Manipulating decision making of typical agents

V.I. Yukalov[1,2] and D. Sornette[1,3]

[1] *Department of Management, Technology and Economics, ETH Zürich, Swiss Federal Institute of Technology, Zürich CH-8032, Switzerland*

[2] *Bogolubov Laboratory of Theoretical Physics, Joint Institute for Nuclear Research, Dubna 141980, Russia*

[3] *Swiss Finance Institute, c/o University of Geneva, 40 blvd. Du Pont d'Arve, CH 1211 Geneva 4, Switzerland*

Abstract

We investigate how the choice of decision makers can be manipulated under the presence of risk and uncertainty. Our analysis is based on the Quantum Decision Theory (QDT) previously introduced by the authors, which we generalize to the case of decision makers that are members of a society. Similarly to the concept of a representative agent in economics, the notion of a typical decision maker, representing the average behavior within a given society, is introduced and characterized. QDT describes an agent's choice as a probabilistic event occurring with a probability that is the sum of a utility factor and of an attraction factor. The attraction factor embodies subjective and unconscious dimensions in the mind of the decision maker. The most efficient manipulation of decision making is realized by influencing the attraction factors of decision makers. This can be done in two ways. One method is to arrange in a special way the payoff weights, which induces the required changes of values of attraction factors. We show that a variation of the payoff weights can inverse the attraction factor values and reverse the decision preferences, even when the prospect utilities are not changed. The second method of manipulation is by providing information to decision makers or by allowing consultations in the society. The attraction factors can be either decreased, when decision makers obtain correct information, or increased, if the delivered information is wrong. The variation of the attraction factors, induced by positive or negative information, can lead to the reversal of preferences. The methods of manipulating decision making are illustrated by several experiments, whose outcomes are compared quantitatively with the prediction of QDT.

JEL classification: C44, D03, D71, D81, D83

Keywords: Decision theory, Decision making under risk and uncertainty, Group consultations, Social interactions, Information and knowledge

Corresponding authors: D. Sornette and V.I. Yukalov
E-mail address: dsornette@ethz.ch and yukalov@theor.jinr.ru

1 Introduction

How to manipulate decision choices made by separate decision makers as well as by societies of many agents is an important and widely studied problem of psychology (Wilson and Schooler 1991; Wilson et al. 2005; Gneezy et al. 2006; Norton et al. 2007; Payne et al. 2008; Koehler and James 2009). This problem is important for a variety of practical applications ranging from medicine (Polister 1989; O'Carrol 2003) to politics (Barber 1984; Gutmann and Thompson 1996; Dryzek 2000; Chambers 2003; Hafer and Landa 2007). Numerous articles are devoted to the effects of manipulation in economics, studying the influence of different framing effects on product evaluation (Schul and Ganzach 1995; Armel et al. 2008; Lerouge 2009), consumer response to price (Busemeyer and Townsend 1993; Heath et al. 1995; Busemeyer and Diedrich 2002; Armel and Rangel 2008), evaluation of retail outlets (Kellaris et al. 1995), market advertizing (Keller 1991; Lee et al. 2009; Goldsmith and Amir 2010), buying decisions (Qualls and Puto 1989; Gibbs 1997), perceptions of control and efficacy (Koehler et al. 1994), distributive justice (Kinsey et al. 1991), performance feedbacks (Hogarth et al. 1991), and so on.

In the present paper, we consider a *mathematical model* describing how decision makers can be manipulated and how it would be possible to *quantitatively* evaluate the influence of such a manipulation. For this purpose, we generalize the Quantum Decision Theory (QDT), developed earlier by the authors (Yukalov and Sornette 2008, 2009a,b,c, 2010a,b, 2011) for individual decision makers, to the case of decision makers in a society. This generalization is formulated in Sec. 2. In experiments, one usually deals with large groups of decision makers with different preferences. In order to compare theoretical results with experimental, we introduce and characterize, in Sec. 3, the notion of typical social agents. In Sec. 4, we explain how it is possible to manipulate the typical decision maker preference by varying the arrangement of prospects. We formulate a criterion for the inversion of the attraction factor leading to the inversion of preferences. In Sec. 5, the results, predicted by our approach, are compared with several classical experiments, demonstrating good quantitative agreement. In Sec. 6, we show how decision makers can be manipulated by providing them additional either correct or wrong information. Section 7 concludes.

2 Quantum decision making in society

In this section, we generalize the QDT approach, whose detailed exposition can be found in our previous publication (Yukalov and Sornette 2012), developed for individual decision makers, to a society of many decision makers. We recall that the decision makers are not quantum objects, but are usual humans. The techniques of quantum theory are employed merely for taking into account the hidden variables, such as emotions and biases of decision makers (Yukalov and Sornette 2011).

Let us consider a society of N agents who are decision makers. The agents are enumerated by $\alpha = 1, 2, \ldots, N$. Each agent is characterized by a set $\{e_{\alpha n} : n = 1, 2, \ldots, d\}$ of d elementary prospects that are represented by vectors $|\alpha n\rangle$ in a Hilbert space. Different elementary prospects are orthonormal to each other,

$$\langle \alpha m | \alpha n \rangle = \delta_{mn} \; ,$$

which symbolizes their mutual independence and incompatibility. The space of mind of an α - decision maker is the Hilbert space

$$\mathcal{H}_\alpha \equiv \text{Span}_n\{|\alpha_n\rangle\} \; . \tag{1}$$

The dimension of this space of mind is d. The space of mind of the whole society is the tensor product

$$\mathcal{H} = \bigotimes_{\alpha=1}^{N} \mathcal{H}_\alpha \; , \tag{2}$$

whose dimension is Nd.

An α - agent deliberates on deciding between L prospects forming a complete lattice

$$\mathcal{L}_\alpha \equiv \{\pi_{\alpha j} : \; j = 1, 2, \ldots, L\} \; . \tag{3}$$

Each prospect $\pi_{\alpha j}$ is represented by a vector $|\pi_{\alpha j}\rangle$ in the space of mind (1). The prospect vectors do not need to be orthonormal, which implies that they are not necessarily incompatible.

The prospect operator

$$\hat{P}(\pi_{\alpha j}) \equiv |\pi_{\alpha j}\rangle\langle\pi_{\alpha j}| \tag{4}$$

acts on the space of mind (1). The set of all these operators is analogous to the algebra of local observables in quantum theory (von Neumann 1955). Respectively, the prospect probabilities are defined as the expectation values of the prospect operators. The expectation values for individual decision makers are given by averaging the prospect operators over a strategic state of the decision maker (Yukalov and Sornette 2011), being a pure state represented by a single vector.

However, for agents in a society, pure states of each agent, generally, do not exist, since the society agents interact with each other by exchanging information. Moreover, the society as a whole may not be completely isolated from the surrounding. Therefore the society state has to be characterized by a statistical operator $\hat{\rho}$ that is a non-negative normalized operator,

$$\text{Tr}_{\mathcal{H}} \hat{\rho} = 1 \; , \tag{5}$$

where the trace operation is over the society space (2). Then the expectation values of the prospect operators are given by the trace

$$p(\pi_{\alpha j}) \equiv \text{Tr}_{\mathcal{H}} \hat{\rho} \hat{P}(\pi_{\alpha j}) \; , \tag{6}$$

defining the probabilities of the corresponding prospects. This definition makes the basic difference in calculating the prospect probabilities, as compared to the averaging over a single strategic state for individual decision makers (Yukalov and Sornette 2012).

Quantity (6), by its construction, is non-negative and defines the prospect probabilities under the normalization condition

$$\sum_{j=1}^{L} p(\pi_{\alpha j}) = 1 \; , \qquad 0 \leq p(\pi_{\alpha j}) \leq 1 \; . \tag{7}$$

Remembering that the prospect operator (4) acts on the space of mind (1) and introducing the reduced statistical operator

$$\hat{\rho}_\alpha \equiv {\rm Tr}_{\mathcal{H}/\mathcal{H}_\alpha} \hat{\rho} \; ,$$

in which the trace is over the partial factor space

$$\mathcal{H}/\mathcal{H}_\alpha \equiv \bigotimes_{\beta(\neq\alpha)}^N \mathcal{H}_\beta \; ,$$

makes it possible to rewrite the prospect probability (6) in the form

$$p(\pi_{\alpha j}) = {\rm Tr}_{\mathcal{H}_\alpha} \hat{\rho}_\alpha \hat{P}(\pi_{\alpha j}) \; , \tag{8}$$

with the trace over the space of mind (1).

Expanding the prospect vectors over the elementary prospect basis, with introducing the matrix elements

$$\rho^\alpha_{mn} \equiv \langle \alpha m | \hat{\rho}_\alpha | \alpha n \rangle \; , \qquad P_{mn}(\pi_{\alpha j}) \equiv \langle \alpha m | \hat{P}(\pi_{\alpha j}) | \alpha n \rangle \; , \tag{9}$$

it is straightforward to get the prospect probability

$$p(\pi_{\alpha j}) = f(\pi_{\alpha j}) + q(\pi_{\alpha j}) \; , \tag{10}$$

consisting of two terms. The first term, called the *utility factor*,

$$f(\pi_{\alpha j}) = \sum_n \rho^\alpha_{nn} P_{nn}(\pi_{\alpha j}) \; , \tag{11}$$

describes the classical objective probability, showing how the considered prospect is useful for the decision maker. While the second term, called the *attraction factor*,

$$q(\pi_{\alpha j}) = \sum_{m\neq n} \rho^\alpha_{mn} P_{nm}(\pi_{\alpha j}) \; , \tag{12}$$

characterizes the subjective influence of subconscious feelings, emotions, and biases and shows to what extent the prospect is attractive for the decision maker.

By its definition, the utility factor (11) is non-negative,

$$0 \leq f(\pi_{\alpha j}) \leq 1 \; , \tag{13}$$

and also it is normalized,

$$\sum_{j=1}^L f(\pi_{\alpha j}) = 1 \; , \tag{14}$$

representing the classical objective probability. In the case, when the prospect utilities $U(\pi_{\alpha j})$ can be evaluated by means of the classical utility theory, the utility factor takes the form

$$f(\pi_{\alpha j}) = \frac{U(\pi_{\alpha j})}{\sum_j U(\pi_{\alpha j})} \; . \tag{15}$$

The attraction factor (12), by its definition, varies in the range

$$-1 \leq q(\pi_{\alpha j}) \leq 1 \; . \tag{16}$$

An important property of the attraction factor, following from conditions (7) and (14), is the *alternation property*

$$\sum_{j=1}^{L} q(\pi_{\alpha j}) = 0 \; . \tag{17}$$

It is worth mentioning that the attraction factor comes into play only for composite prospects experiencing mutual interference (Yukalov and Sornette 2011). But for elementary prospects, it does not occur, being identically zero:

$$q(e_{\alpha n}) = 0 \; .$$

Having defined the prospect probabilities, the prospects become naturally ordered. A prospect $\pi_{\alpha 1}$ is said to be preferred to a prospect $\pi_{\alpha 2}$ if and only if

$$p(\pi_{\alpha 1}) > p(\pi_{\alpha 2}) \qquad (\pi_{\alpha 1} > \pi_{\alpha 2}) \; . \tag{18}$$

The prospects $\pi_{\alpha 1}$ and $\pi_{\alpha 2}$ are indifferent if and only if

$$p(\pi_{\alpha 1}) = p(\pi_{\alpha 2}) \qquad (\pi_{\alpha 1} = \pi_{\alpha 2}) \; . \tag{19}$$

And the prospect $\pi_{\alpha 1}$ is preferred or indifferent to $\pi_{\alpha 2}$ if

$$p(\pi_{\alpha 1}) \geq p(\pi_{\alpha 2}) \qquad (\pi_{\alpha 1} \geq \pi_{\alpha 2}) \; . \tag{20}$$

A prospect π_α^* that corresponds to the maximal probability

$$p(\pi_\alpha^*) = \max_j p(\pi_{\alpha j})$$

is called optimal.

3 Typical behavior of social agents

Let all agents in a society confront the same prospect lattice (3), with the same prospects $\pi_\alpha = \pi_{\alpha j}$. The agents composing the society are different individuals and their decisions, even related to the same set of prospects, can vary, producing different probabilities $p(\pi_{\alpha j})$.

The society as a whole can be characterized by the average probability

$$p(\pi_j) \equiv \frac{1}{N} \sum_{\alpha=1}^{N} p(\pi_{\alpha j}) \; , \tag{21}$$

averaged over all society members, which describes the typical behavior of agents. In view of expression (10), the typical probability (21) reads as

$$p(\pi_j) = f(\pi_j) + q(\pi_j) \; , \tag{22}$$

with the typical utility factor

$$f(\pi_j) \equiv \frac{1}{N} \sum_{\alpha=1}^N f(\pi_{\alpha j}) \tag{23}$$

and typical attraction factor

$$q(\pi_j) \equiv \frac{1}{N} \sum_{\alpha=1}^N q(\pi_{\alpha j}) \; . \tag{24}$$

Because of Eqs. (13) and (14), the typical utility factor, describing the objective probability, satisfies the conditions

$$\sum_{j=1}^L f(\pi_j) = 1 \; , \qquad 0 \leq f(\pi_j) \leq 1 \; . \tag{25}$$

In the case when it is defined by the prospect utilities as in Eq. (15), it reduces to the expression

$$f(\pi_j) = \frac{U(\pi_j)}{\sum_j U(\pi_j)} \; , \tag{26}$$

where we take into account that objective utilities are invariant with respect to agents, so that $U(\pi_{\alpha j}) = U(\pi_j)$.

The attraction factor, generally, is not invariant with respect to different decision makers, but, owing to Eqs. (16) and (17), it preserves the conditions

$$\sum_{j=1}^L q(\pi_j) = 0 \; , \qquad -1 \leq q(\pi_j) \leq 1 \; . \tag{27}$$

Moreover, assuming that the attraction factor for different decision makers is a random quantity, with its modulus $|q(\pi_j)|$ varying in the interval $[0, 1]$, one can show (Yukalov and Sornette 2011) that it satisfies the *quarter law*

$$\frac{1}{L} \sum_{j=1}^L |q(\pi_j)| = \frac{1}{4} \; . \tag{28}$$

In this way, each prospect is evaluated by the society with respect to two points, its utility and its attractiveness. A prospect π_i is more useful than π_j, if $f(\pi_i) > f(\pi_j)$. And a prospect π_i is more attractive than π_j, if $q(\pi_i) > q(\pi_j)$. Therefore, a prospect can be more useful, but not preferred, being less attractive. As follows from expression (22), a prospect π_1 is preferred to a prospect π_2, in the sense of definition (18), when

$$p(\pi_1) > p(\pi_2) \qquad (\pi_1 > \pi_2) \; ,$$

if and only if the inequality

$$f(\pi_1) - f(\pi_2) > q(\pi_2) - q(\pi_1) \tag{29}$$

holds true.

Actually, the comparison of theory with experiment is meaningful only for a sufficiently large pool of decision makers, when the general typical features can be defined. When, in such a large society, the number of agents choosing a prospect π_j is N_j, then the experimentally observed fraction

$$p_{exp}(\pi_j) \equiv \frac{N_j}{N} \tag{30}$$

provides the aggregate frequentist definition of probability that should be compared with the theoretical value (22).

4 Manipulation by reversing attraction factors

In the prospect probability (22), the first term (23) is an objectively defined quantity characterizing the prospect utility. It would, of course, be possible to change the society choice by varying the utility of prospects. This, however, is not what is called manipulation, but it is just an objective shift of preferences caused by the varying prospect utilities.

Under *manipulation*, one understands the possibility of essentially changing the decision maker choice merely by influencing the attractiveness of the considered prospects, without essentially varying their utilities. This means that these are the attraction factors that are to be manipulated.

4.1 Prospect probabilities for binary lattices

The most often and illustrative case is the choice between two prospects forming a binary lattice

$$\mathcal{L} = \{\pi_1, \pi_2\} \; . \tag{31}$$

Suppose that the prospect π_1 is more attractive than π_2, which means that

$$q(\pi_1) > q(\pi_2) \; .$$

According to the alternation property (27), we have

$$q(\pi_1) = -q(\pi_2). \tag{32}$$

Then, taking into account the quarter law (28), we can estimate the attraction factor $q(\pi_1)$ as $1/4$, while the attraction factor $q(\pi_2)$ as $-1/4$. Keeping in mind that a probability, by its meaning, lies in the interval $[0,1]$, the prospect probabilities can be evaluated by the formulas

$$p(\pi_1) = \mathrm{Ret}_{[0,1]}\left\{ f(\pi_1) + \frac{1}{4} \right\} \; , \qquad p(\pi_2) = \mathrm{Ret}_{[0,1]}\left\{ f(\pi_2) - \frac{1}{4} \right\} \; , \tag{33}$$

where the retract function

$$\mathrm{Ret}_{[0,1]}\{z\} \equiv \begin{cases} 0, & z < 0 \\ z, & 0 \le z \le 1 \\ 1, & z > 1 \end{cases}$$

is employed.

4.2 Definition of more attractive prospects

Formulas (33) could be used for evaluating the prospect probabilities of the binary lattice (31). The sole thing that is left for the straightforward application of these formulas to concrete cases is a practical rule allowing one to distinguish the prospects as more and less attractive. Being based on the notion of aversion to uncertainty and risk, or ambiguity aversion (Gollier 2001; Sornette 2003; Malevergne and Sornette 2006; Abdellaoui et al 2011a,b), it is possible to define as more attractive the prospect that provides more certain gain, hence, more uncertain loss (Yukalov and Sornette 2011). However, it is necessary to specify what does mean to be "more certain".

Let us consider two prospects

$$\pi_1 = \{x_n,\ p_1(x_n)\} \; , \qquad \pi_2 = \{x'_n,\ p_2(x'_n)\} \; , \tag{34}$$

characterized by the sets of payoffs $\{x_n\}$ and $\{x'_n\}$ and the corresponding payoff weights, or probabilities, obeying the normalization conditions

$$\sum_n p_1(x_n) = 1 \; , \qquad \sum_n p_2(x'_n) = 1 \; .$$

Let the largest payoffs of each prospect be denoted as

$$x_{max} \equiv \max_n \{x_n\} \; , \qquad x'_{max} \equiv \max_n \{x'_n\} \; . \tag{35}$$

Suppose that the maximal payoff of the first prospect is larger than that of the second prospect:

$$x_{max} > x'_{max}, \tag{36}$$

so that the utility factor of the first prospect be slightly greater than or equal to that of the second prospect:

$$f(\pi_1) \geq f(\pi_2) \; , \qquad |f(\pi_1) - f(\pi_2)| \ll 1 \; . \tag{37}$$

And let the payoff x'_{max} be more probable than x_{max}, so that the related payoff weights would satisfy the inequality

$$p_1(x_{max}) < p_2(x'_{max}) \; . \tag{38}$$

Thus, the first prospect provides a larger possible payoff than the second prospect, but with a smaller payoff weight. Then, which of the prospects is more attractive?

Deciding on this problem, we have to resort to the known empirical fact, showing that people typically pay more attention to payoff weights that are larger than some critical value p_c and pay less attention to the weights that are smaller than this critical value (Kahneman and Tversky 1979; Tversky and Kahneman 1992; van de Kuilen 2007). It is natural to associate this critical value with 1/2, since it is with respect to this value people usually name the probability as being high or low. In line with this experimental observation, we suggest the following simple criterion.

One-half criterion. Among the two prospects defined above that are characterized by (37), the first prospect is less attractive, if the weight of the largest payoff of the second prospect is larger than one-half:

$$p_2(x'_{max}) > \frac{1}{2} \; , \tag{39}$$

which leads to

$$q(\pi_1) < q(\pi_2) \ . \tag{40}$$

Then, the probability $p(\pi_1)$ that the first project is chosen becomes $p(\pi_1) \equiv f(\pi_1) + q(\pi_1) = f(\pi_1) - q(\pi_2)$, by the alternation property (27) simplifying into (32) for two prospects, which must compared with probability $p(\pi_2)$ that the second project is chosen, given by $p(\pi_2) = f(\pi_2) + q(\pi_2)$. This shows mathematically how a prospect with smaller utility factor can be actually preferred due to a large attraction factor associated with a probability of the largest payoff greater than 1/2. In contrast, the first prospect is more attractive when the weight of the largest payoff of the second prospect is smaller than one-half:

$$p_2(x'_{max}) < \frac{1}{2} \ , \tag{41}$$

which leads to

$$q(\pi_1) > q(\pi_2) \ . \tag{42}$$

In the following subsection, we show that this criterion allows us to quantitatively predict the results of experiments. At the same time, this criterion gives a key of how it is possible to manipulate human decisions by arranging the prospects in such a way that would lead to the attraction reversal, and, consequently, to the preference reversal.

5 Illustration of preference reversal by examples

Here we consider several examples of experiments described by Kahneman and Tversky (1979). In these experiments, the total number of decision makers was about or smaller than one hundred, and the corresponding statistical errors were around $10\% - 12\%$. Decision makers had to choose between two prospects having the properties as those discussed above. Payoff were counted in monetary units, say in thousands of schekels, francs, or dollars. The kind of monetary units has no influence on the relative quantities, such as utility factors and prospect probabilities. Calculating the utility factors, we use, for simplicity, a linear utility function.

Example 1. One chooses between the prospects

$$\pi_1 = \{2.5, 0.33 \mid 2.4, 0.66 \mid 0, 0.01\} \ , \qquad \pi_2 = \{2.4, 1\} \ .$$

Using definition (26) gives the utility factors

$$f(\pi_1) = 0.501, \qquad f(\pi_2) = 0.499 \ .$$

To obtain these numbers, we have used

$$f(\pi_1) = \frac{U(\pi_1)}{U(\pi_1) + U(\pi_2)} \ , \quad f(\pi_2) = \frac{U(\pi_2)}{U(\pi_1) + U(\pi_2)} \ ,$$

with

$$U(\pi_1) = 2.5 \times 0.33 + 2.4 \times 0.66 + 0 \times 0.01 \ , \quad U(\pi_2) = 2.4 \ .$$

Since $p_2(x'_{max}) = 1 > 0.5$, the first prospect is less attractive, as follows from the one-half criterion. Then, $q(\pi_1)$ can be estimated as $-1/4$, while $q(\pi_2)$, as $1/4$. Thus, we get the prospect probabilities

$$p(\pi_1) = 0.251, \qquad p(\pi_2) = 0.749 .$$

In experiments, it was found that

$$p_{exp}(\pi_1) = 0.18, \qquad p_{exp}(\pi_2) = 0.82 ,$$

which, within the experimental accuracy, coincides with the theoretical prediction.

Example 2. One considers the prospects

$$\pi_1 = \{2.5, 0.33 \mid 0, 0.67\} , \qquad \pi_2 = \{2.4, 0.34 \mid 0, 0.66\} .$$

The utility factors are practically the same as in the previous example:

$$f(\pi_1) = 0.503, \qquad f(\pi_2) = 0.497 .$$

But now, because $p_2(x'_{max}) = 0.34 < 0.5$, the first prospect becomes more attractive, which gives the prospect probabilities

$$p(\pi_1) = 0.753, \qquad p(\pi_2) = 0.247 .$$

Again, this is in agreement with the experimental values

$$p_{exp}(\pi_1) = 0.83, \qquad p_{exp}(\pi_2) = 0.17 ,$$

being in the corridor of statistical errors.

Comparing the examples 1 and 2, we see that a change in the distribution of payoff weights, under the same payoffs, has lead to the reversal of the attraction factors and, as a result, to the preference reversal.

Example 3. The prospects are

$$\pi_1 = \{4, 0.8 \mid 0, 0.2\} , \qquad \pi_2 = \{3, 1\} .$$

The utility factors (26) become

$$f(\pi_1) = 0.516, \qquad f(\pi_2) = 0.484 .$$

As far as $p_2(x'_{max}) = 1 > 0.5$, the second prospect is more attractive. Then, we have the prospect probabilities

$$p(\pi_1) = 0.266, \qquad p(\pi_2) = 0.734 ,$$

which agree well with the empirical results

$$p_{exp}(\pi_1) = 0.2, \qquad p_{exp}(\pi_2) = 0.8 .$$

Example 4. The prospects

$$\pi_1 = \{4, 0.2 \mid 0, 0.8\} , \qquad \pi_2 = \{3, 0.25 \mid 0, 0.75\}$$

have the same payoffs and the same utility factors

$$f(\pi_1) = 0.516, \qquad f(\pi_2) = 0.484 ,$$

as in the previous case. But now $p_2(x'_{max}) = 0.25 < 0.5$. Hence the first prospect is more attractive. This gives the prospect probabilities

$$p(\pi_1) = 0.766, \qquad p(\pi_2) = 0.234 ,$$

with the reverse preference, as compared to Example 3. The experimental results

$$p_{exp}(\pi_1) = 0.65, \qquad p_{exp}(\pi_2) = 0.35$$

are in agreement with the theoretical prediction.

Example 5. For the prospects

$$\pi_1 = \{6, 0.45 \mid 0, 0.55\} , \qquad \pi_2 = \{3, 0.9 \mid 0, 0.1\} ,$$

the utility factors are equal,

$$f(\pi_1) = 0.5, \qquad f(\pi_2) = 0.5 .$$

Owing to $p_2(x'_{max}) = 0.9 > 0.5$, the second prospect is more attractive. Then

$$p(\pi_1) = 0.25, \qquad p(\pi_2) = 0.75 .$$

The experimental results

$$p_{exp}(\pi_1) = 0.14, \qquad p_{exp}(\pi_2) = 0.86 ,$$

within the statistical errors of 12%, agree with the theoretical prediction.

Example 6. The prospects

$$\pi_1 = \{6, 0.001 \mid 0, 0.999\} , \qquad \pi_2 = \{3, 0.002 \mid 0, 0.998\}$$

lead to the same utility factors

$$f(\pi_1) = 0.5, \qquad f(\pi_2) = 0.5 ,$$

as in the previous example. However, now $p_2(x'_{max}) = 0.002 < 0.5$ together with $x'_{max} < x_{max}$, which makes the second prospect less attractive. As a result, the prospect preference reverses, as compared to Example 5, with the prospect probabilities

$$p(\pi_1) = 0.75, \qquad p(\pi_2) = 0.25 .$$

The experimental data

$$p_{exp}(\pi_1) = 0.73, \qquad p_{exp}(\pi_2) = 0.27$$

practically coincide with the theoretical prediction, again demonstrating the preference reversal.

In the above examples, we have considered prospects that are characterized by different gains. The treatment of prospects, involving losses, is a separate problem. Strictly speaking, in real life, in order to lose something, it is in general the case that one possesses a wealth no less than the loss. However, there are also examples of negative wealth, associated with debts that are larger than present equity. For a firm, this leads in general to bankruptcy. Rationally, agents should also default on their debts, if they can, a situation that often but not always occurs, as for instance exemplified by the many cases of negative equity of homeowners in the USA (Bhutta et al., 2010) and Great Britain (Hellebrandt et al. 2009) following the real estate price collapse and financial crisis[1]. Thus, in general, we should expect that the prospect probabilities depend on the initial richness of decision makers. But in the laboratory experiments, one usually considers artificial situations, with imaginary or unrealistic small losses, when the starting assets are not important. The real and imaginary losses are rather different things and are to be treated differently. However these delicate problems are out of the scope of the present paper.

Our aim has been to demonstrate the fact that, under the same utility, by appropriately arranging the payoff weights, it is possible to realize the reversal of the attraction factors and, as a result, the reversal of decision preferences.

6 Manipulation by varying available information

The standard setup of studying decision making in the laboratory is when decision makers are assumed to give responses without consulting each other and without looking for additional information. However, in a number of experimental studies, it has been found that decisions can essentially change when the agents are allowed to consult with each other, increasing by this their mutual information (Charness and Rabin 2002; Blinder and Morgan 2005; Cooper and Kagel 2005; Charness et al. 2007a,b; Chen and Li 2009; Charness et al. 2010), or when they can get additional information learning from their own experience (Kühberger et al. 2001).

When the objective parts of the prospect probabilities are assumed to remain invariant, the influence on decision making of the obtained information can be realized by varying the attraction factors. Therefore, we have to understand how the latter vary with respect to the change of information available to decision makers.

Let us denote by μ the measure of information available to a decision maker. This measure can be defined according to one of the known ways of measuring information (Arndt 2004). Decision making depends on the amount of this information and varies with the variation of the latter (Dong et al. 2008).

Generalizing the consideration of Sec. 2, we take into account that the society state, represented by the statistical operator $\hat{\rho}(\mu)$, depends on the available information μ. The prospect probabilities of an α - agent take the form

$$p(\pi_{\alpha j}, \mu) = \mathrm{Tr}_{\mathcal{H}} \hat{\rho}(\mu) \hat{P}(\pi_{\alpha j}) \; , \tag{43}$$

[1]The Wall Street Journal reported on 24 Nov. 2009 that 10.7 U.S. million households, or 23% of homeowners with mortgage, had negative equity, according to the First American Home CoreLogic, a real-estate information company based in Santa Ana, California (`http://s.wsj.net/public/resources/documents/info-NEGATIVE_EQUITY_0911.html`)

where all notations are the same as in Sec. 2.

The variation of the society state with information can be described by the information evolution operator $\hat{U}(\mu)$, so that

$$\hat{\rho}(\mu) = \hat{U}(\mu)\hat{\rho}\hat{U}^+(\mu) , \tag{44}$$

where

$$\hat{\rho}(0) = \hat{\rho} . \tag{45}$$

As before, the society state is normalized, such that

$$\mathrm{Tr}_{\mathcal{H}}\hat{\rho}(\mu) = 1 . \tag{46}$$

The initial condition (43) yields

$$\hat{U}(0) = \hat{1}_{\mathcal{H}} , \tag{47}$$

with $\hat{1}_{\mathcal{H}}$ being the unity operator on space (2). And the normalization condition (44) requires that the evolution operator be a unitary operator:

$$\hat{U}^+(\mu)\hat{U}(\mu) = \hat{1}_{\mathcal{H}} . \tag{48}$$

These properties make it possible to represent the evolution operator as

$$\hat{U}(\mu) = e^{-i\hat{H}\mu} , \tag{49}$$

where $\hat{H}$, acting on space (2), is called the evolution generator.

The general form of the evolution generator can be written as the sum of the terms acting on each of the decision makers in the society and the term characterizing the decision makers interactions:

$$\hat{H} = \bigoplus_{\alpha=1}^{N} \hat{H}_\alpha + \hat{H}_{int} , \tag{50}$$

where $\hat{H}_\alpha$ acts on space (1) and $\hat{H}_{int}$, on space (2).

The agents of the society are considered as separate individuals who, though interacting with each other, do not loose their personal identities and are able to take individual decisions. In mathematical language, this means that agents are quasi-isolated (Yukalov 2012; Yukalov and Sornette, 2012). The mathematical formulation of the quasi-isolation state reads as the commutation condition

$$\left[\hat{H}_\alpha \bigotimes \hat{1}_{\mathcal{H}} \; , \; \hat{H}_{int} \right] = 0 . \tag{51}$$

Similarly to Sec. 3, assuming that all agents are confronted with the same prospect lattice, we introduce the notion of a typical agent, whose decisions are described by the average prospect probabilities

$$p(\pi_j, \mu) = \frac{1}{N} \sum_{\alpha=1}^{N} p(\pi_{\alpha j}, \mu) . \tag{52}$$

The property of quasi-isolation (49) makes it possible to show (Yukalov and Sornette 2012) that the prospect probabilities (50) acquire the form

$$p(\pi_j, \mu) = f(\pi_j) + q(\pi_j, \mu) \; , \qquad (53)$$

similar to Eq. (22). Here, the first term is the utility factor that is the same as in Eqs. (23) and (26). The second term is the attraction factor that can be represented as

$$q(\pi_j, \mu) = q(\pi_j) D(\mu) \; , \qquad (54)$$

where

$$q(\pi_j) = q(\pi_j, 0) \qquad (55)$$

is the attraction factor at the initial state, when no additional information has yet been consumed, and $D(\mu)$ is a decoherence factor. The name of the latter comes from the fact that, technically, the attraction factor appears under the interference of composite prospects (Yukalov and Sornette 2011). Decoherence implies that the interference effects fade away, so that the prospect probabilities tend to their classical values defined by the utility factors. In other words, this means (Yukalov and Sornette 2012) that

$$\lim_{\mu \to \infty} p(\pi_j, \mu) = f(\pi_j) \; . \qquad (56)$$

Treating the agent interactions as a scattering process over random scatterers with the width μ_c in the Lorentzian distribution (Yukalov and Sornette 2012), we have

$$D(\mu) = \exp\left(- \frac{\mu}{\mu_c} \right) \; . \qquad (57)$$

The meaning of μ_c is the amount of information required for the exponential reduction of the attraction factors.

The dependence of the attraction factors on the available information suggests that it is admissible to vary these factors by regulating the amount of information. Respectively, by varying the attraction factors, it is possible to manipulate decisions. For instance, suppose that, at $\mu = 0$, the prospect π_1 is preferred to π_2. By providing additional information, one can reduce the attraction factors according to Eq. (55), as a result of which the preference can be reversed, with the prospect π_2 becoming preferable to π_1.

In a series of experimental studies, it has been found that decisions essentially change, when the agents are allowed to consult with each other, increasing by this their mutual information (Charness and Rabin 2002; Blinder and Morgan 2005; Cooper and Kagel 2005; Charness et al. 2007a,b; Chen and Li 2009; Charness et al. 2010), or when they can get additional information learning from their own experience (Kühberger et al. 2001).

Note that it is possible to provide correct information as well as incorrect one, confusing decision makers and forcing them to accept the desired decision. The effect, similar to providing negative information, can be achieved if decision makers are asked to deliberate concentrating of the uncertainty contained in the considered prospects (Waroquier et al. 2010).

7 Conclusion

We have studied how the choice of decision makers can be manipulated under the presence of risk and uncertainty. Our analysis is based on the Quantum Decision Theory that has been previously developed for individual decision makers. We have suggested a generalization of the theory to the case of decision makers that are members of a society. The social decision makers interact with each other by exchanging information. The notion of a typical decision maker, representing the average society behavior, has been introduced and characterized.

Under the given utility of prospects, the typical behavior of agents can be manipulated. The manipulation of decision making is realized by influencing the attraction factor of decision makers. This can be done in two ways. One method is to arrange the payoff weights so as to induce the required changes of values of the attraction factors. The variation of the payoff weights can invert the attraction factor values and reverse the decision preferences. The second method of manipulation is by providing information to decision makers or by allowing consultations between the agents of the society. The attraction factors can be either decreased, when decision makers obtain correct information, or increased, if the delivered information is wrong. The variation of the attraction factors, induced by positive or negative information, can lead to the reversal of preferences. The methods of manipulating decision making are illustrated by several experiments.

The possibility of manipulating decision makers is, of course, not a novelty. What is principally new in the present paper is the *mathematical description* of the process allowing for *quantitative predictions*. By treating several concrete decision problems, we have illustrated that our theory yields theoretical predictions that, within experimental accuracy, coincide with empirical results.

Acknowledgement

We are very grateful to E.P. Yukalova for many useful discussions. Financial support from the Swiss National Science Foundation is appreciated.

References

Abdellaoui, M., Baillon, A., Placido, L., and Wakker, P.P. (2011a). The rich domain of uncertainty: source functions and their experimental implementation. *American Economic Review, 101*, 695–723.

Abdellaoui, M., Driouchi, A., and L'Haridon, O. (2011b). Risk aversion elicitation: reconciling tractability and bias minimization. *Theory and Decision, 71*, 63–80.

Armel, K.C., Beaumel, A., and Rangel, A. (2008). Biasing simple choices by manipulating relative visual attention. *Judgement and Decision Making, 3*, 396–403.

Armel, K.C., and Rangel, A. (2008). The impact of computation time and experience on decision values. *American Economic Review, 98*, 163–168.

Arndt, C. (2004). *Information Measures.* Berlin: Springer.

Barber, B.R. (1984). *Strong Democracy.* Berkeley: University of California.

Bhutta, N., Shan, H. and Dokko, J.K. (2010) The Depth of Negative Equity and Mortgage Default Decisions, FEDS Working Paper No. 2010-35. Available at SSRN: `ttp://ssrn.com/abstract=1895493}or{\url`ttp://dx.doi.org/10.2139/ssrn.1895493

Blinder, A., and Morgan, J. (2005). Are two heads better than one? An experimental analysis of group versus individual decision-making. *Journal of Money Credit Banking, 37*, 789–811.

Busemeyer, J.R., and Diederich, A. (2002). Survey of decision field theory. *Mathematical Social Sciences, 43*, 345–370.

Busemeyer, J.R., and Townsend, J.T. (1993). Decision field theory: a dynamic cognition approach to decision making. *Psychological Review, 100*, 432–459.

Chambers, S. (2003). Deliberative democratic theory. *Annual Review of Political Science, 6*, 307–326.

Charness, G., and Rabin, M. (2002). Understanding social preferences with simple tests. *Quarterly Journal of Economics, 117*, 817–869.

Charness, G., Karni, E., and Levin, D. (2007a). Individual and group decision making under risk: an experimental study of Bayesian updating and violations of first-order stochastic dominance. *Journal of Risk and Uncertainty, 35*, 129–148.

Charness, G., Rigotti, L., and Rustichini, A. (2007b). Individual behavior and group membership. *American Economic Review, 97*, 1340–1352.

Charness, G., Karni, E., and Levin, D. (2010). On the conjunction fallacy in probability judgement: new experimental evidence regarding Linda. *Games and Economic Behavior, 68*, 551–556.

Chen, Y., and Li, S. (2009). Group identity and social preferences. *American Economic Review, 99*, 431–457.

Cooper, D., and Kagel, J. (2005). Are two heads better than one? Team versus individual play in signaling games. *American Economic Review, 95*, 477–509.

Dong, D., Chen, C., Li, H., and Tarn, T.J. (2008). Quantum reinforcement learning. *IEEE Transactions on Systems, Man, and Cybernetics B, 38*, 1207–1220.

Dryzek, J.K. (2000). *Deliberative Democracy and Beyond.* Oxford: Oxford University.

Gibbs, B.J. (1997). Predisposing the decision maker versus framing the decision: a consumer-manipulation approach to dynamic preference. *Marketing Letters, 8*, 71–83.

Goldsmith, K., and Amir, O. (2010). Can uncertainty improve promotions? *Journal of Marketing Research, 47*, 1070–1077.

Gollier, G. (2001) *Economics of Risk and Time*. Cambridge: MIT.

Gneezy, U., List, J.A., and Wu, G. (2006). The uncertainty effect: when a risky prospect is valued less than its worst possible outcome. *Quarterly Journal of Economics, 121*, 1283–1309.

Gutmann, A., and Thompson, D. (1996) *Democracy and Disagreement.* Cambridge: Harvard University.

Hafer, C., and Landa, D. (2007). Deliberation as self-discovery. *Journal of Theoretical Politics, 19*, 329–360.

Heath, T.B., Chatterjee, S., and France, K.R. (1995). Mental accounting and changes in price: the frame dependence of reference dependence. *Journal of Consumer Research, 22*, 90–97.

Hellebrandt, T., Kawar, S. and Waldron, M. (2009) The Economics and Estimation of Negative Equity, Bank of England Quarterly Bulletin 2009 Q2. Available at SSRN: `ttp://ssrn.com/abstract=1420033}\vskip2mmHogart`, R.M., Gibbs, B.J., McKenzie, C.R., and Marquis, M.A. (1991). Learning from feedback: exactingness and incentives. *Journal of Experimental Psychology: Learning Memory and Cognition, 17*, 734–752.

Kahneman, D., and Tversky, A. (1979). Prospect theory: an analysis of decision under risk. *Econometrica, 47*, 263–292.

Kellaris, J.J., Frank, R.K., and DiNovo, T. (1995). Exploring the boundaries of the framing effect: the moderating roles of disparate expected values and perceived costs of judgemental errors. *Marketing Letters, 6*, 175–182.

Keller, K.L. (1991). Cue compatibility and framing in advertising. *Journal of Marketing Research, 28*, 42–57.

Kinsey, K.A., Grasmick, H.G., and Smith, K.W. (1991). Framing justice: taxpayer evaluations of personal tax burdens. *Law and Society Review, 25*, 845–873.

Koehler, J.J., Gibbs, B.J., and Hogarth, R.M. (1994). Shattering the illusion of control: multi-shot versus single-shot gambles. *Journal of Behavioral Decision Making, 7*, 183–191.

Koehler, J.J., and James, G. (2009). Probability matching in choice under uncertainty: intuition versus deliberation. *Cognition, 113*, 123–127.

Kühberger, A., Komunska, D., and Perner, J. (2001). The disjunction effect: does it exist for two-step gambles? *Organizational Behavior and Human Decision Processes, 85*, 250–264.

Lee, L., Amir, O., and Ariely, D. (2009). In search of homo economicus: cognitive noise and the role of emotion in preference consistency. *Journal of Consumer Research, 36*, 173–187.

Lerouge, D. (2009). Evaluating the benefits of distraction on product evaluation: the mind-set effect. *Journal of Consumer Research, 36*, 367–379.

Malevergne, Y., and Sornette, D. (2006). *Extreme Financial Risks*. Heidelberg: Springer.

Norton, M.I., Frost, J.H., and Ariely, D. (2007). Less is more: the lure of ambiguity, or why familiarity breeds contempt. *Journal of Personality and Social Psychology, 92*, 97–105.

O'Carrol, R.E., and Papps, B.P. (2003). Decision making in humans: the effect of manipulating the central noradrenergetic system. *Journal of Neurology Neurosurgery and Psychiatry, 74*, 376–378.

Payne, J.W., Samper, A., Bettman, J.R., and Luce, M.F. (2008). Boundary conditions on unconscious thought in decision making. *Psychological Science, 19*, 1118–1123.

Polister, P.E. (1989). Cognitive guidelines for simplifying medical information: data framing and perception. *Journal of Behavioral Decision Making, 2*, 149–165.

Qualls, W.J., and Puto, C.P. (1989). Organizational climate and decision framing: an integrated approach to analyzing industrial buying decisions. *Journal of Marketing Research, 26*, 179–192.

Schul, Y., and Ganzach, Y. (1995). The effects of accessibility of standards and decision framing on product evaluations. *Journal of Consumer Psychology, 4*, 61–83.

Sornette, D. (2003) *Why Stock Markets Crash*. Princeton: Princeton University.

Tversky, A., and Kahneman, D. (1992). Advances in prospect theory: cumulative representation of uncertainty. *Journal of Risk and Uncertainty, 5*, 297–323.

van de Kuilen, G. (2009). Subjective probability weighting and the discovered preference hypothesis. *Theory and Decision, 67*, 1–22.

von Neumann, J. (1955). *Mathematical Foundations of Quantum Mechanics*. Princeton: Princeton University.

Waroquier, L., Marchiori, D., Klein, O., and Cleeremans, A. (2010). Is it better to think unconsciously or to trust your first impression? *Social Psychological and Personality Science, 1*, 111–118.

Wilson, T.D., and Schooler, J.W. (1991). Thinking too much: introspection can reduce the quality of preferences and decisions. *Journal of Personality and Social Psychology, 60*, 181–192.

Wilson, T.D., Centerbar, D.B., Kermer, D.A., and Gilbert, D.T. (2005). The pleasures of uncertainty: prolonging positive moods in ways people do not anticipate. *Journal of Personality and Social Psychology, 88*, 5–21.

Yukalov, V.I. (2012). Equilibration of quasi-isolated quantum systems. *Physics Letters A, 376*, 550–554.

Yukalov, V.I., and Sornette, D. (2008). Quantum decision theory as quantum theory of measurement. *Physics Letters A, 372*, 6867–6871.

Yukalov, V.I., and Sornette, D. (2009a). Physics of risk and uncertainty in quantum decision making. *European Physical Journal B, 71*, 533–548.

Yukalov, V.I., and Sornette, D. (2009b). Processing information in quantum decision theory. *Entropy, 11*, 1073–1120.

Yukalov, V.I., and Sornette, D. (2009c). Scheme of thinking quantum systems. *Laser Physics Letters, 6*, 833–839.

Yukalov, V.I., and Sornette, D. 2010a. Entanglement production in quantum decision making. *Physics of Atomic Nuclei, 73*, 559–562.

Yukalov, V.I., and Sornette, D. (2010b). Mathematical structure of quantum decision theory. *Advances in Complex Systems, 13*, 659–698.

Yukalov, V.I., and Sornette, D. (2011). Decision theory with prospect interference and entanglement. *Theory and Decision, 70*, 283–328.

Yukalov, V.I., and Sornette, D. (2012). Quantum decision making by social agents. http://ssrn.com/abstract=2018270.