J. H. (1980). Prospect theory's reflection hypothesis: A critical examination. *Organizational Behavior and Human Performance* **25,** 395–418.

Kahneman, D., Tversky, A. (1979). Prospect theory: An analysis of decision under risk. *Econometrica* **47(2),** 263–292.

Kobberling, V., Wakker, P. P. (2005). An index of loss aversion. *Journal of Economic Theory* **122,** 119–131.

Koszegi, B. (2010). Utility from anticipation and personal equilibrium. *Economic Theory* **44,** 415–444.

Koszegi, B., Rabin, M. (2006). A model of reference-dependent preferences. *The Quarterly Journal of Economics* **121(4),** 1133–1165.

Koszegi, B., Rabin, M. (2007). Reference-dependent risk attitudes. *The American Economic Review* **97(4),** 1047–1073.

Post, T., van den Assem, M. J., Baltussen, G., Thaler, R. H. (2008). Deal or no deal? Decision making under risk in a large-payoff game show. *American Economic Review* **98(1),** 38–71.

Quiggin, J. (1982). A theory of anticipated utility. *Journal of Economic Behavior and Organization* **3(4),** 323–343.

Schmeidler, D. (1989). Subjective probability and expected utility without additivity. *Econometrica* **57(3),** 571–587.

Siegel, S. (1957). Level of aspiration and decision making. *Psychological Review* **64(4),** 253–262.

Shalev, J. (2000). Loss aversion equilibrium. *International Journal of Game Theory* **29,** 269–287.

Tu, Q. (2005). Empirical analysis of time preferences and risk aversion. *CentER PhD Thesis* **142,** Tilburg University.

Tversky, A., Kahneman, D. (1991). Loss aversion in riskless choice: A reference-dependent model. *Quarterly Journal of Economics* **106(4),** 1039–1061.

Tversky, A., Kahneman, D. (1992). Advances in prospect theory: cumulative representation of uncertainty. *Journal of Risk and Uncertainty* **5,** 297–323.

Wakker, P. P. (2008). Explaining the characteristics of the power (CRRA) utility family. *Health Economics* **17,** 1329–1344.