A quantum approach for determining a state of the opinion

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Abstract. – We propose to define a notion of state of the opinion in order to link politician popularity estimations and voting intentions. We present two ways of modelling: a classical approach and quantumm modelling. We test this ideas on French data obtained during spring 2012.

Keywords: opinion polls, voting.

1) Introduction

• Electoral periods are favorable to opinion polls. We keep in mind that opinion polls are intrinsically complex (see *e.g.* Gallup [15], Tillé [30] or the introduction of Bar-Hen and Chiche [6]) and give an approximates picture of a possible social reality. They are traditionnally of two types: popularity polls for various outstanding political personnalities and voting intentions polls when a list of candidates is known. We remark that in

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the fisrt case, a grid of appreciation is given by the questionnaire, typically of the type "very good" $\succ$ "good" $\succ$ "no opinion" $\succ$ "bad" $\succ$ "very bad".

• We have two different informations and to construct a link between them is not an easy task. In particular, the determination of the voting intensions is a quasi intractable problem! Predictions of votes classically use of so-called "voting functions". Voting functions have been developed for the prediction of presidential elections. They are based on correlations between economical parameters, popularity polls and other technical parameters. We refer to Abramowitz [1], Lewis-Beck [25], Campbell [12], Lafay [23] and the survey paper proposed by Auberger [2].

• In this contribution, we make the hypothesis that there exists some global "state of the opinion" that can be determined with the help of the given grid of analysis, denoted by G in the following. Moreover, the responses to popularity polls can be considered as a reflection of what the opinion thinks about himself. We propose in the following to determine as much information as possible about this state of the opinion, in the case where voting intentions are also available. In the second section, we propose a mathematical model founded on a classical framework. The state of the opinion is described by a law of probability and the double information of popularity polls and opinion polls give the input information.

• We adopt afterwards quantum modelling (see *e.g.* Bitbol *et al* [7] for an introduction), in the spirit of authors like Khrennikov and Haven [22], La Mura and Swiatczak [24] and Zorn and Smith [31] concerning voting processes. We recall two voting models developed in previous contributions [13, 14], founded on "range voting" and "first run" of an election, having implicitely in mind the case of the French presidential election. We do not recall the mathematical difficulties associated with the question of voting when the number of candidates is grether than three [8, 11, 3]. They conduct to present-day researches like range voting, independently proposed by Balinski and Laraki [4, 5] and by Smith [29, 28]. It is composed by two steps: grading and ranking. In the grading step, all the candidates are evaluated by all the electors. This first step is quite analogous to a popularity investigations and we will merge the two notions in this contribution. The second step of range voting is a majority ranking; it consists of a successive extraction of medians. Note that this theory is applied for wine testing, as described in Peynaud and Blouin [27].

• Then we propose in section 4 to link our two models and introduce for doing this the state of the opinion. Then we test in section 5 our previous ideas with two sets of data coming from 2012 French presidential elections and propose preliminary results.

2) A classical reconstruction of the state of the opitnion

• We consider a grid G of m types of opinions as one of the two following ones:

(1) $$\text{"very good"} \succ \text{"good"} \succ \text{"no opinion"} \succ \text{"bad"} \succ \text{"very bad"}$$

$$\text{“good”} \succ \text{“no opinion”} \succ \text{“bad”}\,. \tag{2}$$

We have $m = 5$ for the grid (1) and $m = 3$ for (2). These grids are typically used for popularity polls [17, 18, 21]. We assume also that a ranking grid like (1) or (2) is a basic tool to represent a “state of the opinion”. If some political personnality has a great proportion of “very good” opinion, we suppose here that this fact is a kind of “mirror effect” of an existing state of social opinion. The reflection that the opinion is for a certain proportion in a “very good” state.

• We adopt in this section a “classical” point of view for taking into account the variety of possibles underlyings. The state of opinion is mathematically modelized by a discrete law of probability $\big(p_\nu\big)_{\nu \in G}$. We suppose that the opinion ν (with $\nu \in G$) is present in the entire population with a probability p_ν. We have also two type of data, as explained in the introduction. We denote by Γ the set of candidates and we denote by n their number. We suppose also that

$$\text{the number of candidates} \equiv n \;>\; m \equiv \text{size of the grid } G\,. \tag{3}$$

On one side the result of a popularity poll for the n candidates is given. We have a matrix of data $\big(S_{\gamma\,\nu}\big)_{\gamma \in \Gamma,\, \nu \in G}$ with an hypothesis of coherence:

$$S_{\gamma\,\nu} \geq 0\,, \quad \sum_{\nu \in G} S_{\gamma\,\nu} = 1\,, \quad \gamma \in \Gamma\,. \tag{4}$$

On the other side, we have the voting intentions β_γ for each candidate $\gamma \in \Gamma$. We have at our disposal a vector $\big(\beta_\gamma\big)_{\gamma \in \Gamma}$ satisfying

$$\beta_\gamma \geq 0\,, \quad \sum_{\gamma \in \Gamma} \beta_\gamma \,\leq\, 1\,. \tag{5}$$

• If the global opinion is considered as a given state with associated probabilities p_ν, the opinion for a candidate with a label γ is equal to $S_{\gamma\,\nu}$ and the associated voting attention is equal to β_γ. How to link the unknown $p \equiv \big(p_\nu\big)_{\nu \in G}$ to the data $S_{\gamma\,\nu}$ and β_γ ? A naive unswer could be

$$p_\nu \,=\, \sum_{\gamma \in \Gamma} \beta_\gamma \; S_{\gamma\,\nu}\,, \quad \nu \in G\,. \tag{6}$$

The relation (6) is explicit and due to (4) and (5), is coherent with natural constraints

$$p \,\in\, K_m \,\equiv\, \Big\{ q \in \mathbb{R}^m,\; q_j \geq 0 \;\sum_{j=1}^{m} q_j \,=\, 1 \Big\} \tag{7}$$

that express that we have a discrete law of probability. But at our opinion, the relation (6) does not describe a state of the opinion and just compute *a posteriori* numbers that do not express a real *a priori* state of the opinion.

• We think coherent to express that the expectation of the family $S_{\gamma\,\nu}$ for ν running in G is equal to the voting intention β_γ. We can say also that the correlation of the probability vectors p and $s_\gamma \equiv \big(S_{\gamma\,\nu}\big)_{\nu \in G}$ is equal to the voting intention β_γ. In algebraic terms,

(8) $$\sum_{\nu \in G} S_{\gamma\,\nu}\, p_\nu \,=\, \beta_\gamma\,, \quad \gamma \in \Gamma\,.$$

Of course, the system (8) is in general not correctly posed if the hypothesis (3) is satisfied. We have n equations and only m unknowns. We adopt a least square approach and replace the system (8) by the minimization of some squared functional, say

(9) $$J(p) \,=\, \frac{1}{2} \sum_{\gamma \in \Gamma} \Big(\sum_{\nu \in G} S_{\gamma\,\nu}\, p_\nu \,-\, \beta_\gamma \Big)^2$$

to fix the ideas. The constraint (7) has to be satisfied because the family of numbers $\big(p_\nu\big)_{\nu \in G}$ is a probability distribution. We have to solve a quadratic optimization problem with linear inequalities

(10) $$J(p) \,=\, \inf \big\{ J(q),\ q \in K_m \big\}\,.$$

If the matrix $S_{\gamma\,\nu}$ introduced at the relation (4) is of maximal rank m (and we do this hypothesis in the following), the problem (10) is the minimization of a coercive quadratic functional inside a closed non empty convex set. This problem has a unique solution; we have solved it using the Uzawa algorithm (see *e.g.* the book of Gondran and Minoux [16].

3) Two quantum models for voting process

• The fact to consider quantum modelling induces a specific vision of probabilities. We refer *e.g.* to the classical treatise on quantum mechanics of Cohen-Tannoudji *et al* [10], to the approach of Mugur-Schächter MMS08 or to the elementary introduction proposed by Busemeyer and Trueblood [9] in the context of statistical inference.

• In a first tentative [13], we have proposed to introduce an Hilbert space H_Γ formally generated by the candidates $\gamma \in \Gamma$. In this space, a canditate γ is represented by a unitary vector $|\,\gamma >$ and this family of vectors is supposed to be orthogonal. Then an elector ℓ can be decomposed in the space H_Γ of candidates according to

(11) $$|\,\ell > \,=\, \sum_{\gamma \in \Gamma} \theta_{\ell\gamma}\, |\,\gamma > \ .$$

The vector $|\,\ell > \in H_\Gamma$ is supposed also to be a unitary vector to fix the ideas. According to Born's rule, the probability for a given elector ℓ to give his voice to the particular candidate γ is equal to $|\,\theta_{\ell\gamma}\,|^2$. The violence of the quantum measure is clearly visible with this example: the opinions of elector ℓ never coincidate with the program of any candidate. But with a voting system where an elector has to choice only one candidate among n, his social opinion is *reduced* to the one of a particular candidate.

• Our second model [14] is adapted to the grading step of range voting [4, 29]. We introduce an other space H_P of political opinions associated with a grading family G. The space H_P is formally generated by the orthogonal vectors $|\,\nu >$ relative to the opinions. Then we suppose that the candidates γ are now decomposed by each elector ℓ on the basis $\mid \nu >$:

$$\mid \gamma > = \sum_{\nu \in G} \alpha_{\gamma\nu} \mid \nu > \ . \tag{12}$$

With this notation (where we have omitted the index ℓ), the probability for a given elector ℓ to give an opinion ν to a candidate γ is simply a consequence of the Born's rule and this probability is equal to $\mid \alpha_{\gamma\nu} \mid^2$.

4) State of the opinion: a link between quantum voting models

• We wish now to represent the candidate γ inside the space H_P of opinions, generated by the orthogonal vectors $\mid \nu >$. We suppose a relation of the type (12) and the candidate γ is still represented by a unitary vector:

$$\sum_{\nu \in G} \mid \alpha_{\gamma\nu} \mid^2 \ = \ 1 \,, \quad \gamma \in \Gamma \,. \tag{13}$$

We will denote by A the matrix with n lines and m colums and generic element $\alpha_{\gamma\nu}$. We construct a link between the Born rule and the popularity polls: the mean statistical expectation of a given opinion ν for a candidate γ is equal to $\mid \alpha_{\gamma\nu} \mid^2$ on one hand and is given by $S_{\gamma\nu}$ on the other hand. Consequently,

$$\mid \alpha_{\gamma\nu} \mid^2 \ = \ S_{\gamma\nu} \,, \quad \gamma \in \Gamma \,, \ \nu \in G. \tag{14}$$

• The candidates are modelized now by n orthogonal vectors in a space of dimension m. The relation (11) is not simple to generalize. We introduce for this reason a density operator ρ instead of the relation (11):

$$\rho \ = \sum_{\gamma \in \Gamma} \theta_\gamma^2 \mid \gamma > < \gamma \mid \tag{15}$$

The coefficient of statistical mixing θ_γ^2 is be supposed to be positive (an we can suppose $\theta_\gamma \geq 0$ without loss of generality). It is easy to see that the operator ρ is a convex sum of projectors onto the orthogonal vectors $\mid \gamma >$. The operator ρ has a unity trace under the condition

$$\sum_{\gamma \in \Gamma} \theta_\gamma^2 \ = \ 1 \,. \tag{16}$$

• We introduce the state of opinion ζ in space H_P in order to have a natural relation between the decompositions (13) and the density matrix ρ :

$$\mid \zeta > \in \ H_P \,, \quad \parallel \zeta \parallel = 1 \,. \tag{17}$$

The mean value $< \zeta >$ of this global opinion vector measured by the density matrix ρ is given according to the relation

$$< \zeta > \, = \, < \zeta \,, \, \rho \bullet \zeta > \ . \tag{18}$$

It is natural to make the hypothesis that the voting process determines the decomposition (15) when the state of opinion is given in order to maximize the mean value $< \zeta >$. Then the voting allows to determine the vector $\big(\theta_\gamma\big)_{\gamma\in\Gamma}$ solution of the following optimization problem

$$\left\{ \begin{array}{c} \max \quad < \zeta \,,\, \rho \bullet \zeta > \,. \\ \parallel \theta \parallel = 1 \end{array} \right. \tag{19}$$

After some lines of elementary algebra, it is easy to determine the vector θ :

$$\theta \,=\, \frac{1}{\parallel A^{\rm t} \bullet \zeta \parallel} \, A^{\rm t} \bullet \zeta \tag{20}$$

With this relation, we link a real intricate state ζ and a statistical mixing ρ operated by the election. As a consequence, the voting proportion β_γ for each candidate is equal to the square of θ :

$$\beta_\gamma \,=\, |\zeta_\gamma|^2 \,, \quad \gamma \in \Gamma \,. \tag{21}$$

5) Spring 2012 preliminary results

	+	0	−	voting
Ba	.55	.14	.31	.125
Ho	.52	.08	.40	.30
Jo	.29	.13	.58	.03
LP	.28	.06	.66	.175
Mé	.38	.20	.42	.085
Sa	.33	.00	.67	.25

Table 1. Popularity and sounding polls, february 2012 [17, 19, 21].

	++	+	0	−	−−	voting
Ba	.08	.62	.03	.23	.23	.12
Ho	.09	.45	.00	.30	.16	.275
Jo	.02	.34	.02	.40	.22	.03
LP	.10	.24	.01	.26	.39	.17
Mé	.11	.46	.03	.31	.09	.11
Sa	.10	.31	.00	.29	.30	.28

Table 2. Popularity and sounding polls, march 2012 [18, 20].

• We have a first family of data obtained in february 2012. Populatity data [17, 21] and result of voting intentions [17, 21] that conduct to Table 1, when the names of the candidates are proposed with transparent abbreviations. Similar data are proposed in Table 2 for march 2012 [18, 20]. We will present our results at the conference and compare the "classical" and "quantum" approaches for determing the state of the opinion.

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