Calibration without reduction for nonexpected utility

David Freeman*

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Abstract

Evidence from the lab and the field shows that most people exhibit substantial risk aversion over stakes of hundreds of dollars. Expected utility cannot capture nonnegligible risk aversion over such small stakes without producing implausible risk aversion over large stakes, and under the reduction of compound lotteries axiom, neither can nonexpected utility preferences. Motivated by experimental evidence, this paper assumes that compound lotteries are evaluated recursively and shows that popular nonexpected utility models can be consistent with empirically plausible risk aversion over both small and large stakes.

*PhD Candidate, Department of Economics, University of British Columbia. Address: 997 - 1873 East Mall, Vancouver, BC, V6T 1Z1. E-mail: freemand@interchange.ubc.ca. I would like to especially thank my advisor, Yoram Halevy, for substantial guidance, feedback, and encouragement. For conversations and comments that lead to substantive and expositional improvements, I would also like to thank Nicholas Barberis, Eddie Dekel, David Dillenberger, Eran Hanany, Chad Kendall, Luba Petersen, Matthew Rabin, Zvi Safra, and Lanny Zrill. The author was supported by a SSHRC Canada Graduate Scholarship while this paper was written.

Recent calibration critiques of Rabin (2000) and Safra and Segal (2008) show that whenever expected utility (EU) and non-expected utility (non-EU) define utility over final wealth states, they cannot simultaneously exhibit nonnegligible risk aversion over small stakes and can only exhibit moderate risk aversion over large stakes. Introspection and empirical evidence suggest that even if the stakes are small, most people would rather not take a small risk with a positive expected value if it could involve a loss of money. Yet most people still take substantial risks over large stakes, for instance, by investing in stocks. As a result, these calibration critiques have been widely understood as suggesting the demise of descriptive theories that define utility over final wealth except as a normative benchmark—further suggesting that descriptive models must define utility over gains and losses. But defining utility over final wealth gives non-EU theories a tractability and modelling discipline that more psychologically based theories such as prospect theory lack.

This paper shows that nonexpected utility can generate both nonnegligible small-stakes risk aversion as well as the moderate large-stakes risk aversion. The crucial assumption made here is that a decision maker (DM) who faces preexisting risks treats a gamble that is offered as the first stage of a two-stage *compound lottery*, which is then not treated as equivalent to the one-stage lottery that gives the same probability distribution over final wealth but is evaluated recursively (Segal, 1990). This contrasts sharply with existing proposals for solving the Rabin critique, which have all relied on abandoning consequentialism—the assumption that utility is defined over final wealth levels—assuming instead that utility is also evaluated over gains and losses or lab income.

The intuition for the rank-dependent utility (RDU) (Quiggin, 1982) case of the main results of the paper is as follows: without background risk, RDU can produce descriptively reasonable risk aversion at a range of stakes through probability weighting even if utility is defined over wealth levels. Now suppose DM faces background risk $\tilde{w}$, and the utility-for-wealth function u is linear. Under recursive RDU, a compound lottery is evaluated by a folding back procedure, and DM evaluates the offered gamble $(-L, .5; +G, .5)$ by folding back the compound lottery $[\tilde{w} - L, .5; \tilde{w} + G, .5]$ to $[c(\tilde{w} - L), .5; c(\tilde{w} + G), .5]$ where c is DM's certainty-equivalent function. When u is linear, $c(\tilde{w} + x) = c(\tilde{w}) + x$; this compound lottery is evaluated as $[c(\tilde{w}) - L, .5; c(\tilde{w}) + G, .5]$, and probability weighting produces small-stakes risk aversion over the offered gamble the way it would without background risk. With a linear u, DM turns down $(-L, .5; +G, .5)$ if and only if DM turns down $(-tL, .5; +tG, .5)$ for all $t > 0$; therefore, small-stakes risk aversion due to probability weighting is compatible with reasonable large stakes risk aversion.

Relation to previous literature Since EU maximizers are approximately risk neutral over small stakes, they would only be willing to pay a trivial amount to avoid small risks. Popular alternatives to EU that define utility over final wealth levels are either (i) 'smooth' as in (Machina, 1982), locally risk neutral, and subject to the same criticism as EU (Safra and Segal, 2008), or (ii) obtain nonnegligible risk aversion over small stakes because they weigh probabilities nonlinearly (as suggested by the Allais paradox) and are hence immune from Rabin's critique.

However, most people face substantial lifetime wealth risk (e.g., employment-income risk and ownership of risky assets). The combination of a gamble offered in the lab in the presence of preexisting wealth risk is naturally viewed as a two-stage compound lottery in which the offered gamble resolves first. When DM reduces compound lotteries to single-stage lotteries by multiplying out probabilities, any small-stakes gamble offered adds only minimally to lifetime wealth risk; therefore, probability weighting is mostly determined by preexisting lifetime wealth risk and does not produce substantial risk aversion over offered small-stakes gambles (Safra and Segal, 2008; Barberis, Huang and Thaler, 2006). This argument relies crucially on the assumption that DM satisfies reduction of compound lotteries—an assumption that is not consistent with experimental evidence. Instead, this paper assumes recursive preferences over compound lotteries.

Recursive non-EU (RNEU) preferences over compound lotteries are used in this paper as a descriptive model of decision making, following Segal (1990). The theoretical distinction between compound versus single-stage lotteries was first suggested by Samuelson (1952). RNEU preferences have been applied by Segal (1987*b*) to explain ambiguity aversion and by Dillenberger (2010) to explain preferences for one-shot resolution of uncertainty. Dillenberger also remarks that an RNEU DM behaves as if they frame narrowly; section 1.5 of this paper makes a precise connection between RNEU and risk aversion with narrow framing.

The theoretical tradition of RNEU preferences following Segal (1990) is related to but distinct from the use of recursive utility due to preferences over the timing of resolution of uncertainty (Kreps and Porteus, 1978; Epstein and Zin, 1989). When recursive preferences are used only because of preferences over the resolution of uncertainty then DM applies reduction of compound lotteries to an offered delayed risk combined with income risk that resolves at the same time, and will not demonstrate small-stakes risk aversion over such gambles (Barberis, Huang and Thaler, 2006).

Existing theoretical approaches that avoid a calibration critique (Kahneman and Tversky (1979); Cox and Sadiraj (2006);

Barberis, Huang and Thaler (2006)) directly incorporate narrow framing by assuming that the value function is defined over the outcomes of a gamble as well as (possibly) over final wealth states. RNEU is formally very different from these nonconsequentialist models in that the utility function is only defined over final wealth states and not directly over the outcomes of a gamble.

Under RDU, the RNEU preferences studied in this paper are still subject to calibration arguments by Neilson (2001) and Cox and Sadiraj (2011); however, the assumptions that drive their calibration critiques lack strong experimental support or obvious intuitive appeal. For a literature review of calibration critiques, see Section 8.6 of Wakker (2010).

Experimental evidence on compound lotteries Halevy (2007) finds that 80 percent of subjects violate reduction of compound lotteries, while 59 percent of subjects' choices are best explained by RNEU. Previous experimental work also found substantial violations of reduction of compound lotteries that suggest the use of RNEU preferences; for example, Carlin (1992); Camerer and Ho (1994). Recursive preferences over compound lotteries are also consistent with experimental findings that randomly picking one of the subject's many decisions to determine payment is an incentive compatible mechanism for eliciting preferences (Cubitt and Sugden (1998); Laury (2005)).

1 Theory: RNEU risk preferences with background wealth risk

1.1 Nonexpected utility over lotteries

Notation Let $W = \Re_+$ denote the set of feasible final wealth levels, and consider a preference over one-stage lotteries, $V : \Delta(W) \to \Re$ with utility-for-wealth function $u : W \to \Re$. A *one-stage lottery* over W can be written as $q = [w_1, q_1; ...; w_n, q_n] \in \Delta(W)$ whenever q has finite support, where q_i denotes the probability of receiving prize w_i. Assume V is increasing in the sense of first-order stochastic dominance. Adopt the convention that $w_1 \leq ... \leq w_n$. Say that V is *risk averse* if it is averse to mean-preserving spreads.

Popular models The two most commonly used non-EU theories are RDU (Quiggin (1982), Yaari (1987), Segal (1990)), and disappointment aversion (DA) (Gul, 1991). Table 1 reviews these and EU; it notes conditions under which RDU and DA demonstrate the Allais paradox and small-stakes risk aversion not present under EU. DA preferences are a special case of the larger class of betweenness-satisfying preferences (Dekel (1986), Chew (1989)).

1.2 Recursive nonexpected utility

RNEU extends non-EU preferences over single-stage lotteries to the domain of compound lotteries.

Define a *compound lottery* as a finite lottery over lotteries over final wealth levels; a compound lottery can be written as $Q = [q^1, p_1; ...; q^m, p_m]$ where $q^i \in \Delta(W)$ and p_i is the probability of receiving lottery q^i; let $\Delta(\Delta(W))$ denote the set of compound lotteries. The utility function U is defined over com-

Table 1: Nonexpected utility theories

Theory	$V([w_1, q_1; ...; w_n, q_n])$	Allais?	Small-stakes risk averse?
EU	$\sum_{i=1}^{n} q_i u(w_i)$	No	Not if u' exists
RDU	$\sum_{i=1}^{n} [g(\sum_{j=1}^{i} q_j) - g(\sum_{j=1}^{i-1} q_j)]u(w_i)$	g concave	g concave
DA	$\sum_{i=1}^{n} \frac{1+\beta \mathbb{I}_{V(w_i)<V(q)}}{1+\beta \sum_{i=1}^{n} q_j \mathbb{I}_{V(w_j)<V(q)}} q_i u(w_i)$	$\beta > 0$	$\beta > 0$

Sources: Gul (1991); Segal and Spivak (1990); Segal (1987*a*)

pound lotteries over final wealth levels. Without loss of generality, adopt the convention that for a compound lottery Q as above, $V([q^1, 1]) \leq ... \leq V([q^m, 1])$.[1]

A RNEU maximizer evaluates a compound lottery Q via a simple two step procedure:

1. Compute the certainty equivalent of each lottery q^i that is a possible prize of Q:

$$c(q^i) = u^{-1} \circ V(q^i)$$

2. Recursively compute the value of the compound lottery as the nonexpected utility of the one-step lottery $[c(q^1), p_1; ...; c(q^m), p_m]$:

$$U(Q) = V([c(q^1), p_1; ...; c(q^m), p_m]) \qquad (1.1)$$

An alternative to the recursivity assumption is that a DM immediately reduces the compound lottery to a single-stage lottery,

[1] Readers familiar with Segal (1990) will note that I assume time neutrality here. This is not essential for the conclusions.

which is then evaluated according to V. Such a reduction of compound lotteries assumption is *not* consistent with evidence that subjects fail to reduce compound lotteries to one-stage lotteries presented earlier. If a non-EU DM reduces compound lotteries, compound lottery Q is evaluated as equivalent to the one-stage lottery $Q^R = [w_1, \sum_{i=1}^{m} p_i q_1^i; ...; w_K, \sum_{i=1}^{m} p_i q_K^i]$. For purposes of comparison, a *nonexpected utility with reduction* DM evaluates a compound lottery p by:

$$U^{ROCL}(Q) = V(Q^R) \tag{1.2}$$

1.3 Wealth risk as a compound lottery

A one-time choice is never the only thing going on in a DM's life. Empirical work shows that people face substantial risks in their lives (Guiso, Jappelli and Pistaferri, 2002). If we want to retain the modeling discipline of defining utility over final wealth levels, then we have to make a choice about how to model a DM's attitude to risk from a one-time gamble and from everything else in life. The combination of a one-time gamble offered (like those offered in lab experiments) and background wealth risk constitutes a compound lottery composed of two distinct and independent sources of risk in which the one-time gamble resolves first, and the rest of life's uncertainties resolve in due course.

Consider a DM who faces background wealth risk described by the random variable $\tilde{w} = [w_1, q_1; ...; w_m, q_m]$, which is not the subject of choice, and who is offered the gamble over prizes $\hat{p} = (y_1, p_1; ...; y_n, p_n)$ where $y_i \in Y \subset \Re$ is a monetary prize added to or taken away from the DM's final wealth after lottery $\hat{p}$ resolves. Let $\hat{p} \oplus \tilde{w}$ denote the compound lottery formed by simple gamble over prizes $\hat{p}$, which resolves first, and independent background risk $\tilde{w}$, which resolves second. The compound lottery $\hat{p} \oplus \tilde{w}$ is given by:

$$\hat{p} \oplus \tilde{w} = [\tilde{w} + y_1, p_1; ...; \tilde{w} + y_n, p_n] \tag{1.3}$$

where $\tilde{w} + y_i = [w_1 + y_i, q_1; ...; w_n + y_i, q_n]$ denotes the lottery over final wealth states that the DM faces if– prize y_i is won in the gamble $\hat{p}$. The compound lottery $\hat{p} \oplus \tilde{w}$ is well defined whenever $w + y_i \in W$ for each w in the support of $\tilde{w}$ and each y_i in the support of $\hat{p}$.[2]

Say that a DM with utility function U defined on $\Delta(\Delta(W))$ treats an offered gamble $\hat{p}$ in the presence of background risk $\tilde{w}$ as a compound lottery in which $\hat{p}$ resolves first if for any offered gambles $\hat{p} \in \Delta(Y)$, DM evaluates the utility of $\hat{p}$ according to $U(\hat{p} \oplus \tilde{w})$.

1.4 Small-stakes risk aversion in recursive non-expected utility: calibration results

Suppose we only assume that DM is a recursive non-EU decision maker and V is a risk-averse. When combined with the observation that DM turns down a given gamble $\hat{p}$ at different background risk distributions, assuming risk aversion alone does not allow us to conclude anything stronger than that DM will turn down mean-preserving spreads of $\hat{p}$. This is the content of Remark 1.

Remark 1. Suppose a recursive non-EU decision maker treats an offered gamble in the presence of background risk as a compound lottery as in (1.3), and always turns down an actuarially favorable

[2] An alternative but less intuitive assumption is that DM distinguishes between one-stage and compound risks but treats the gamble $\hat{p}$ as resolving in the second stage. The main results of the paper are not sensitive to this assumption.

gamble $\hat{p}$ for any distribution of background risk $\tilde{w}$. Knowing only that V is globally risk averse, the strongest conclusion that can be drawn is that DM will always turn down any mean-preserving spread of $\hat{p}$.

Perhaps if we were to assume more about V, such as that V is RDU (or alternatively, DA) and DM is risk averse, the stronger set of assumptions would allow us to draw a stronger calibration result. Theorem 2 shows that the strongest possible calibration result that can be obtained from RNEU preferences under these assumptions is that if a DM turns down the offered gamble, any version of the gamble with the stakes scaled up by a factor of $t > 1$ will also be turned down.

Theorem 2. *Suppose a recursive non-EU decision maker treats an offered gamble in the presence of background risk as a compound lottery as in (1.3), and always turns down a gamble* $\hat{p} = (y_1, p_1; ...; y_n, p_n)$ *for any distribution of background risk* $\tilde{w}$. *Knowing only that* V *is risk averse and is RDU, then the strongest restriction on large-stakes gambles that can be drawn without further assumptions is that DM will turn down any gamble* $\hat{p}^t = (ty_1, p_1; ...; ty_n, p_n)$ *for all* $t > 1$ *and for all* $\tilde{w}$. *The same result applies if "RDU" is replaced with "DA".*

Proof. DM turns down $\hat{p}^t$ if $U(\hat{p}^t \oplus \tilde{w}) - V(\tilde{w}) < 0$. Turning down $\hat{p}$ implies $U(\hat{p} \oplus \tilde{w}) - V(\tilde{w}) < 0$. If u is linear, then under DA and RDU this implies that DM turns down $\hat{p}^t$ for all t but cannot rule out accepting more favorable gambles. If u is concave, then it can be shown that under risk aversion and either DA or RDU that $U(\hat{p}^t \oplus \tilde{w})$ is concave in t; therefore, DM will still turn down $\hat{p}^t$ for any $t > 1$. However, EU remains a special case of DA and RDU, and if DM had EU preferences, $\hat{p}^t$ would be accepted for a sufficiently small t whenever $0 \notin \text{support}(\tilde{w})$.

Under DA and RDU, if V is globally risk averse u must be weakly concave, so the strongest calibration result possible comes from the case where u is linear for $t > 1$. That is, DM will turn down $\hat{p}^t$ for all $t > 1$. $\square$

I interpret Theorem 2 as demonstrating that recursive RDU and DA are immune to Rabin-style calibration critiques. Since recursive is a special case of the more general class of betweenness preferences (Dekel, 1986; Chew, 1989), if recursive DA is immune to calibration critiques, then the class of recursive betweenness preferences is immune to Rabin-style calibration critiques, as are more general classes of preferences. Thus, recursive versions of a wide range of non-EU theories are not susceptible to calibration critiques, and are potentially suitable for modeling risk preferences over both small and large stakes.

An alternative way of thinking about small-stakes risk aversion is in terms of whether DM maintains aversion to a risk as its stakes are uniformly shrunk to zero, (i.e., demonstrates *first-order risk aversion*) or is neutral to arbitrarily small risks (*second-order risk aversion*) (Segal and Spivak, 1990). The Supplementary Appendix characterizes small-stakes risk aversion of a broader class of models under recursive non-EU in terms of first- versus second-order risk aversion.

1.5 Nonreduction versus narrow framing

Segal (1990) first suggested replacing the reduction of compound lotteries axiom with recursivity as a consequentialist alternative to prospect theory that captures Kahneman and Tversky's (1979) "isolation effect" (a particular example of a failure of the reduction of compound lotteries axiom). Rabin (2000) noted that prospect theory is immune to his calibration critique. Existing theoretical approaches that avoid a calibration critique (Kahneman and Tversky (1979); Cox and Sadiraj (2006); Barberis, Huang and Thaler (2006)) directly incorporate narrow framing by assuming that the value function is defined over the outcomes of a gamble as well as (possibly) over final wealth states. RNEU is formally very different from these nonconsequentialist models in that the value function is only defined over final wealth states and not directly over the outcomes of a gamble.

While RNEU does not assume that a gamble is framed narrowly, Theorem 3 shows that to a first-order approximation (i.e., over arbitrarily small stakes), DM's attitudes to a gamble under recursive non-EU are independent of background risk—consistent with narrow framing over small stakes. The intuition for this is that if u is differentiable, then it is locally linear over arbitrarily small stakes, and DM's local risk attitudes are determined entirely by probability weighting.

Theorem 3. *Suppose a recursive non-EU decision maker treats an offered gamble in the presence of background risk as a compound lottery as in (1.3):*

1. *When V is RDU, DM takes on a gamble $\hat{p}^t = (ty_1, p_1; ...; ty_n, p_n)$ as $t \to 0^+$ if and only if $\sum_{i=1}^{n}[g(\sum_{j=1}^{i} p_j) - g(\sum_{j=1}^{i-1} p_j)]y_i > 0$, regardless of the other risks faced.*

2. *When V is DA, DM takes on a gamble $\hat{p}^t = (ty_1, p_1; ...; ty_n, p_n)$ as $t \to 0^+$ if and only if $\sum_{i=1}^{n} p_i[1 + \beta \mathbb{I}_{y_i<0}]y_i > 0$, regardless of the other risks faced, so long as the background wealth risk does not have a mass point at its certainty equivalent.*

3. *If u is linear, then the above results hold for all t such that DM never faces wealth constraints.*

4. If $u(w) = 1 - \exp(-\gamma w)$ for $\gamma > 0$, then under RDU DM accepts $\hat{p}^t$ if and only if $\sum_{i=1}^{n}[g(\sum_{j=1}^{i} p_j) - g(\sum_{j=1}^{i-1} p_j)][1 - \exp(-\gamma y_i)] > 0$ for all t such that DM never faces wealth constraints.

A recent study by Barseghyan et al. (2010) estimated RDU preferences with an exponential u while ignoring background wealth risk; they found substantial support for probability weighting as an explanation for insurance choices. Their specification is inconsistent with RDU with reduction of compound lotteries, but Result 4 of Theorem 3 shows that their empirical specification is consistent with RNEU preferences, lending nonexperimental support to the use of RNEU.

2 Calibration

What constitutes descriptively reasonable risk aversion is a quantitative question. This section calibrates a version of recursive RDU and shows that this calibration can produce descriptively reasonable risk aversion, while EU and RDU with reduction cannot.

Convenient functional forms for g and u should have as few parameters as possible to calibrate and should be easily comparable to commonly used models. I adopt the standard power utility-for-wealth function:

$u(w) = \frac{w^{1-\gamma}}{1-\gamma}$

and the probability weighting function:

$g(p) = p^{\nu}$

used in Safra and Segal (2008) since it is only one parameter richer than EU, is consistent with small-stakes risk aversion and Allais-type choices when $0 < \nu < 1$, and captures expected utility as a special case when $\nu = 1$.

Chetty (2006) points out that the curvature of the utility-for-wealth function also governs how an individual makes trade-offs between labour and leisure. I use $\gamma = .71$, suggested by Chetty based on previous studies of labor supply responses to wage changes.[3] I then calibrate ν to match modal choices in Holt and Laury's (2002) experimental data on small-stakes risk aversion to the extent possible. While EU cannot avoid mispredicting the modal choice in their data when most subjects demonstrate risk aversion, if $\nu \in [.5, .64]$, the calibrated recursive RDU model fits the data reasonably but not perfectly.

While the risk in $\tilde{w}$ only has a second-order effect on decisions among offered gambles in recursive RDU, the risk in $\tilde{w}$ reduces risk aversion RDU with reduction. To allow for comparison, take $\tilde{w} = U[\$100000,\ \$500000]$ to capture background wealth risk.[4]

Table 2 summarizes how different calibrated models discussed above would predict that a DM would make choices in $(-L, .5; G, .5)$ gambles. In each row of the table, L is fixed at the level in the left-hand column, while the entry in the table lists the G at which a DM would be indifferent to either taking or turning down the listed gamble.

Table 2 (Columns 1 and 2) indicates that for $\nu = .5$, $.64$ recursive RDU can produce descriptively reasonable risk aversion over both small and large stakes. RDU with reduction produces barely any risk aversion over small stakes (Column 3). Even for stakes into the thousands of dollars, EU induces preferences over gambles

[3] While Chetty assumes expected utility in his calculations, the approach he takes fully carries through to RDU in the case where utility is separable in consumption and leisure; I use Chetty's estimates from this case.

[4] I derive quantitative results using a discrete approximation of the uniform distribution. I assume that lifetime wealth has an expected present value of \$300,000, since this figure is emphasized in Rabin (2000), but the quantitative results are not particularly sensitive to this assumption.

Table 2: Calibration results - small and large stakes risk aversion

Loss	$\nu = .5$	$\nu = .64$	$\nu = .5$, reduction	$\nu = 1$ (EU)
10	24.14	17.91	10.10	10.00
100	241.60	179.21	103.36	100.03
200	483.59	358.62	209.61	200.13
500	1211.84	898.12	538.77	500.83
1000	2433	1801	1112	1003
2000	4904	3623	2328	2013
5000	12555	9219	6403	5084
10000	26133	18992	14364	10343
15000	40820	29358	23704	15787
20000	56714	40361	34461	21425
25000	73931	52052	46738	27270
30000	92602	64492	60685	33334
40000	134957	91889	94395	46180
50000	185392	123239	137678	60105

Gain required for DM to take (-Loss, .5; Gain, .5) that are extremely close to expected value maximization (Column 4). Even with a higher value for γ, EU would induce preferences over gambles that are extremely close to expected value maximization over stakes of hundreds of dollars. Table 2 demonstrates quantitatively what I showed qualitatively in Section 1.4: recursive RDU can produce descriptively reasonable risk aversion, while RDU with reduction and EU cannot. These quantitative results are not sensitive to the choice of a distribution for background wealth risk.

3 Conclusion

This paper has shown that nonexpected utility theories can produce nonnegligible small-stakes risk aversion without implying ridiculous large-stakes risk aversion. A calibration exercise demonstrated that recursive RDU can be calibrated to provide descriptively reasonable levels of risk aversion in the small and in the large. The nonexpected utility theories studied in this paper are attractive for two reasons. Nonexpected utility theories, rank-dependent utility and disappointment aversion in particular, have been well studied and have clear axiomatic foundations that make them amenable to further applications. Nonexpected utility does not relax the assumption of consequentialism, which has made it easy to apply expected utility to a variety of economic problems without having to pay attention to the 'context' or the 'frame' present in each application. Furthermore, the two departures this model does make from expected utility theory are each well supported by experimental evidence on the Allais paradox and nonreduction of compound lotteries.

4 Appendix

Proof of Theorem 2.

Completing the proof of Theorem 2 requires showing that if DM is risk averse and V is RDU/DA, then $V(\tilde{w}+x)$ is concave in x and $U(\hat{p}^t \oplus \tilde{w})$ is concave in t.

Under RDU, $V(\tilde{w}+x) = \int u(w+x)dg(F_{\tilde{w}}(w))$ where $F_{\tilde{w}}$ is the CDF associated with $\tilde{w}$. Since u is concave, $V(\tilde{w}+x)$ is concave in x.

Now under RDU, $U(\hat{p}^t \oplus \tilde{w}) = \int V(\tilde{w}+ty)dg(F_{\hat{p}}(y))$. Since $V(\tilde{w}+x)$ is concave in x, $U(\hat{p}^t \oplus \tilde{w})$ is concave in t. This implies that $\frac{1}{t}(U(\hat{p}^t \oplus \tilde{w}) - V(\tilde{w})) \leq U(\hat{p} \oplus \tilde{w}) - V(\tilde{w}) < 0$ for $t > 1$, so DM will also turn down $\hat{p}^t$.

Under DA, the same argument applies, except that it is messier to prove that $V(\tilde{w}+x)$ is concave in x and $U(\hat{p}^t \oplus \tilde{w})$ is concave in t - this is proven below.

Proof that $V(\tilde{w}+x)$ is concave in x under DA A DA DM is globally risk averse in the sense of weakly not preferring mean-preserving spreads if and only if u is concave and $\beta \geq 0$.

I will show that $(1+\beta)[V(\tilde{w}+x) - V(\tilde{w})] \leq (1+\beta)[V(\tilde{w}) - V(\tilde{w}-x)]$ to prove concavity.

We can write out the left- and right- hand sides of the above equation as:

$$\begin{aligned}
&(1+\beta)[V(\tilde{w}+x) - V(\tilde{w})] \\
&= \int \{u(w+x) - u(w) + \beta \min[u(w+x), V(\tilde{w}+x)] - \beta \min[u(w), V(\tilde{w})]\} dF_{\tilde{w}}(w)
\end{aligned} \tag{4.1}$$

$$
\begin{aligned}
&(1+\beta)[V(\tilde{w}) - V(\tilde{w}-x)] \\
&= \int \{u(w)-u(w-x)+\beta\min[u(w),V(\tilde{w})]-\beta\min[u(w-x),V(\tilde{w}-x)]\}dF_{\tilde{w}}(w)
\end{aligned}
\tag{4.2}
$$

To compare (4.1) and (4.2), compare the term inside the integral for each w. First note $u(w+x)-u(w) \leq u(w)-u(w-x)$. Second, compare the remaining parts of the integrals by working with four different regions/cases that depend on w and V.

Case A $\min[V(\tilde{w}+x), u(w+x)] - \min[V(\tilde{w}), u(w)] = u(w+x)-u(w)$ and $\min[V(\tilde{w}), u(w)] - \min[V(\tilde{w}-x), u(w-x)] = u(w)-u(w-x)$. By concavity of u for these w, the (4.1) term is smaller than the (4.2) term.

Case B $\min[V(\tilde{w}+x), u(w+x)] - \min[V(\tilde{w}), u(w)] = V(\tilde{w}+x)-V(\tilde{w})$ and $\min[V(\tilde{w}), u(w)] - \min[V(\tilde{w}-x), u(w-x)] = V(\tilde{w})-V(\tilde{w}-x)$. We can cancel these terms from $(1+\beta)[V(\tilde{w}+x)-V(\tilde{w})]$ and $(1+\beta)[V(\tilde{w})-V(\tilde{w}-x)]$.

Case C Suppose neither of the above two cases applies and $u(w) \leq V(\tilde{w})$. Then, applying concavity of u,

$$
\begin{aligned}
\min[u(w+x),V(\tilde{w}+x)]-\min[u(w),V(\tilde{w})]\} &= \min[u(w+x),V(\tilde{w}+x)]-u(w) \\
&\leq u(w+x)-u(w) \\
&\leq u(w)-u(w-x) \\
\leq u(w)-\min[u(w-x),V(\tilde{w}-x)] &= \min[u(w),V(\tilde{w})]-\min[u(w-x),V(\tilde{w}-x)]\}
\end{aligned}
$$

so in this case, this term of the (4.1) is smaller than the corresponding term in (4.2).

Case D Suppose neither of the above three cases applies, so $u(w) > V(\tilde{w})$. Then,

$$\begin{aligned}\min[u(w{+}x), V(\tilde{w}{+}x)] - \min[u(w), V(\tilde{w})]\} &= \min[u(w{+}x), V(\tilde{w}{+}x)] - V(\tilde{w}) \\ &\leq V(\tilde{w}+x) - V(\tilde{w})\end{aligned}$$

and

$$\begin{aligned}\min[u(w), V(\tilde{w})] - \min[u(w{-}x), V(\tilde{w}{-}x)]\} &= V(\tilde{w}) - \min[u(w{-}x), V(\tilde{w}{-}x)]\} \\ &\geq V(\tilde{w}) - V(\tilde{w}-x)\end{aligned}$$

Plugging in these terms and cancelling out as case B establishes the desired inequality.

Proof that $U(\hat{p}^t \oplus \tilde{w})$ is concave in t under DA Define $I_V(y) = 1$ if $V(\tilde{w}+ty) - V(\tilde{w}) < U(\hat{p}^t \oplus \tilde{w}) - V(\tilde{w})]$ and zero otherwise, and $I_U(y) = 1$ if $V(\tilde{w}+ty) - V(\tilde{w}) \geq U(\hat{p}^t \oplus \tilde{w}) - V(\tilde{w})]$ and zero otherwise. For $t > 1$,

$\frac{1+\beta}{t}\{U(\hat{p}^t \oplus \tilde{w}) - V(\tilde{w})\}$

$= \frac{1}{t}\int\{V(\tilde{w}+ty) - V(\tilde{w}) + \beta \min[V(\tilde{w}+ty) - V(\tilde{w}), U(\hat{p}^t \oplus \tilde{w}) - V(\tilde{w})]\}dF_{\hat{p}}(y)$

$\leq \frac{1}{t}\int\{V(\tilde{w}+ty) - V(\tilde{w}) + \beta I_V(y)[V(\tilde{w}+ty) - V(\tilde{w})] + \beta I_U(y)[U(\hat{p}^t \oplus \tilde{w}) - V(\tilde{w})]\}dF_{\hat{p}}(y)$

Let $\bar{p} = 1 - \int I_U(y)dF_{\hat{p}}(y)$. Then, rearranging the above expression yields the inequality:

$\frac{1+\beta\bar{p}}{t}\{U(\hat{p}^t \oplus \tilde{w}) - V(\tilde{w})\} \leq \frac{1}{t}\int\{V(\tilde{w}+ty) - V(\tilde{w}) + \beta I_V(y)[V(\tilde{w}+ty) - V(\tilde{w})]\}dF_{\hat{p}}(y)$

$\leq \int\{V(\tilde{w}+y) - V(\tilde{w}) + \beta I_V(y)[V(\tilde{w}+y) - V(\tilde{w})]\}dF_{\hat{p}}(y)$

$= \frac{1+\beta\bar{p}}{t}\{U(\hat{p} \oplus \tilde{w}) - V(\tilde{w})\}$

$\square$

Proof of Theorem 3.

Proof of 1. Define: $V_x(\tilde{w}) = \lim_{x\to 0} \frac{V(\tilde{w}+x)-V(\tilde{w})}{x}$

$= \int u'(w)dg(F_{\tilde{w}}(w))$.

DM evaluates taking $\hat{p}^t$ based on:

$U(\hat{p}^t \oplus \tilde{w}) - V(\tilde{w}) = \sum_{i=1}^m [g(\sum_{j=1}^i p_j) - g(\sum_{j=1}^{i-1} p_j)]\{V(\tilde{w} + ty_i) - V(\tilde{w})\} \gtrless 0$

Dividing by t and taking $t \to 0$, this is equivalent to evaluating the gamble over arbitrarily small stakes based on:

$\lim_{t\to 0} \frac{1}{t}\{U(\hat{p}^t \oplus \tilde{w}) - V(\tilde{w})\}$

$= \sum_{i=1}^m [g(\sum_{j=1}^i p_j) - g(\sum_{j=1}^{i-1} p_j)]V_x(\tilde{w})y_i$

which is equivalent to accepting the small-stakes gamble if :

$= \sum_{i=1}^m [g(\sum_{j=1}^i p_j) - g(\sum_{j=1}^{i-1} p_j)]y_i > 0$.

Proof of 2. In DA, $(1+\beta)V_x(\tilde{w}) = \lim_{x\to 0} \frac{1}{x}\{\int\{u(w+x) + \beta\min[u(w+x), V(\tilde{w}+x)]\}dF_{\tilde{w}}(w) - V(\tilde{w})\}$

If $\tilde{w}$ has no mass on $c(\tilde{w})$, then $(1+\beta)V_x(\tilde{w}) = \int\{u'(w) + \beta\mathbb{I}_{w<c(\tilde{w})}u'(w) + \beta\mathbb{I}_{w>c(\tilde{w})}V_x(\tilde{w})\}dF_{\tilde{w}}(w)$

Using a similar argument as in the RDU case,

$(1+\beta)\frac{1}{t}\{U(\hat{p}^t \oplus \tilde{w}) - V(\tilde{w})\}$

$= \frac{1}{t}\sum_{i=1}^n \{V(\tilde{w}+ty_i) - V(\tilde{w}) + \beta\min[V(\tilde{w}+ty_i) - V(\tilde{w}), U(\hat{p}^t \oplus \tilde{w}) - V(\tilde{w})]\}p_i$

So:

$\lim_{t\to 0} \frac{1}{t}(1 + \beta\sum_{i:y_i\geq 0} p_i)\{U(\hat{p}^t \oplus \tilde{w}) - V(\tilde{w})\}$

$= \sum_{i=1}^n p_i(1+\beta\mathbb{I}_{y_i<0})V_x(\tilde{w})y_i$

and so DM accepts the gamble for small stakes if $\sum_{i=1}^{n} p_i(1+\beta\mathbb{I}_{y_i<0})y_i > 0$.

Proof of 3 and 4. Under RDU or DA, $c(\tilde{w}+x) = c(\tilde{w}) + x$ when u is linear or exponential.

When $u(w) = w$, $U(\hat{p}^t \oplus \tilde{w}) - U(\tilde{w}) = t[U(\hat{p} \oplus \tilde{w}) - U(\tilde{w})]$ so part 3 of the theorem holds.

When $u(w) = 1 - \exp(-\gamma w)$ for some $\gamma > 0$, equivalently we could have $u(w) = -\exp(-\gamma w)$. Then,

$U(\hat{p}^t \oplus \tilde{w}) = \sum_i [g(\sum_{j=1}^{i} p_j) - g(\sum_{j=1}^{i-1} p_j)]V(\tilde{w} + ty_i)$

$= -\sum_i [g(\sum_{j=1}^{i} p_j) - g(\sum_{j=1}^{i-1} p_j)] \int \exp(-\gamma(w+ty_i))dg(F_{\tilde{w}}(w))$

$= -\sum_i [g(\sum_{j=1}^{i} p_j) - g(\sum_{j=1}^{i-1} p_j)] \int \exp(-\gamma w)\exp(-\gamma t y_i)dg(F_{\tilde{w}}(w))$

$= -\sum_i [g(\sum_{j=1}^{i} p_j) - g(\sum_{j=1}^{i-1} p_j)] \exp(-\gamma t y_i)V(\tilde{w})$

So DM accepts the gamble if

$-\sum_i [g(\sum_{j=1}^{i} p_j) - g(\sum_{j=1}^{i-1} p_j)] \exp(-\gamma t y_i) > 1$

or equivalently, if

$\sum_i [g(\sum_{j=1}^{i} p_j) - g(\sum_{j=1}^{i-1} p_j)][1 - \exp(-\gamma t y_i)] > 0$.

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5 Supplementary Appendix

Results using first vs. second order risk aversion

Segal and Spivak's (1990) concept of *first-order risk aversion* provides an alternative way of characterizing a model's small-stakes risk preferences by looking at how preferences behave over (arbitrarily) small stakes. Define a decision maker's *risk premium* for a gamble by $\pi^{\hat{p},w} = w - c(\hat{p}+w)$. Say that V is *first-order risk averse* (FORA) if V is risk averse and for any actuarially fair gamble $\hat{p}^t = (ty_1, p_1; ...; ty_n, p_n)$ with positive variance, $\lim_{t\to 0^+} \frac{\pi^{\hat{p}^t,w}}{t} > 0$. Say that V is *second-order risk averse* (SORA) if it is risk averse and $\lim_{t\to 0^+} \frac{\pi^{\hat{p}^t,w}}{t} = 0$ but $\lim_{t\to 0^+} \frac{\pi^{\hat{p}^t,w}}{t^2} > 0$. Intuitively, a FORAdecision maker who faces no risk has a positive marginal willingness to pay to avoid actuarially fair risk. If u is differentiable, EU is only SORA, while risk averse RDU and DA are FORA.

It is possible to work with non-EU preferences without imposing a specific functional form on V. I use the tool of local utility analysis, pioneered by Machina (1982). Say that V admits *local utility analysis relative to norm* $||_a$ if, given a sequence of lotteries $q^t \to q$, there is a function $v(\cdot, q)$ such that $V(q^t) - V(q) = \int v(w, q)[dF_{q^t}(w) - dF_q(w)] + o(||F_{q^t} - F_q||_a)$. Wang (1993) shows that under some technical assumptions, many RDU and betweenness satisfying (Dekel (1986), Chew (1989)) preferences admit local utility analysis relative to the L_ρ norm. Theorem 4 thus gives an alternative and model-independent result on small-stakes risk aversion in nonexpected utility theories under RNEU.

Theorem 4. *Suppose a decision maker has a risk averse non-EU representation V with a differentiable u; let U be the corresponding utility function over compound lotteries. Suppose the decision-maker treats an offered gamble in the presence of background risk as a compound lottery as in (1.3) and evaluates it according to U.*

(i) If V is a FORA RDU or DA non-EU function, then for any distribution of background risk $\tilde{w}$ with support bounded below by some $a > 0$, the decision-maker will be FORA over offered gambles.

(ii) If V is FORA and admits local utility analysis relative to an L_ρ norm for some $\rho > 1$, then the decision maker is FORA over offered gambles.

The results of Theorem 4 suggest that any FORA non-EU V coupled with RNEU will be capable of producing non-trivial small-stakes risk aversion without producing ridiculous large-stakes risk aversion.

Proof of Theorem 4

RDU case. The assumptions of the theorem imply DM has a recursive RDU representation with a concave and differentiable u and a concave and nonlinear g.

I start by proving that the decision maker is FORA over two-outcome gambles.

Let $\hat{p}^t = (ty_1, p; ty_2, 1-p)$ with $py_1 + (1-p)y_2 = 0$.

Since u' exists, $\lim_{t\to 0} \frac{c(\tilde{w}) - c(\hat{p}\oplus\tilde{w})}{t} > 0$ if and only if $\lim_{t\to 0^+} \frac{V(\tilde{w}) - U(\hat{p}^t\oplus\tilde{w})}{t} > 0$.

$U(\hat{p}^t \oplus \tilde{w}) = g(p)V(\tilde{w} - ty_1) + [1-g(p)]V(\tilde{w} + ty_2)$

$= g(p)\int u(w+ty_1)dg(F_{\tilde{w}}(w)) + (1-g(p))\int u(w+ty_2)dg(F_{\tilde{w}}(w))$

Taking limits:

$\lim_{t\to 0^+} \frac{V(\tilde{w})-U(\hat{p}^t\oplus\tilde{w})}{t}$
$= \lim_{t\to 0^+}[g(p)\int \frac{[u(w)-u(w+ty_1)]}{t}dg(F_{\tilde{w}}(w))+(1-g(p))\int \frac{u(w)-u(w+ty_2)}{t}dg(F_{\tilde{w}}(w))]$
$= -[g(p)y_1 + [1-g(p)]y_2]\int u'(w)dg(F_{\tilde{w}}(w)) > 0$

This proof generalizes immediately to the case where there are an arbitrary number of outcomes in p^t, since it relies only on $\sum y_i[g(\sum_{j\leq i} p_j) - g(\sum_{j<i} p_j)] < 0$ if $\sum y_i p_i = 0$, which follows by the concavity and nonlinearity of g.

DA case Write the DA utility for a lottery as: $V(p) = \int(u(w) + \beta\min[u(w), V(p)])dF_p(w)$

So:

$U(\hat{p}^t\oplus\tilde{w}) = \int[u(c(\tilde{w}+ty))+\beta\min[U(\hat{p}^t\oplus\tilde{w}), u(c(\tilde{w}+ty))]]dF_{\hat{p}}(y)$

$\lim_t \frac{U(\hat{p}^t\oplus\tilde{w})-V(\tilde{w})}{t} = \lim_t \int[\frac{u(c(\tilde{w}+ty))-u(c(\tilde{w}))}{t}+\beta(\min[U(\hat{p}^t\oplus\tilde{w}), u(c(\tilde{w}+ty))] - u(c(\tilde{w})))]dF_{\hat{p}}(y)$

$= \int[yV_x(\tilde{w})(1+\beta\mathbb{I}_{y<0}) + \beta\mathbb{I}_{y\geq 0}\lim_t \frac{U(\hat{p}^t\oplus\tilde{w})-V(\tilde{w})}{t}]dF_{\hat{p}}(y)$

Since $\int ydF_{\hat{p}}(y) = 0$, and $1+\beta\mathbb{I}_{y<0}$ is decreasing in y and nonconstant for $\beta > 0$, it follows that $\int yV_x(\tilde{w})(1+\beta\mathbb{I}_{y<0})dF_{\hat{p}}(y) < 0$ hence $\lim_t \frac{U(\hat{p}^t\oplus\tilde{w})-V(\tilde{w})}{t} < 0$ hence the decision-maker is FORA over offered gambles in this case.

Local utility analysis Let $V_x(\tilde{w}) = \lim_{x\to 0}\frac{V(\tilde{w}+x)-V(\tilde{w})}{x}$. When V is amenable to local utility analysis, $\frac{V(\tilde{w}+x)-V(\tilde{w})}{x} = \int \frac{1}{x}[v(w+x,\tilde{w}) - v(w,\tilde{w})]dF_{\tilde{w}}(w) + \frac{o(||(\tilde{w}+x)-\tilde{w}||)}{x}$

$||F_{(\hat{p}^t\oplus\tilde{w})^{REC}} - F_{c(\tilde{w})}||_\rho = (\sum_i |c(\tilde{w}+ty_i) - c(\tilde{w})|[\mathbb{I}_{y_i<0}\sum_{j=1}^i q_j + \mathbb{I}_{y_i>0}(1-\sum_{j=1}^i q_j)]^\rho)^{\frac{1}{\rho}}$.

Since $\frac{c(\tilde{w}+ty_i)-c(\tilde{w})}{t} \to \frac{1}{u'(c(\tilde{w}))}\lim \frac{V(\tilde{w}+ty)-V(\tilde{w})}{y} < \infty$, it follows that $\lim_{t\to 0^+}\frac{||F_{(\hat{p}^t\oplus\tilde{w})^{REC}}-F_{c(\tilde{w})}||_\rho}{t}$ is finite. Now write:

$V((\hat{p}^t\oplus\tilde{w})^{REC}) - V(c(\tilde{w})) = \int[v(c(\tilde{w}+ty), F_{c(\tilde{w})}) - v(c(\tilde{w}), F_{c(\tilde{w})})]dF_{\hat{p}}(y) + o(||(\hat{p}^t\oplus\tilde{w})^{REC} - \tilde{w}||)$

$\lim \frac{1}{t}\int[v(c(\tilde{w}+ty), F_{c(\tilde{w})}) - v(c(\tilde{w}), F_{c(\tilde{w})})]dF_{\hat{p}}(y) + \frac{o(||(\hat{p}^t\oplus\tilde{w})^R-\tilde{w}||_\rho)}{||(\hat{p}^t\oplus\tilde{w})^R-\tilde{w}||_\rho}\frac{||(\hat{p}^t\oplus\tilde{w})^R-\tilde{w}||_\rho}{t}$

$$= \lim \frac{1}{t} \int [v(c(\tilde{w} + ty), F_{c(\tilde{w})}) - v(c(\tilde{w}), F_{c(\tilde{w})})] dF_{\hat{p}}(y) \qquad (5.1)$$

From Machina (1982) and Wang (1993), $v(\cdot, F_{c(\tilde{w})})$ is concave since V is risk averse, so $\lim_{\epsilon \to 0^+} \frac{v(c(\tilde{w})+\epsilon, F_{c(\tilde{w})}) - v(x, F_{c(\tilde{w})})}{\epsilon} = v'^{+}(c(\tilde{w}), F_{c(\tilde{w})})$ and $\lim_{\epsilon \to 0^-} \frac{v(c(\tilde{w})+\epsilon, F_{c(\tilde{w})}) - v(x, F_{c(\tilde{w})})}{\epsilon} = v'^{-}(c(\tilde{w}), F_{c(\tilde{w})})$ both exist. Segal and Spivak (1997) show since V is FORA, $v(\cdot, F_{c(\tilde{w})})$ is not differentiable at $c(\tilde{w})$ and hence $v'^{+}(c(\tilde{w}), F_{c(\tilde{w})}) < v'^{-}(c(\tilde{w}), F_{c(\tilde{w})})$. Thus, (5.1) can be rewritten as

$= \int y v'^{\text{sign}(y)}(c(\tilde{w}), F_{c(\tilde{w})}) \frac{V_x(\tilde{w})}{u'(c(\tilde{w}))} dF_{\hat{p}}(y) < 0$. Since $\int y dF_{\hat{p}}(y) = 0$, it is also the case that $\int y v'^{\text{sign}(y)}(c(\tilde{w}), F_{c(\tilde{w})}) dF_{\hat{p}}(y) < 0$.