

Multiattribute Utility Theory, Intertemporal Utility and Correlation Aversion

by

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ABSTRACT. Convenient assumptions about qualitative properties of the intertemporal utility function have generated counter-intuitive implications for the relationship between atemporal risk aversion and the intertemporal elasticity of substitution. If the intertemporal utility function is additively separable then the latter two concepts are the inverse of each other. We review a simple theoretical specification with a long lineage in the literature on multi-attribute utility, and demonstrate the critical role of a concept known as intertemporal risk aversion or intertemporal correlation aversion. This concept is the intertemporal analogue of a more general concept applied to two attributes of utility, but where the attributes just happen to be the time-dating of the good. In the context of intertemporal utility functions, the concept provides an intuitive explanation of possible differences between (the inverse of) atemporal risk aversion and the intertemporal elasticity of substitution. We use this theoretical structure to guide the design of a series of experiments that allow us to identify and estimate intertemporal correlation aversion. Our results show that subjects are correlation averse over lotteries with intertemporal income profiles, and that the convenient additive specification of the intertemporal utility function is not an appropriate representation of preferences over time.

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Convenient assumptions about qualitative properties of the intertemporal utility function have generated counter-intuitive implications for the relationship between atemporal risk aversion and the intertemporal elasticity of substitution. If the intertemporal utility function is additively separable then the latter two concepts are the inverse of each other. To explain the apparent lack of substitutability of consumption between periods one is led to assume a low intertemporal elasticity of substitution, implying absurdly high levels of atemporal risk aversion. This is no technical side issue: untangling it is central to the general understanding of savings behavior (e.g., Hall [1988]), the analysis of insurance decisions by extremely poor households in developing countries (e.g., Townsend [1994]), and the behavior of asset prices over time (e.g., Hansen and Singleton [1983]).

In section 1 we review a simple theoretical specification which actually has a long lineage in the literature on multi-attribute utility, and demonstrate the critical role of a concept known as intertemporal risk aversion or intertemporal correlation aversion. This concept is the intertemporal analogue of a more general concept applied to two *attributes* of utility, but where the attributes just happen to be the time-dating of the good. In the context of intertemporal utility functions, the concept provides an intuitive explanation of possible differences between (the inverse of) atemporal risk aversion and the intertemporal elasticity of substitution. We also compare this approach to the alternative, “exotic preferences” reviewed by Backus, Routledge and Zin [2004].

In section 2 we use this theoretical structure to guide the design of a series of experiments that will allow us to identify the core parameters of the latent structural models. We also discuss our specific experiments, conducted throughout Denmark in 2009 using a representative sample of the adult Danish population. In section 3 we review econometric models used to estimate the core parameters of the models. Section 4 contains basic results, and *provide the first estimates of the extent of correlation aversion*. Our results show that subjects are indeed correlation averse over lotteries with intertemporal income profiles, and that the convenient additive specification of the intertemporal

utility function is not an appropriate representation of preferences over time. We also show that *correlation aversion contributes significantly to the overall risk premium that characterizes risky intertemporal profiles*. To take one example from our detailed results, the risk premium for one temporal income profile is equal to 2,086 kroner in the unconstrained model, and is equal to 1,512 kroner when correlation neutrality is imposed, so the difference of 574 kroner is entirely due to correlation aversion. Thus one would have understated the risk premium by 27.5% by ignoring correlation aversion.

1. Theory

A. The Idea of Multi-Attribute Utility

The notion of a multi-attribute utility function is designed to capture the intuition that some commodity might have two or more attributes that cannot be aggregated without reference to the preferences of the individual. A particular car might have certain safety attributes and performance attributes, and there might be no simple way to trade these off smoothly in terms of technology or market transactions. Another car might have different safety attributes and performance attributes, and the individual asked to compare the two cars. If the safety attributes and the performance attributes are comparable, which seems reasonable enough *a priori*, then one can characterize these preferences in terms of a multi-attribute utility function (Fishburn [1965] and Keeney and Raiffa [1976]). Although the original axiomatizations of multi-attribute utility theory naturally evolved from the additive specifications of von Neumann and Morgenstern [1944], extensions to non-additive axiomatizations are now well established (Fishburn [1984] and Miyamoto and Wakker [1996]).

The earliest thinking about multi-attribute utility, although formal and abstract, always used non-monetary attributes as motivation. Applications to attributes of environmental policies, or health, naturally motivated this approach, and it complemented more general thinking about the “characteristics” of goods as being the object of preference (Lancaster [1966]). If one starts to think

of monetary objects as attributes, the multi-attribute approach would seem to collapse since the attributes can be readily aggregated. Outcomes of “four \$5 bills” and “two \$10 bills” are hardly the sorts of thing we would think to model as two distinct attributes. If we did, as a formal matter, one would expect perfect substitutability between them, and treat them as one composite good; so nothing is lost by the multi-attribute formalism, but nothing seems to be gained.

But now think of one outcome as “four \$5 bills in my pocket” and the other as “two \$10 bills, for certain, from participating in an experiment lasting two hours.” Is it *a priori* obvious that these are perfect substitutes? No, since the opportunity cost of time is not zero. But then can’t we just convert the money in one’s pocket into an equivalent amount of money for two hours of time? Yes, but for that one has to know the preferences of the individual over two hours of (hard) labor in an experiment, compared to the alternatives. This brings us back to the need for a multi-attribute utility function in order to be able to evaluate the two outcomes.

This example might appear forced, but it is at the heart of debates about the integration of “assets” and the “proper argument” of the utility function. If the individual is assumed to perfectly integrate the wealth in his pocket, or bank, with the income flow in an experiment, then significant consequences flow for the characterization of risk attitudes (Rabin [2000]). But if we allow these to be two attributes of a general multi-attribute utility function, we can allow perfect asset integration and null asset integration as special cases, and let the preferences of the individual tell us which applies in a particular setting (Cox and Sadiraj [2006]). And when one actually has access to data on individual wealth and experimental income, and can directly test this hypothesis, it turns out that behavior is indeed best characterized by “partial asset integration” rather than perfect asset integration (Andersen, Cox, Harrison, Lau, Rutström and Sadiraj [2011]). In turn, this characterization of a two-attribute utility function with substitutability between attributes that is a matter of preference leads to estimates of risk attitudes that are “plausible” at all interesting wealth

levels. The same point can be argued from the perspective of some bundles of consumption being viewed by the individual as different from other bundles of consumption (Chetty and Szeidl [2007]) or from the perspective of “mental accounts” (Thaler [1985]).

Now think of the two outcomes being “\$20 today” or “\$20.50 in 91 days,” and assume credibility of payment in each case. If there is a perfect capital market available with an annual effective interest rate of 10%, these are identical, and we do not even have to think about the time preferences of the individual. Whether the individual has exponential or hyperbolic time preferences, he can go to the capital market and convert the one into the other perfectly. Which trade the individual does depends on time preferences over final consumption, but from a modeling perspective we can view these two as perfect substitutes in defining the intertemporal budget constraint. This is the idea of Fisherian separation, widely used in life-cycle models (Fisher [1930] and Modigliani and Brumberg [1954]).

When perfect capital markets do not exist, in the weak sense of equal borrowing and lending rates, then there does not in general exist a “utility free” investment rule (Pye [1966], Hirshleifer [1970; ch.7]). That is, there is no way to ascertain the intertemporal budget constraint of the individual without knowing his preferences over time-dated bundles. So, again, one comes back to the need to specify an intertemporal utility function defined over the time-dated bundles.

B. Intertemporal Utility as Multi-Attribute Utility

Assume that intertemporal utility is defined over the present atemporal utility of money in some sooner time period t , denoted \mathbf{X} , and the present atemporal utility of money in some later time period $t+\tau$, denoted \mathbf{Y} . Let \tilde{x} and \tilde{X} denote the magnitudes of money associated with the elements of \mathbf{X} , such that $\tilde{x} < \tilde{X}$, and let \tilde{y} and \tilde{Y} denote any elements of \mathbf{Y} such that $\tilde{y} < \tilde{Y}$. Let \mathbf{x} be a typical element of \mathbf{X} , so that it can represent either the present atemporal utility of \tilde{x} or of \tilde{X} , and let \mathbf{y} be a

typical element of \mathbf{Y} in the same sense. We use this notation device of x and y to make it apparent that the issues are more general than the application to intertemporal utility, as important as that is by itself. Thus, in general, think of x and y as two goods, or two attributes of some good.

Define the lottery α as a 50:50 mixture of $\{x, Y\}$ and $\{X, y\}$, and the lottery β as a 50:50 mixture of $\{x, y\}$ and $\{X, Y\}$. So α is a 50:50 mixture of bad and good outcomes in time t and $t+\tau$, and good and bad outcomes in the two time periods; and β is a 50:50 mixture of all-bad outcomes and all-good outcomes in the two time periods. These lotteries α and β are defined over all possible “good” and “bad” outcomes.

If the individual is indifferent between α and β we say that he is intertemporally neutral towards correlated payoffs in the two time periods. If the individual prefers α to β we say that he is intertemporally averse to correlated payoffs: it is better to have a given chance of being lucky in one of the two periods than to have the same chance of being very unlucky or very lucky in both periods. The correlation averse individual prefers to have non-extreme payoffs *across* periods, just as the risk averse individual prefers to have non-extreme payoffs *within* periods. One can also view the correlation averse individual as preferring to avoid correlation-increasing transformations of payoffs in different periods.

Keeney [1973] defined this concept as conditional risk aversion, Richard [1975] defined it as bivariate risk aversion, and Epstein and Tanny [1980] defined it as correlation aversion.¹ Since we interpret the two attributes as referring to different time periods, we call it intertemporal risk aversion or intertemporal correlation aversion.² There are direct parallels in the older literature on multi-attribute utility (Fishburn [1965], Keeney [1968][1971][1972], Meyer [1972] and Pollack

¹ Several studies note that the core concept appeared as early as de Finetti [1952], but this was written in Italian and we cannot verify that claim.

² Everything we say about intertemporal risk aversion or intertemporal correlation aversion applies symmetrically to behavior that exhibits intertemporal risk loving or intertemporal correlation loving.

[1967]): in fact, there is more than just a parallel logic, as the motivating example from Richard [1975; p.13] illustrates.³ The concept of correlation aversion provides insight into properties of univariate risk aversion, such as properness, prudence and temperance (Denuit, Eeckhoudt and Rey [2010]). There are also parallels in the older literature on multivariate risk aversion (Kihlstrom and Mirman [1974], Rothblum [1975], Duncan [1977] and Karni [1979]), as demonstrated by Eeckhoudt, Rey and Schlesinger [2007] and Dorfleitner and Krapp [2007].

The correlation neutral individual has additive preferences over time-dated money flows, and the correlation averse individual has non-additive preferences over time-dated money flows. Let $U(\mathbf{X}, \mathbf{Y})$ denote the intertemporal utility function. Richard [1975] demonstrated that a necessary and sufficient condition for correlation aversion was that $\partial^2 U / \partial \mathbf{x} \partial \mathbf{y}$ be non-positive. The decision-maker that is correlation averse can be risk averse, risk neutral or risk seeking in terms of atemporal payoffs defined over \mathbf{x} or \mathbf{y} . These concepts therefore break the connection between atemporal risk aversion and intertemporal risk aversion.⁴

Following the exposition of Bommier [2007], we can define a number of important concepts using this structure. The marginal rate of substitution between money in periods t and $t+\tau$ can be defined as

$$\text{MRS}_{t,t+\tau} = (\partial U / \partial \mathbf{x}) / (\partial U / \partial \mathbf{y}) \quad (1)$$

The coefficient of relative risk aversion in period t can be defined by

$$\text{RRA}_t = -\mathbf{x} [\partial^2 U / (\partial \mathbf{x})^2] / (\partial U / \partial \mathbf{x}) \quad (2)$$

and a similar definition in period $t+\tau$ as

$$\text{RRA}_{t+\tau} = -\mathbf{y} [\partial^2 U / (\partial \mathbf{y})^2] / (\partial U / \partial \mathbf{y}) \quad (3)$$

³ Harvey [1993] explicitly uses the concept of multiattribute risk aversion to restate formal conditions used to characterize a wide class of empirically tractable, additive multattribute utility functions.

⁴ If the temporal attributes are perfect substitutes, and the discount factor is 1, then there is only atemporal risk aversion.

The (direct) elasticity of substitution between money in periods t and $t+\tau$ is

$$\sigma_{t,t+\tau} = \{1/\mathbf{x}(\partial U/\partial \mathbf{x}) + 1/\mathbf{y}(\partial U/\partial \mathbf{y})\} / \{a + b + c\} \quad (4)$$

where

$$a = -[\partial^2 U / (\partial \mathbf{x})^2] / (\partial U / \partial \mathbf{x})^2 \quad (4')$$

$$b = 2 [\partial^2 U / (\partial \mathbf{x} \partial \mathbf{y})] / [(\partial U / \partial \mathbf{x})(\partial U / \partial \mathbf{y})] \quad (4'')$$

and

$$c = -[\partial^2 U / (\partial \mathbf{y})^2] / (\partial U / \partial \mathbf{y})^2 \quad (4''')$$

Finally, a coefficient of correlation aversion with respect to money flows in periods t and $t+\tau$ can be defined as

$$\varrho_{t,t+\tau} = -2 [\partial^2 U / (\partial \mathbf{x} \partial \mathbf{y})] / [\partial U / \partial \mathbf{x} + \partial U / \partial \mathbf{y}] \quad (5)$$

Clearly $\varrho_{t,t+\tau} \geq 0$ if $\partial^2 U / \partial \mathbf{x} \partial \mathbf{y} \leq 0$, since $\partial U / \partial \mathbf{x} \geq 0$ and $\partial U / \partial \mathbf{y} \geq 0$, connecting this coefficient to the definition of correlation aversion proposed by Richard [1975].

With these concepts defined, there is a remarkable relationship between them noted by Bommier [2007; Proposition 1]:

$$1/\sigma_{t,t+\tau} (1 + \text{MRS}_{t,t+\tau} \mathbf{x}/\mathbf{y}) = (\text{RRA}_t + \mathbf{x}/\mathbf{y} \text{MRS}_{t,t+\tau} \text{RRA}_{t+\tau}) - \varrho_{t,t+\tau} \mathbf{x} (1 + \text{MRS}_{t,t+\tau}) \quad (6)$$

From (6) we see formally that correlation aversion breaks the nexus between the intertemporal elasticity of substitution and atemporal relative risk aversion. If we allow τ to get arbitrarily small, and assume that the only difference between the typical elements \mathbf{x} and \mathbf{y} is in the discount factor, then as the discount factor goes to 1 it is reasonable to assume that $\mathbf{x} \approx \mathbf{y}$, $\text{RRA}_t \approx \text{RRA}_{t+\tau}$ and $\text{MRS}_{t,t+\tau} \approx 1$. Then (6) collapses to

$$1/\sigma_{t,t+\tau} \approx \text{RRA}_t - \varrho_{t,t+\tau} \mathbf{x} \quad (7)$$

and further to

$$1/\sigma_{t,t+\tau} \approx \text{RRA}_t \quad (8)$$

when $\varrho_{t,t+\tau} = 0$ and the individual is correlation neutral or intertemporally risk neutral. Expression (8) reflects the nexus between atemporal risk attitudes and the intertemporal elasticity of substitution

that has frustrated generations of macroeconomists for decades (and led them to do “exotic” things). Expression (7) makes it clear that (6) breaks that nexus, and builds a conceptual bridge between the two concepts.⁵ The exact quantitative relationship between atemporal risk attitudes and the intertemporal elasticity of substitution when $\tau \gg 0$ depends on more than $\rho_{t, t+\tau}$ and \mathbf{x} , but can be evaluated in specific cases.

Consider some parametric functional forms for the atemporal and intertemporal utility functions. The most widely used model of intertemporal choice was developed by Ramsey [1928] and widely popularized by Samuelson [1937], and proposes that the intertemporal utility function at time $t=0$ is written as:

$$U(\mathbf{x}, \mathbf{y}) = 1/(1+\delta)^t u(\tilde{x}) + 1/(1+\delta)^{t+\tau} u(\tilde{y}) \quad (9)$$

where $u(\tilde{x})$ and $u(\tilde{y})$ are the atemporal utilities of money at each time period, and δ is the exponential discount rate. The variable \tilde{x} is the magnitude of the monetary payment, while \mathbf{x} is the element from the attribute choice set \mathbf{X} , such that $\mathbf{x} \equiv D_t u(\tilde{x})$; similarly, $\mathbf{y} \equiv D_{t+\tau} u(\tilde{y})$. This notation allows us to move back and forth between the general multiattribute specification and our particular application where the attributes are just time-dating. This specification assumes that intertemporal utility is equal to a weighted sum of atemporal utility flows, where the weights are determined by the discount rate. Let the atemporal utility function be the constant relative risk aversion (CRRA) specification:

$$u(\tilde{x}) = \tilde{x}^{1-r}/(1-r) \quad (10)$$

$$u(\tilde{y}) = \tilde{y}^{1-r}/(1-r) \quad (11)$$

for $r \neq 1$, where r is the CRRA coefficient, assumed for simplicity to be the same for periods t and $t+\tau$. With this functional form, $r=0$ denotes risk neutral behavior, $r>0$ denotes risk aversion, and

⁵ Building on the concept of multivariate risk aversion from Kihlstrom and Mirman [1974], Kihlstrom [2009] establishes the same qualitative result about the separation of atemporal risk attitudes and the intertemporal elasticity of substitution.

$r < 0$ denotes risk seeking behavior, all defined over atemporal tradeoffs in t or $t + \tau$.

The specification of inter-temporal utility in (9) is additively separable, which implies that the inverse of the inter-temporal elasticity of substitution is equal to the coefficient of atemporal risk aversion. This assumption is made of convenience and is popular in models of inter-temporal choice.

A simple extension of the additively separable model in (9) is to consider a CRRA specification of the inter-temporal utility function:

$$U(\mathbf{x}, \mathbf{y}) = [1/(1+\delta)^t u(\tilde{\mathbf{x}}) + 1/(1+\delta)^{t+\tau} u(\tilde{\mathbf{y}})]^{(1-\eta)/(1-\eta)} = [\xi]^{(1-\eta)/(1-\eta)} \quad (12)$$

where η is the inter-temporal relative risk aversion parameter ($\eta \neq 1$), and the expression $[\xi]$ is useful below.⁶ This utility function is separable but not additive when $\eta \neq 0$, and collapses to (9) when there is intertemporal risk neutrality at $\eta = 0$. The monotonic transformation of (9) into (12) implies that the intertemporal preferences are additively separable, but the intertemporal utility function is no longer additive.⁷

Given the parametric structure we have assumed, we can restate the marginal rate of substitution between money in periods t and $t + \tau$ using (1) as

$$MRS_{t, t+\tau} = D_t \tilde{\mathbf{x}}^{-\tau} / (D_{t+\tau} \tilde{\mathbf{y}}^{-\tau}) \quad (1')$$

⁶ See Epstein and Zin [1989], Farmer [1990], Weil [1990], and Bommier and Rochet [2006] for a discussion of alternative specifications of the inter-temporal utility function and implications for inter-temporal risk aversion. The specification in (12) can generate negative discount rates unless we assume that $r < 1$ and $\eta < 1$ to ensure that $\delta > 0$. This assumption is empirically innocuous, as we demonstrate later with estimated parameter values in Section 3. Andersen, Harrison, Lau and Rutström [2008a] add background consumption ω to a CRRA specification of atemporal utility and apply the function $u(\tilde{\mathbf{x}}) = (\omega + \tilde{\mathbf{x}})^{1-\tau}/(1-\tau)$, where $u(\omega) > 0$. They show that this function is well behaved when the intertemporal utility function is additively separable, in the sense that the addition of background consumption is a sufficient condition to avoid negative discount rates. However, adding background consumption to experimental income is problematic in the estimation of the non-additive function in (12) because one has to specify the atemporal utility of background consumption at every point in time and not only when the sooner and later payments are made.

⁷ See Deaton and Muellbauer [1980; p.137] for a discussion of strong separability and additive preferences. Tsetlin and Winkler [2009] provide a rich, general characterization of the class of utility functions that allow correlation aversion, in the form of mixtures of exponential functions.

the relative risk aversion in period t using (2) as

$$RRA_t = (\eta/[\xi]) D_t \tilde{x}^{1-r} + r, \quad (2')$$

the relative risk aversion in period $t+\tau$ using (3) as

$$RRA_{t+\tau} = (\eta/[\xi]) D_{t+\tau} \tilde{y}^{1-r} + r, \quad (3')$$

the (direct) elasticity of substitution between money in periods t and $t+\tau$ using (4) as

$$\sigma_{t,t+\tau} = 1/r, \quad (4'')$$

and finally the coefficient of correlation aversion with respect to money flows in periods t and $t+\tau$ using (5) as

$$\rho_{t,t+\tau} = \{(2\eta/[\xi]) D_{t+\tau} \tilde{y}^{-r} D_t \tilde{x}^{-r}\} / \{D_{t+\tau} \tilde{y}^{-r} + D_t \tilde{x}^{-r}\}, \quad (5')$$

where $D_t = 1/(1+\delta)^t$. Hence, $\rho_{t,t+\tau}$ is positive (negative) when η is positive (negative). The specific functional forms for these concepts will vary with the parametric assumption assumed, of course.

If we again allow τ to get arbitrarily small, and assume that the only difference between the typical elements \mathbf{x} and \mathbf{y} is in the discount factor⁸, then as the discount factor goes to 1 it is reasonable to assume that $\mathbf{x} \approx \mathbf{y}$, $RRA_t \approx RRA_{t+\tau}$, and $MRS_{t,t+\tau} \approx 1$, then equation (5') collapses to

$$\rho_{t,t+\tau} = \eta(1-r)/(2\tilde{x}). \quad (5'')$$

The coefficient of correlation aversion is further reduced to

$$\rho_{t,t+\tau} = \eta/(2\tilde{x}) \quad (5''')$$

if $r = 0$. In this special case the coefficient of correlation aversion is a function of the inter-temporal relative risk aversion parameter, η , and income in the two time periods. The inter-temporal elasticity of substitution goes towards infinity when the atemporal relative risk aversion parameter, r , goes towards 0, which is just to say that income in the two time periods become perfect substitutes, and hence the coefficient of correlation aversion is no longer a function of r . In this case relative risk

⁸ This is one point where the notational difference between the *attribute* \mathbf{x} and the *value* \tilde{x} helps. Recall that $\mathbf{x} \equiv D_t \tilde{x}$ and $\mathbf{y} \equiv D_{t+\tau} \tilde{y}$.

aversion, RRA, is simply η .

To elicit inter-temporal risk aversion one would have to present subjects with choices over lotteries that have different income profiles over time.⁹ Proper identification of inter-temporal risk aversion (η) thus requires that one controls for atemporal risk aversion (α) and the individual discount rate (δ). The three are intrinsically, conceptually connected as a matter of theory, unless one makes strong assumptions otherwise. Our experimental design and econometric logic follow from this theoretical point.

C. Connections to the “Exotic Preferences” Literature

We can directly connect the multi-attribute utility structure to the vast literature on “exotic preferences” in macroeconomics.

One class, due to Kreps and Porteus [1978], also abandons additivity of the intertemporal utility function. The original motivation was to be able to characterize preferences over time-dated risk resolution, but the formal “recursive utility” specification has found much wider application (e.g., in some “smooth” characterizations of ambiguity aversion). Their specification posits a “time aggregation” function that allows for substitutability between a-temporal utility at time t and the certainty-equivalent of the future stream of a-temporal utilities. That certainty-equivalent is generated in a standard, additive manner, consistent with EUT. This specification is essentially the same as our (12).

The most general class, due to Epstein and Zinn [1989][1991], relaxes *both* additivity of the

⁹ Keeney [1977] illustrates this in a one-on-one, conversational elicitation with a decision-maker. These lottery comparisons are used to justify the use of a correlation-neutral multiattribute utility function, and not to elicit the degree of correlation aversion. Delquie and Luo [1997] show how one can *test for* the tractable correlation-neutral class of functions using only two indifference judgements, rather than the set of lottery comparisons that can be used to *estimate the extent of* multiattribute risk aversion.

intertemporal utility function *and* additivity of the a-temporal utility functional generating the certain-equivalent. Their relaxation of additivity of the intertemporal utility function follows Kreps and Porteus [1978], and their relaxation of additivity of the certainty-equivalent functional is comparable to our (14') below in §4.D.¹⁰

We agree with Backus, Routledge and Zin [2004; p.382] that these specifications have proven useful:

We think several varieties of exotic preferences have already proved themselves. Applications of Kreps-Porteus and Epstein-Zin preferences to asset pricing, precautionary saving, and risk-sharing are good examples. While these preferences have not solved all of our problems, they have become a frequent source of insight. Their ease of use in econometric work is another point in their favor.

Of course, these approaches have “proved themselves” by being *sufficient* to characterize risk and time preferences in a flexible manner.¹¹ Nobody has claimed that they are necessary, or the only way to characterize preferences flexibly. Habit persistence in preferences and/or durable consumption goods can generate non-separable utility representations that can also allow risk and time preferences to be teased apart (e.g., Constantinides [1990] and Dunn and Singleton [1986]). Our contribution is to demonstrate that separable, non-additive representations, built from a well-known theoretical foundation of multi-attribute utility, can be evaluated in a controlled environment. Their relative value in characterizing naturally-occurring microeconomic or macroeconomic data, such as the tests of alternative habit persistence models in Ferson and Constantinides [1991], is not something we consider. However, just as it has proven valuable in the behavioral literature on risk to have alternative specifications to evaluate (Starmer [1990]), we believe it is valuable to have

¹⁰ Our non-additivity derives from a Rank Dependent Utility specification, and theirs from a Chew-Dekel specification. These are not the same: Backus, Routledge and Zin [2004; p.332].

¹¹ Experiments studying these models, and considering the delayed resolution and/or payments of lottery outcomes, include Noussair and Wu [1996] and Coble and Lusk [2010]. There have also been experiments over non-salient or hypothetical rewards, such as Ahlbrecht and Weber [1997].

alternative specifications with respect to risk and time preferences.

2. Experiments

There are several critical components of experimental procedures that need to be addressed when eliciting choices over time-dated monetary flows. Some are behavioral, and some are theory-driven. These components guide the specific experimental design we developed.

A. Essential Characteristics of the Experiments

The first consideration is the importance of the tradeoffs being presented in a transparent manner to subjects, rather than as a jumble of different principal amounts, horizons, front end delays, and implied interest rates. The “multiple price list” procedure for discount rate choices that was proposed by Coller and Williams [1999] is an important advance here. In this procedure the individual gets to choose between a list of options that provide a principal at some sooner date, and a larger amount of money at some future date. The list is ordered in increasing order of the larger amounts of money, to make it easy for the individual to see the tradeoffs. The intuitive aspect of this presentation is that no subject would be expected to defer payment for the first rows, where the implied return is negligible, but that every subject might be expected to defer in the last rows, where the implied return is large. Of course, “negligible” and “large” are in the eyes of the decision-maker, but annualized interest rates of less than a percentage point or more than 100 percentage points would be expected to generally fit the bill.

The second consideration, and related to the need to provide a cognitively transparent task, is the provision of annualized interest rates implied by each alternative. In many countries such rates are required to be provided as part of a regulatory requirement for most consumer loans, but one might also provide them in order to avoid testing hypotheses about whether individuals can calculate

them concurrently with the effort to elicit their preferences. On the other hand, there are many settings in which real decisions with real consequences in the future do not enjoy the cognitive benefit of having implied annualized rates displayed clearly: for example, decisions to smoke, eat bad foods, engage in unsafe sex, have children, get married or divorced, and so on. Again following Coller and Williams [1999], we evaluate the provision of annualized interest rates as a treatment and study its effect on decisions.

The third component is to control for the credibility of payment. This is addressed in large part by using payment procedures that are familiar and credible, and wherever possible by adding some formal legality to the contract between experimenter and subject to pay funds in the future. Coller and Williams [1999] and Coller, Harrison and Rutström [2010] used promises to pay by a permanent faculty member that had been legally notarized; Harrison, Lau and Williams [2002] and Andersen, Harrison, Lau and Rutström [2008a] conducted experiments under the auspices, and actual letterhead, of a recognized government agency. One device for controlling for credibility, albeit at some cost in terms of identifying certain discounting models, is to employ a front end delay on the sooner and later payments: one argument for this procedure is to equalize the credibility of future payment for the two dated payments used to infer discount rates.¹² On the other hand, some would argue that the credibility of payment is one component of the “passion for the present” that generates non-constant discounting behavior, and that it should not be neutered by the use of a front end delay. Moreover, and critical for the present design, if the non-constancy occurs primarily within the front end delay horizon, then one might incorrectly infer constant discounting simply because the design “skipped over it.” In our design we therefore want to consider as a treatment the

¹² Another argument is many, if not all, choices that involve future consequences naturally have a front end delay. Hence the front end delay is not as artefactual a procedure as one might initially think. Although this is true, it is not the case for all such choices.

use of a front end delay or not.

The fourth component is to control for the *atemporal* utility of time-dated monetary flows. All experimental designs prior to Andersen, Harrison, Lau and Rutström [2008a] assumed that utility was linear in experimental income, and defined discount rates in terms of monetary flows instead of utility flows. This assumption had been clearly recognized earlier, such as in Keller and Strazzera [2002, p. 148] and Frederick, Loewenstein, and O'Donoghue [2002, p. 381ff.], but the importance for inferred discount rates not appreciated. A direct application of Jensen's Inequality to (12) shows that a more concave atemporal utility function (10) and (11) must lower inferred discount rates for given choices between the two monetary options. The only issue for experimental design then is how to estimate or induce the non-linear utility function. The approach of Andersen, Harrison, Lau and Rutström [2008a] was to have one experimental task to identify the utility function, another task to identify the discount rate conditional on knowing the utility function, and jointly estimate the structural model defined over the parameters of the utility function and discount rate. Thus the general principle is a recursive design, combined with joint estimation of all structural parameters so that uncertainty about the parameters defining the utility function propagates in a "full information" sense into the uncertainty about the parameters defining the discount function. Intuitively, if the experimenter only has a vague notion of what r is in (10) and (11), then one cannot make precise inferences about δ (or η) in (12).

The existing literature suggests that the front end delay and the correction for non-linear utility are the most significant treatments in terms of their quantitative impact on elicited discount rates. Coller and Williams [1999] were the first to demonstrate the effect of a front end delay; their estimates show a drop in elicited discount rates over money of just over 30 percentage points from an average 71% with no front end delay. Using the same experimental and econometric methods, and with all choices having a front end delay, Harrison, Lau and Williams [2002] estimated average

discount rates over money of 28.1% for the adult Danish population. Andersen, Harrison, Lau and Rutström [2008a] were the first to demonstrate the effect of correcting for non-linear utility; their estimates show a drop in elicited discount rates of 15.1 percentage points from a discount rate over money of 25.2%. These results would lead us to expect discount rates around 10% with a front end delay, with a significantly higher rate when there is no front end delay.

B. The Experimental Design

Subjects are presented with three tasks. The first task identifies individual discount rates, the second task identifies atemporal risk attitudes, and the third task identifies intertemporal risk attitudes. We use tasks with real monetary incentives. Observed choices from all three tasks are then used to jointly estimate structural models of the discounting function defined over utility.

Individual Discount Rates

Individual discount rates will be examined by asking subjects to make a series of choices over two certain outcomes that differ in terms of when they will be received. For example, one option can be 3000 kroner in 1 month, and another option can be 3300 kroner in 13 months. If the subject picks the earlier option we can infer that their discount rate is below 10% for 12 months, starting in 1 month, and if the subject picks the later option we can infer that their discount rate is above 10% for that horizon and start date. By varying the amount of the later option we can identify the discount rate of the individual, conditional on knowing the utility of those amounts to this individual. One can also vary the time horizon to identify the discount rate function, and of course one can vary the front end delay. This method has been widely employed in the United States (e.g., Coller and Williams [1999]), Denmark (e.g., Harrison, Lau and Williams [2002]), and Canada (e.g., Eckel, Johnson and Montmarquette [2005]).

We ask subjects to evaluate choices over several time horizons. We consider time horizons between 2 weeks and 1 year. Each subject is presented with choices over four time horizons, and those horizons are drawn at random, without replacement, from a set of thirteen possible horizons (2 weeks, and 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 and 12 months). This design will allow us to obtain a smooth characterization of the discount rate function across the sample for horizons up to one year. We also over-sampled the first three horizons, since this very short-term is clearly of great significance for the alternative specification. Hence each subject was twice as likely to get a horizon of 2 weeks, 1 month or 2 months as any of the later horizons.¹³

We also varied the time delay to the early payment option on a between-subjects basis: roughly half of the sample had no front end delay, and the other half had a 30-day front end delay. It would be possible to consider more variations in the front end delay, but we wanted to keep the treatment as sharp as possible before examining the tradeoff. Similarly, we varied the provision of implied interest rates for each choice on a between-subjects basis, and independently of the front end delay treatment. We also varied the order in which the time horizon was presented to the subject: either in ascending order or descending order.

Another treatment, to examine the “magnitude effect” that suggests that exponential discounting is more prevalent with larger stakes, is to vary the principal.¹⁴ We employ two levels of the principal on a between-subjects basis, again to assess the significance of the hypothesized fixed monetary cost of delay.

These four treatments, the front end delay, information on implied interest rates, the level of

¹³ The shorter horizons were each chosen with probability $2/16 = 0.125$, compared to the $1/16 = 0.0625$ probability for each of the others.

¹⁴ See Loewenstein and Prelec [1992; p. 575] and Frederick, Loewenstein, and O’Donoghue [2002, p. 363] for statements of the magnitude effect. Scholten and Read [2010] claim that the magnitude effect “... is probably the most robust anomaly in intertemporal choice.” We evaluate this treatment in Andersen, Harrison, Lau and Rutström [2011b] and conclude that it is not.

the principal, and the order of presentation of the horizon, result in a $2 \times 2 \times 2 \times 2$ design. Roughly 1/16 of the sample was assigned at random to any one particular combination.

Risk Attitudes

Risk attitudes were evaluated by asking subjects to make a series of choices over outcomes that involve some uncertainty. To be clear, risk attitudes are elicited here simply as a convenient vehicle to estimate the non-linear utility function of the individual. The theoretical requirement, from the definition of the discount factor in (12), is for us to know the utility function over income if we are to correctly infer the discount rate the individual used. The discount rate choices described above are not defined over lotteries.

Our design poses a series of binary lottery choices. For example, lottery A might give the individual a 50-50 chance of receiving 1600 kroner or 2000 kroner to be paid today, and lottery B might have a 50-50 chance of receiving 3850 kroner or 100 kroner today. The subject picks A or B. One series of 10 choices would offer these prize sets with probabilities on the high prize in each lottery starting at 0.1, then increasing by 0.1 until the last choice is between two certain amounts of money. In fact, these illustrative parameters and design was developed by Holt and Laury [2002][2005] to elicit risk attitudes in the United States, and has been widely employed. Their experimental procedures provided a decision sheet with all 10 choices arrayed in an ordered manner on the same sheet; we used the procedures of Hey and Orme [1994], and presented each choice to the subject as a “pie chart” showing prizes and probabilities. We gave subjects 40 choices, in four sets of 10 with the same prizes. The prize sets employed are as follows: [A1: 2000 and 1600; B1: 3850 and 100], [A2: 1125 and 750; B2: 2000 and 250], [A3: 1000 and 875; B3: 2000 and 75] and [A4: 2250 and 1000; B4: 4500 and 50]. The order of these four sets was random for each subject, but within each set the choices were presented in an ordered manner, with increments of the high prize

probability of 0.1.

The typical findings from lottery choice experiments of this kind are that individuals are generally averse to risk, and that there is considerable heterogeneity in risk attitudes across subjects: see Harrison and Rutström [2008] for an extensive review. Much of that heterogeneity is correlated with observable characteristics, such as age and education level.

Intertemporal Risk Attitudes

Intertemporal risk attitudes were evaluated by asking subjects to make a series of choices over uncertain profiles of outcomes that are paid out at different points in time.¹⁵ For example, lottery A might give the individual a 10% chance of receiving 3850 kroner to be paid today *and* 100 kroner to be paid in 12 months and a 90% chance of receiving 100 kroner to be paid today *and* 3850 kroner to be paid in 12 months. Lottery B might give the individual a 10% chance of receiving 3850 kroner to be paid today *and* 3850 kroner to be paid in 12 months and a 90% chance of receiving 100

¹⁵ Other experiments with real incentives have examined choices over time-delayed risk, but where the time delay is the same for all outcomes within a given lottery (Noussair and Wu [1996], Coble and Lusk [2010], Baucells and Heukamp [2010] and Abdellaoui, Diecidue and Öncüler [2011]). This is an important domain of choice to examine, but is not intended to address the issue of correlation aversion. The only previous experiments with real incentives that we know of that explicitly test for *a-temporal* multiattribute risk aversion were due to von Winterfeldt [1980]. He considered lottery choices of 18 subjects defined over 36 consumption bundles of gallons of gasoline and pounds of ground beef. For example, the lottery α might be a 50% chance of {16 gallons of gas and 10 pounds of ground beef} and a 50% chance of {no gas and no beef}, and the lottery β defined as a 50% chance of {10 pounds of ground beef} and a 50% chance of {16 gallons of gas}. He used three different methods of eliciting preference: direct statement of preference, including the option of indifference; a rating normalized between 0 and 100; and cash-equivalents elicited using an incentive-compatible Becker, DeGroot and Marschak [1964] procedure with buying prices elicited from between \$0 and \$20. It appears that only the choices in the last elicitation procedure were played out for real, although the exposition is not completely clear (contrast the top of page 73 and the top of page 70). There were some response mode effects, of the kind now known as “preference reversals,” and some violations of non-satiation. But the general conclusion is that “Multiattribute risk aversion showed very clearly for all except two subjects” (p. 80). He also concluded that “Multiattribute risk aversion appeared unrelated to [...] single attribute risk aversion” (p. 81). Pliskin, Shepard and Weinstein [1980] and Payne, Laughhunn and Crum [1984] report experiments with *hypothetical* incentives that test for multiattribute risk neutrality, and respectively report results that support and reject that characterization.

kroner to be paid today *and* 100 kroner to be paid in 12 months. The subject picks A or B. We gave subjects 40 choices, in four sets of 10 with the same prizes. Each series of 10 choices would offer the prize sets with probabilities on the high, sooner prize in each lottery starting at 0.1, then increasing by 0.1 until the last choice is between two certain amounts of money. In this example, the last choice would be a choice between receiving 3850 kroner now and 100 kroner in 12 months later (lottery A) or receiving 3850 kroner now and 3850 kroner in 12 months (lottery B).

We present each choice to the subject as a “pie chart” showing prizes and probabilities. The prize sets employed are as follows (with the sooner payment first and the later payment second): [A1: (3850, 100) and (100, 3850); B1: (3850, 3850) and (100, 100)], [A2: (2000, 250) and (250, 2000); B2: (2000, 2000) and (250, 250)], [A3: (2000, 75) and (75, 2000); B3: (2000, 2000) and (75, 75)] and [A4: (4500, 50) and (50, 4500); B4: (4500, 4500) and (50, 50)]. One of these four sets was selected at random for each subject.

Each subject is presented with choices over four time horizons between 2 weeks and 1 year in the discount rate tasks, and the same four time horizons were applied in the intertemporal risk aversion tasks. We also varied the time delay to the early payments on a between-subjects basis, and this treatment again followed from the discount rate tasks. If there was no time delay to the early payment in the discount rate tasks, then the early payments in the intertemporal risk aversion tasks would also be paid out immediately, and similarly if the delay to the early payment option in the discount rate tasks was 1 month.

C. The Experiments

Between September 28 and October 22, 2009, we conducted experiments with 413 Danes. The sample was drawn to be representative of the adult population as of January 1, 2009, using sampling procedures that are virtually identical to those documented at length in Harrison, Lau,

Rutström and Sullivan [2005]. We received a random sample of the population aged between 18 and 75, inclusive, from the Civil Registration Office and sent out 1969 invitations.¹⁶

With a sample of 413, on average 25.8 subjects were assigned to each of the 16 treatments for the discounting tasks. We did not develop this experimental design to estimate models at the level of the individual subject or treatment condition, although obviously we will control for these factors.

Our experiments were all conducted in hotel meeting rooms around Denmark, so that travel logistics for the sample would be minimized. Various times of day were also offered to subjects, to facilitate a broad mix of attendance. The largest session had 15 subjects, but most had fewer. The procedures were standard: Appendix A documents an English translation of the instructions, and shows typical screen displays. Subjects were given written instructions, which were also read out, and then made choices in a trainer task, which was “played out” so that the full set of consequences of each choice were clear. In fact, subjects were paid *Big Ben* caramels instead of money for all trainers, and the payments were happily consumed when delivered. All interactions were by computer. The order of the block of discount rate tasks and the block of risk attitudes tasks was randomized for each session. After all choices had been made the subject was asked a series of standard socio-demographic questions.

There were 40 discounting choices, 40 atemporal risk attitude choices and 40 intertemporal risk attitude choices, and each subject had a 10% chance of being paid for one choice in each set of

¹⁶ That recruiting sample was drawn by us from a random sample of 50,000 adult Danes obtained from the Civil Registration Office, which includes information on sex, age, residential location, marital status, and whether the individual is an immigrant. We also randomized the fixed recruitment show-up fee across subjects. All of this information can be used to evaluate the possibility of sample selection biases in the manner of Harrison, Lau and Rutström [2009]. At a very broad level our sample was representative on average: the sample of 50,000 had an average age of 49.8, 50.1% of them were married, and 50.7% were female; our final sample of 413 had an average age of 48.7, 56.5% of them were married, and 48.2% were female.

40 choices. Average payments on the first block were 201.4 kroner (although some were for deferred receipt), on the second block the average was 242.5 kroner, and average payments on the third block were 270.7 kroner for a combined average of 714.6 kroner. The exchange rate at the time was close to 5 kroner per U.S. dollar, so earnings averaged approximately 143 dollars per 2 two-hour session for these tasks. Subjects were also paid a fixed show-up fee of 300 kroner or 500 kroner.¹⁷

For payments to be made in the future, the following language explained the procedures:

You will receive the money on the date stated in your preferred option. If you receive some money today, then it is paid out at the end of the experiment. If you receive some money to be paid in the future, then it is transferred to your personal bank account on the specified date. In that case you will receive a written confirmation from Copenhagen Business School which guarantees that the money is reserved on an account at Danske Bank. You can send this document to Danske Bank in a prepaid envelope, and the bank will transfer the money to your account on the specified date.

Payments by way of bank transfer are common in Denmark, Copenhagen Business School is a well-known educational institution in Denmark, and Danske Bank is the largest financial enterprise in Denmark as measured by total assets.

3. Econometrics

Our objective is to evaluate alternative discounting functions reviewed in section 1. The approach we adopt is direct estimation by maximum likelihood of some structural model of a latent choice process in which the core parameters defining risk attitudes and discounting behavior can be estimated. We review the basic inferential logic for estimating risk attitudes, and discuss the extension to discounting behavior. Extensions to consider mixture specifications and random coefficient models are considered in section 5. Our experimental design was not intended to allow

¹⁷ An extra show-up fee of 200 kroner was paid to 35 subjects who had received invitations stating 300 kroner, but then received a final reminder that accidentally stated 500 kroner.

estimation at the level of the individual, but could be extended to allow that. Instead we focus on estimates pooled over individuals, allowing for individual heterogeneity (Appendix B). We use joint estimation of all preference parameters, recognizing that we have observations on all tasks for each individual.

A. Estimating the Atemporal Utility Function

Assume that the atemporal utility of income is defined over monetary payments \tilde{z} and \tilde{Z} to be paid at the end of the session, where $\tilde{z} < \tilde{Z}$. Just as \mathbf{x} and \mathbf{y} are elements of \mathbf{X} and \mathbf{Y} , \mathbf{z} is a typical element of \mathbf{Z} . Since there is no temporal dimension of the set \mathbf{Z} , $\mathbf{z} = u(\tilde{z})$. In general \mathbf{x} and \mathbf{y} are defined over payments to be made in the future, although in some cases, with no front end delay, payments may be made at the end of the session. Let the utility function defined over the typical element \mathbf{z} be the same as the utility functions (10) and (11) defined over \mathbf{x} and \mathbf{y} :

$$u(\tilde{z}) = \tilde{z}^{1-r}/(1-r). \quad (13)$$

Then, if $p(\mathbf{z})$ denotes the objective probability of receiving \mathbf{z} , we can define an expected utility of a lottery defined over the two elements \tilde{z} and \tilde{Z} as:

$$EU = [p(\tilde{z}) \times U(\tilde{z})] + [p(\tilde{Z}) \times U(\tilde{Z})] \quad (14)$$

The EU for each lottery pair on the right and left of the display is calculated for a candidate estimate of r , and the index

$$\nabla EU = EU_B - EU_A \quad (15)$$

calculated, where EU_A is Option A and EU_B is Option B as presented to subjects. This latent index, based on latent preferences, is then linked to observed choices using the cumulative logistic distribution function $\Lambda(\nabla EU)$. This “logit” function takes any argument between $\pm\infty$ and transforms it into a number between 0 and 1. Thus we have the logit link function,

$$\text{prob}(\text{choose lottery B}) = \Lambda(\nabla EU) \quad (16)$$

The index defined by (15) is linked to the observed choices by specifying that the B lottery is chosen when $\Lambda(\nabla EU) > 1/2$, which is implied by (16).

Thus the likelihood of the observed responses, conditional on the EUT and CRRA specifications being true, depends on the estimates of r given the above statistical specification and the observed choices. The conditional log-likelihood is then

$$\ln L(r; \mathbf{c}, \mathbf{C}) = \sum_i [(\ln \Lambda(\nabla EU)) \times \mathbf{I}(c_i = 1) + (\ln (1 - \Lambda(\nabla EU))) \times \mathbf{I}(c_i = -1)] \quad (17)$$

where $\mathbf{I}(\cdot)$ is the indicator function, $c_i = 1(-1)$ denotes the choice of the Option B (A) lottery in risk aversion task i , and \mathbf{C} is a vector of individual characteristics reflecting age, sex, race, and so on. The parameter r is defined as a linear function of the characteristics in vector \mathbf{C} .

Harrison and Rutström [2008; Appendix F] review procedures and syntax from the popular statistical package *Stata* that can be used to estimate structural models of this kind. The goal is to illustrate how experimental economists can write explicit maximum likelihood (ML) routines that are specific to different structural choice models. It is a simple matter to correct for stratified survey responses, multiple responses from the same subject (“clustering”), or heteroskedasticity, as needed.

Extensions of the basic model are easy to implement, and this is the major attraction of the structural estimation approach. For example, one can easily extend the functional forms of utility to allow for varying degrees of relative risk aversion (RRA).¹⁸ It is also simple matter to generalize this ML analysis to allow any core parameter to be a linear function of observable characteristics of the individual or task. For example, we would extend the model for the parameter r in (13) to be $r = r_0 + R \times \mathbf{C}$, where r_0 is a fixed parameter and R is a vector of effects associated with each characteristic in the variable vector \mathbf{C} . In effect the unconditional model assumes $r = r_0$ and just estimates r_0 . This

¹⁸ Consider, as one important example, the Expo-Power (EP) utility function proposed by Saha [1993]. Following Holt and Laury [2002], the EP function is defined as $U(\mathbf{z}) = [1 - \exp(-\alpha \tilde{z}^{1-\hat{r}})]/\alpha$, where α and \hat{r} are parameters to be estimated. RRA is then $\hat{r} + \alpha(1-\hat{r})\tilde{z}^{1-\hat{r}}$, so RRA varies with income if $\alpha \neq 0$. This function nests CRRA (as $\alpha \rightarrow 0$) and CARA (as $\hat{r} \rightarrow 0$).

extension significantly enhances the attraction of structural ML estimation, particularly for responses pooled over different subjects, since one can condition estimates on observable characteristics of the task or subject.

An important extension of the core model is to allow for subjects to make some errors. The notion of error is one that has already been encountered in the form of the statistical assumption that the probability of choosing a lottery is not 1 when the EU of that lottery exceeds the EU of the other lottery. This assumption is clear in the use of a link function between the latent index ∇EU and the probability of picking one or other lottery; in the case of the logistic CDF, this link function is $\Lambda(\nabla EU)$. If there were no errors from the perspective of the decision-making model under risk, this function would be a step function: zero for all values of $\nabla EU < 0$, anywhere between 0 and 1 for $\nabla EU = 0$, and 1 for all values of $\nabla EU > 0$.

The problem with this CDF is immediate: it predicts with probability one or zero. The likelihood approach asks the model to state the probability of observing the actual choice, conditional on some trial values of the parameters of the theory. Maximum likelihood then locates those parameters that generate the highest probability of observing the data. For binary choice tasks, and independent observations, the likelihood of the sample is just the product of the likelihood of each choice conditional on the model and the parameters assumed, and that the likelihood of each choice is just the probability of that choice. So if we have any choice that has zero probability, and it might be literally 1-in-a-million choices, the likelihood for that observation is not defined. Even if we set the probability of the choice to some arbitrarily small, positive value, the log-likelihood zooms off to minus infinity. We can reject the theory without even firing up any statistical package.

This implication is true for any theory that makes deterministic predictions, including Expected Utility Theory. This is why one needs some formal statement about how the deterministic prediction of the theory translates into a probability of observing one choice or the other, and then

perhaps also some formal statement about the role that structural errors might play. In short, one *cannot divorce the job of the theorist from the job of the econometrician*, and some assumption about the process of linking latent preferences and observed choices is needed. That assumption might be about the mathematical form of the link, as in (16), but it cannot be avoided. Even the very definition of risk aversion needs to be specified using stochastic terms unless we are to impose absurd economic properties on estimates (Wilcox [2008][2011]).

We employ the error specification originally due to Fechner and popularized by Hey and Orme [1994]. This error specification posits the latent index

$$\nabla EU = (EU_B - EU_A)/\mu \quad (16')$$

instead of (16), where μ is a structural “noise parameter” used to allow some errors from the perspective of the deterministic model of decision-making under risk. This is just one of several different types of error story that could be used, and Wilcox [2008] provides a masterful review of the implications of the alternatives.¹⁹ As $\mu \rightarrow 0$ this specification collapses to the deterministic choice model, where the choice is strictly determined by the EU of the two lotteries; but as μ gets larger and larger the choice essentially becomes random. When $\mu=1$ this specification collapses to (16), where the probability of picking one lottery is given by the ratio of the EU of one lottery to the sum of the EU of both lotteries. Thus μ can be viewed as a parameter that flattens out the link functions as it gets larger. We then extend the likelihood function to include the behavioral parameter:

$$\ln L^{RA}(r, \mu; c, \mathbf{C}) = \sum_i [(\ln \Lambda(\nabla EU)) \times \mathbf{I}(c_i = 1)] + (\ln (1 - \Lambda(\nabla EU))) \times \mathbf{I}(c_i = -1)] \quad (17')$$

and calculate ML values of r and μ by maximizing (17').

An important contribution to the characterization of behavioral errors is the “contextual

¹⁹ Some specifications place the error at the final choice between one lottery or after the subject has decided which one has the higher expected utility; some place the error earlier, on the comparison of preferences leading to the choice; and some place the error even earlier, on the determination of the expected utility of each lottery.

error” specification proposed by Wilcox [2011]. It is designed to allow robust inferences about the primitive “more stochastically risk averse than.” It posits the latent index

$$\nabla EU = ((EU_B - EU_A)v)/\mu \quad (16'')$$

instead of (16'), where v is a new, normalizing term for each lottery pair A and B. The normalizing term v is defined as the maximum utility over all prizes in this lottery pair minus the minimum utility over all prizes in this lottery pair. The value of v varies, in principle, from lottery choice to lottery choice: hence it is said to be “contextual.” For the Fechner specification, dividing by v ensures that the *normalized* EU difference $[(EU_B - EU_A)/v]$ remains in the unit interval.

B. Estimating the Discounting Function

For the moment, consider the intertemporal utility function (9), which is the same as assuming that the agent is intertemporally risk neutral or intertemporally correlation neutral. A subject is indifferent between two time-dated income options \tilde{x} and \tilde{y} if and only if

$$D_t u(\tilde{x}) = D_{t+\tau} u(\tilde{y}) \quad (18)$$

where $u(\tilde{x})$ is the utility at time t of monetary outcome \tilde{x} for delivery at time t , τ is the horizon for delivery of the later monetary outcome \tilde{y} at time $t+\tau$, and D_t is the discount factor at time t . Thus (18) is an indifference condition and D is the discount factor that equalizes the present value of the *utility* of the two monetary outcomes \tilde{x} and \tilde{y} .

We can write out the likelihood function for the choices that our subjects made and jointly estimate the risk parameter r in equation (13) and the discount rate parameter δ in (18). We use the same stochastic error specification as in (16'), albeit with a different Fechner error term μ' for the discount choices.²⁰ Instead of (16') we have

²⁰ We do not need to apply the contextual utility correction v for these choices since they are over deterministic monetary amounts.

$$\nabla PV = (PV_B - PV_A)/\mu', \quad (19)$$

where the discounted utility of Option A is given by

$$PV_A = (1/(1+\delta)^t) \tilde{x}^{1-r} \quad (20)$$

and the discounted utility of Option B is

$$PV_B = (1/(1+\delta)^{t+\tau}) \tilde{y}^{1-r}, \quad (21)$$

and \tilde{x} and \tilde{y} are the monetary amounts in the choice tasks presented to subjects for delivery at time t and time $t+\tau$, respectively. The parameter μ' captures noise for the discount rate choices, just as μ was a noise parameter for the risk aversion choices. We assume here that the utility function is stable over time and is perceived *ex ante* to be stable over time.²¹

Thus the likelihood of the discount rate responses, conditional on the EUT, CRRA and exponential discounting specifications being true, not to mention the assumption of intertemporal risk neutrality, depends on the estimates of r , δ , μ and μ' , given the assumed value of ω and the observed choices. The conditional log-likelihood is

$$\ln L^{DR}(r, \delta, \mu, \mu'; c, \mathbf{C}) = \sum_i [(\ln \Lambda(\nabla PV)) \times \mathbf{I}(c_i=1) + (\ln (1-\Lambda(\nabla PV))) \times \mathbf{I}(c_i=-1)] \quad (23)$$

where $c_i = 1(-1)$ denotes the choice of Option B (A) in discount rate task i , and \mathbf{C} is again a vector of individual characteristics.

The joint likelihood of the risk aversion and discount rate responses, under the maintained assumption for now of intertemporal risk neutrality, can then be written as

$$\ln L(r, \delta, \mu, \mu'; c, \mathbf{C}) = \ln L^{RA} + \ln L^{DR} \quad (24)$$

where L^{RA} is defined by (17') and L^{DR} is defined by (23). This expression can then be maximized

²¹ Direct evidence for the former proposition is provided by Andersen, Harrison, Lau and Rutström [2008b], who examine the temporal stability of risk attitudes in the Danish population. The second proposition is a more delicate matter: even if utility functions are stable over time, they may not be subjectively perceived to be, and that is what matters for use to assume that the same r that appears in (1) appears in (9) and (10). When there is no front end delay, this assumption is immediate for (9), but not otherwise.

using standard numerical methods.

Nothing in this inferential procedure relied on the use of EUT, or the CRRA functional form. Nor did anything rely on the use of the exponential discounting function. These methods generalize immediately to alternative multiattribute models of decision making under risk, such as those presented in Miyamota and Wakker [1996]. They also extend to specifications that use alternative discounting functions, such as presented in Andersen, Harrison, Lau and Rutström [2011a]. The key innovation here, however, is to replace (18') and (23) with non-additive specifications that allow for correlation aversion.

C. Estimating the Intertemporal Utility Function

The next step is to consider non-additive separable specifications of the intertemporal utility function and estimate the coefficient of intertemporal risk aversion. Equation (12) is a simple extension of the additively separable model in (9) and implies that the expected utility of Option A in the intertemporal risk aversion task is given by

$$PEU_A = p(\tilde{X}, \tilde{y}) \times [D_t u(\tilde{X}) + D_{t+\tau} u(\tilde{y})]^{(1-\eta)} / (1-\eta) + p(\tilde{x}, \tilde{Y}) \times [D_t u(\tilde{x}) + D_{t+\tau} u(\tilde{Y})]^{(1-\eta)} / (1-\eta) \quad (25)$$

and the expected utility of Option B is given by

$$PEU_B = p(\tilde{X}, \tilde{Y}) \times [D_t u(\tilde{X}) + D_{t+\tau} u(\tilde{Y})]^{(1-\eta)} / (1-\eta) + p(\tilde{x}, \tilde{y}) \times [D_t u(\tilde{x}) + D_{t+\tau} u(\tilde{y})]^{(1-\eta)} / (1-\eta) \quad (26)$$

where $p(\mathbf{x}, \mathbf{y})$ is the probability of receiving \mathbf{x} in period t and \mathbf{y} in period $t+\tau$. We can again write out the likelihood function for the choices that the subjects made and jointly estimate the risk parameter η , the discount rate parameter δ , and the intertemporal risk parameter η . We employ the contextual error specification, and the latent index is specified by

$$\nabla PEU = ((PEU_B - PEU_A)v') / \mu'' \quad (27)$$

where μ'' is a noise parameter for the intertemporal risk aversion choices. The normalizing term v' is

defined as the maximum intertemporal utility over all prize profiles in this lottery pair (\tilde{X}, \tilde{Y}) minus the minimum utility over all prize profiles in this lottery pair (\tilde{x}, \tilde{y}) . The maximum intertemporal utility over all prize profiles in the lottery pair is $[D_t u(\tilde{X}) + D_{t+\tau} u(\tilde{Y})]^{(1-\eta)}/(1-\eta)$, and the minimum intertemporal utility is $[D_t u(\tilde{x}) + D_{t+\tau} u(\tilde{y})]^{(1-\eta)}/(1-\eta)$.

The likelihood of the intertemporal risk aversion responses, conditional on the specification of intertemporal utility being true, depends on the estimates of $r, \delta, \eta, \mu, \mu'$ and μ'' , given the observed choices. The conditional log-likelihood is

$$\ln L(r, \delta, \eta, \mu, \mu', \mu''; \mathbf{c}, \mathbf{C}) = \sum_i [(\ln \Lambda(\nabla EV) \times \mathbf{I}(c_i=1)) + (\ln (1-\Lambda(\nabla EV)) \times \mathbf{I}(c_i=-1))] \quad (28)$$

where $c_i = 1(-1)$ denotes the choice of Option B (A) in intertemporal risk aversion task i , and \mathbf{C} is again a vector of individual characteristics.

The joint likelihood of the atemporal risk aversion, discount rate and intertemporal risk aversion responses can then be written as

$$\ln L(r, \delta, \eta, \mu, \mu', \mu''; \mathbf{c}, \mathbf{C}) = \ln L^{\text{RA}} + \ln L^{\text{DR}} + \ln L^{\text{SDR}} \quad (29)$$

where L^{RA} is defined by (20'), L^{DR} is defined by (24) and L^{SDR} is defined by (28).

The recursive nature of this joint likelihood function is matched by our experimental design. Ignoring the objective parameters of the tasks, the lottery choices over stochastic lotteries paid out today (RA) depend on r and μ ; the discounting tasks over non-stochastic outcomes paid out today or some time in the future (DR) depend on r, μ, δ and μ' ; and the discounting tasks over stochastic outcomes paid out today or some time in the future (SDR) depend on $r, \mu, \delta, \mu', \eta$ and μ'' . Putting the behavioral error terms aside, if we were to try to estimate r and δ using either the RA or the DR choices, we would be unable to identify both parameters. Similarly, if we were to try to estimate r, δ and η using only two of three tasks, we would face an identification problem. These identification problems are inherent to the theoretical definitions of the discount rate and correlation aversion, and demand an experimental design that combines multiple types of choices and an econometric

approach that recognizes the complete structural model. The general principle is a recursive design, combined with joint estimation of all structural parameters so that uncertainty about the parameters defining the utility function propagates in a “full information” sense into the uncertainty about the parameters defining the discount function *and* the intertemporal utility function. Intuitively, if the experimenter only has a vague notion of what $u(\cdot)$ is, because of poor estimates of r , then one cannot make precise inferences about δ or η are. Similarly, poor estimates of δ , even if r is estimated relatively precisely, imply that one cannot make precise inferences about η .²²

4. Results

A. Basic Results

Table 1 reports maximum likelihood estimates of the specification with the CRRA atemporal utility function defined by (13).²³ There is evidence of a concave atemporal utility function ($r > 0$), with r estimated to be 0.55. The discount rate is estimated to be 7.7% on an annualized basis.²⁴ The main novelty here is evidence of intertemporal risk aversion, with η estimated to be 0.44 and statistically different from 0. The implication is that there should be a difference between the inverse of RRA and the intertemporal elasticity of substitution IES (which is equal to $1/r$), and this is confirmed by the implied estimates in Panel D of Table 1. The IES is estimated to be 1.81 with a

²² This experimental design principle applies more broadly. It is *sometimes* possible to design experimental procedures that do not require two or more experimental tasks, and embed the identification of the utility function into one task. We do not know how to do that with respect to intertemporal risk aversion.

²³ Virtually identical results are obtained if one uses the Expo-Power atemporal utility function.

²⁴ The estimates of r and δ are comparable to those reported for the specification in Andersen, Harrison, Lau and Rutström [2008a] that sampled the same adult Danish population in 2003, but assumed intertemporal risk neutrality. In that case the point estimate of $r = \text{RRA}$ was 0.74 and the discount rate δ was estimated to be 10.1% (Table III, p.601). If we impose the constrain $\eta=0$ in our analysis, the log-likelihood drops significantly from -26347.1 to -26568.5, although the estimates for r and δ in Table 1 are virtually identical.

standard error of 0.10, and it exceeds the inverse of RRA by 0.27.²⁵ This difference between the IES and the inverse of RRA is statistically significant with a p -value of less than 0.001.

We can derive the certainty equivalents for each lottery in Option A and Option B of the intertemporal risk aversion tasks using (12), and then evaluate the risk premia associated with different prize sets.²⁶ Option A pays (\tilde{X}, \tilde{y}) with probability $p(\tilde{X}, \tilde{y})$ and (\tilde{x}, \tilde{Y}) with probability $(1-p(\tilde{X}, \tilde{y}))$. The decision tasks are designed such that $\tilde{x}=\tilde{y}$ and $\tilde{X}=\tilde{Y}$. If we assume, for simplicity, that the discount rate is equal to zero, then the present value of Option A is $\tilde{X}+\tilde{x}$ kroner. If we define the certainty equivalent as either (\tilde{X}, \tilde{x}) or (\tilde{x}, \tilde{X}) then the risk premium is equal to zero for the lotteries in Option A. However, if we define the certainty equivalent as the same certain amount to be paid out in both time periods, instead of high amount \tilde{X} and a low amount \tilde{x} , then the certainty equivalent is

$$CE_A = [(\tilde{X}^{(1-r)} + \tilde{x}^{(1-r)})/2]^{1/(1-r)} \quad (30)$$

where CE_A is paid out in period t and in period $t+\tau$. We can then define the risk premium of the lotteries in Option A as

$$RP_A = (\tilde{X} + \tilde{x}) - 2CE_A \quad (31)$$

The subject prefers to smooth consumption over time if the atemporal utility function is concave, which is just to say that the risk premium is positive when $r>0$. Using the estimated model from Table 1, the estimated risk premium is 1,508 kroner for prize set A1, 462 kroner for prize set A2, 723 kroner for prize set A3, and 2,014 kroner for prize set A4.

If we allow the discount rate to be positive then the certainty equivalent for Option A is

²⁵ We evaluate RRA in the special case where $MRS_{t,t+\tau}(\mathbf{x}/\mathbf{y}) = 1$. In this case $RRA_t = RRA_{t+\tau} = \eta(1-r)/2 + r$.

²⁶ The *matrix* of risk premia in the multiattribute case is characterized by Duncan [1977] and Karni [1979]. Kihlstrom and Mirman [1974; §2.2] derived a “directional risk premium” which takes on as many values as there are possible “directions,” and so is also multi-valued. But they pointed out their measure allowed unique comparisons of utility functions representing the same ordinal preferences.

$$CE_A = [((1-\eta) PEU_A)^{1/(1-\eta)} (1-r) / (D_t + D_{t+\tau})]^{1/(1-r)} \quad (30')$$

where PEU_A is the expected utility of Option A given by (25). The risk premium is then derived as

$$RP_A = p(\tilde{X}, \tilde{y}) \times [D_t \tilde{X} + D_{t+\tau} \tilde{y}] + (1-p(\tilde{X}, \tilde{y})) \times [D_t \tilde{x} + D_{t+\tau} \tilde{Y}] - (D_t + D_{t+\tau}) \times CE_A \quad (31')$$

The estimated risk premium for prize set A1 varies between 1,451 kroner when $p(\tilde{X}, \tilde{y})=0.1$ and 1,455 kroner when $p(\tilde{X}, \tilde{y})=1$. The estimated risk premium for A2 is 445 and 446 kroner for $p(\tilde{X}, \tilde{y})$ equal to 0.1 and 1, respectively. Similarly, the risk premium interval is [695; 697] for A3 and [1,936; 1,943] for A4. Hence, the estimated risk premium for Option A falls slightly when we consider positive discount rates.

The lotteries in Option B pay (\tilde{X}, \tilde{X}) with probability $p(\tilde{X}, \tilde{X})$ and (\tilde{x}, \tilde{x}) with probability $(1-p(\tilde{X}, \tilde{X}))$. The certainty equivalent of Option B is

$$CE_B = [p(\tilde{X}, \tilde{X}) \times \tilde{X}^{(1-r)(1-\eta)} + (1-p(\tilde{X}, \tilde{X})) \times \tilde{x}^{(1-r)(1-\eta)}]^{1/(1-r)(1-\eta)} \quad (32)$$

where CE_B is again a certain amount that is paid out in both period t and period $t+\tau$. This definition of certainty equivalence implies that CE_B is independent of the discount rate and is equal to \tilde{X} if $p(\tilde{X}, \tilde{X})=1$ and equal to \tilde{x} if $p(\tilde{X}, \tilde{X})=0$. The risk premium is then

$$RP_B = p(\tilde{X}, \tilde{X}) \times \tilde{X} + (1-p(\tilde{X}, \tilde{X})) \times \tilde{x} - CE_B, \quad (33)$$

which is equal to 0 if $p(\tilde{X}, \tilde{X})$ is equal to 0 or 1.

Figure 1 displays the estimated risk premium as a function of $p(\tilde{X}, \tilde{X})$ for each of the four prize sets in Option B of the intertemporal risk aversion task. The solid line is based on the estimated parameter values for r and η in Table 1, and the dashed line is based on a constrained model in which we assume that η is equal to 0. Hence the risk premium when $\eta=0$, and the decision maker is assumed to be correlation neutral (CN), derives entirely from the a-temporal risk aversion r of the decision maker. When η and r are positive, and the decision maker is correlation averse (CA) as well as being atemporally risk averse, the risk premium derives from both types of risk aversion. The results show that *intertemporal risk aversion accounts for a substantial amount of the estimated risk premium.*

For example, the upper left panel shows that the risk premium for prize set B1 is equal to 2,086 kroner in the unconstrained model when $p(\tilde{X}, \tilde{X})=0.5$ and is equal to 1,512 kroner when the intertemporal risk aversion parameter η is constrained to be 0, so the difference of 574 kroner is due to correlation aversion.

It is a simple matter to extend the econometric model to allow structural parameters to depend on observed demographics and experimental treatments. Appendix B documents those estimates. Figure 2 displays the implied distributions of the intertemporal elasticity of substitution and the inverse of relative risk aversion. If the subjects are intertemporally risk neutral, then the predicted values of the two coefficients would be identical and the two curves be identical. The results show that there is some heterogeneity across subjects in the sample with respect to atemporal risk attitudes and the willingness to substitute consumption over time. The predicted values of IES have an estimated mean of 2.07 and standard deviation of 0.84, the population distribution of the inverse of RRA has a mean of 1.92 and standard deviation of 1.51. Figure 3 shows the estimated distribution of the difference between the IES and inverse of RRA. This distribution has an estimated mean of 0.16, with a standard deviation of 1.20, and the 95% confidence interval is between -1.32 and 1.38. Despite the relatively wide confidence interval more than 75% of the observations are positive, and the general tendency away from intertemporal risk neutrality is observed.

B. The Coefficient of Correlation Aversion

Figure 4 displays the predicted distribution of the coefficient of correlation aversion (CCA), evaluated using (5') and the four prize sets in the intertemporal risk aversion tasks. The upper left panel shows predicted CCA values for the income profile with high sooner payments and low later payments. The predicted mean is 0.10 ($\div 1000$), with a standard deviation of 0.08 ($\div 1000$). There is

clear evidence of correlation aversion in general, although roughly 5% of the sample exhibits correlation neutrality or correlation loving preferences. The lower left panel displays the predicted distribution for the income profile with low sooner payments and high later payments. If the individual discount rate is equal to 0 then the predicted values of CCA in the upper and lower left panels would be the same. The predicted mean of the individual discount rate for the sample is 0.076 with a standard deviation of 0.022, and we therefore see small differences between the estimated means and standard deviations across the two income profiles in the upper and lower left panels. The upper and lower right panels show the predicted distributions for the income profiles with two high payments and two low payments respectively. The general pattern is the same as before, although the estimated means and standard deviations are lower (higher) for the income profile with two high (low) payments. Thus we observe correlation aversion in general across the four income profiles in the intertemporal risk aversion tasks.

We can also consider a more flexible specification of the inter-temporal utility function and replace the CRRA specification in (12) with a two-parameter version of the cumulative gamma distribution. There are very few *a priori* restrictions on the shape of the gamma distribution other than those of a cumulative density function. The estimated coefficients of r and δ are the same as before: the estimated IES is equal to 1.81 and the discount rate is estimated to be 7.7% on an annualized basis. However, we find that the estimated CCA is higher when the CRRA specification is replaced by the gamma distribution. We evaluate the CCA using the four prize sets in the intertemporal risk aversion tasks, as before, and find that CCA is 1.2 ($\div 1000$) for the income profile with low sooner payments and low later payments, and 8.3 ($\div 1000$) for the income profile with high sooner payments and high later payments. The estimated CCA are significantly different from 0, and we observe correlation aversion in general when the gamma function is used.

C. Non-Exponential Discounting

The concept of intertemporal correlation aversion does not depend on the use of the exponential discounting model. To illustrate the generality of the results, we consider the effect of using two popular alternative discounting models.²⁷ The exponential discounting model may be viewed as assuming a *constant variable utility cost* per time period of delay. These two alternatives are (a) the Quasi-Hyperbolic specification that allows for a *fixed utility cost* as well as a *constant variable utility cost*, and (b) the Weibull specification that allows for a *non-constant variable utility cost*.

The discount factor for the Quasi-Hyperbolic (QH) specification is defined as

$$D^{QH}(t) = 1 \quad \text{if } t = 0 \quad (34a)$$

$$D^{QH}(t) = \beta/(1+\delta)^t \quad \text{if } t > 0 \quad (34b)$$

where $\beta < 1$ implies quasi-hyperbolic discounting and $\beta = 1$ is exponential discounting. Although the δ in (34b) may be estimated to be a different value than the δ in (9), we use the same notation to allow comparability of functional forms. The defining characteristic of the QH specification is that the discount factor has a jump discontinuity at $t=0$, and that is thereafter exactly the same as the exponential specification. The discount rate for the QH specification is the value of d^{QH} that solves $D^{QH} = 1/(1+d^{QH})$, so it is

$$d^{QH}(t) = 1/ [\beta/(1+\delta)^t]^{(1/t)} - 1 \quad (35)$$

for $t > 0$. Thus for $\beta < 1$ we observe sharply declining discount rates in the very short run, and then discount rates asymptoting towards δ as the effect of the initial drop in the discount factor diminishes. The drop β can be viewed as fixed utility cost of discounting anything relative to the present, since it does not vary with the horizon t once $t > 0$. The QH specification was introduced by Phelps and Pollak [1968] for a social planning problem, and applied to model individual behavior by

²⁷ There are many variants from the exponential model, and most are evaluated by Andersen, Harrison, Lau and Rutström [2011a] using the separable and additive intertemporal utility function (9).

Elster [1979; p.71] and then Laibson [1997].

The QH performs poorly in our model with intertemporal risk aversion, in the sense that the coefficient β is not significantly different from 1, which of course is the exponential case. We estimate the value to be 1.003, with a 95% confidence interval between 0.993 and 1.01. The estimated values for r , δ and η are virtually identical to those shown in Table 1 for the exponential specification (which is not surprising if $\beta \approx 1$). It is worth noting that the QH model performs poorly with these data even when one assumes intertemporal risk neutrality: see Andersen, Harrison, Lau and Rutström [2011a].

The discount factor for the Weibull distribution from statistics²⁸ is defined as

$$D^W(t) = \exp(-\acute{r}t^{1/\acute{s}}) \quad (36)$$

for $\acute{r} > 0$ and $\acute{s} > 0$. For $\acute{s}=1$ this collapses to the exponential specification, and hence the parameter \acute{s} can be viewed as reflecting the “slowing down” or “speeding up” of time as perceived by the individual. This specification is due to Read [2001; p. 25, equation (16)], although he noted (p.25, equation (15)) that the same point about time perception was implicit in an earlier generalization of the hyperbolic due to Mazur [1984; p.427].²⁹ The discount rate at time t in the Weibull specification is then

$$d^W(t) = \exp(\acute{r}t^{(1-\acute{s})/\acute{s}}) - 1 \quad (37)$$

Thus one can again see the exponential emerge as a special case when $\acute{s}=1$.

²⁸ Any probability density function $f(t)$ defined on $[0, \infty)$ can form the basis of a discounting function by taking the integral of $f(t)$ between t and ∞ . Indeed, discounting functions are formally identical to the “survivor functions” that labor and health economists routinely estimate in duration models, and are also known as “reliability functions” in the applied statistics literature on failure. Hence familiar and flexible families of probability density functions, such as the Gamma or Weibull, can be used to directly define discounting functions. This has the attraction of allowing the analyst to rely on a large literature in statistics on the properties of these functions for different inferential purposes.

²⁹ The Weibull specification is the same as the simple functional form in Prelec [2004; p. 526] and applied in Ebert and Prelec [2007; p. 1424ff.] and Andersen, Harrison, Lau and Rutström [2008a; p. 607].

The Weibull model also performs poorly in our data, in the sense that the key parameter \acute{s} is estimated to be 1.047, with a standard error of 0.140 and a 95% confidence interval between 0.77 and 1.32. This uncertainty in the estimate of \acute{s} does translate into some uncertainty about discount rates in the short-run, but not in a significant way. For horizons of 1 week the implied discount rate is 0.092, for 3 months it is 0.082, and for 1 year it is 0.077; the confidence intervals for these estimates are $0 \leftrightarrow 0.19$, $0.05 \leftrightarrow 0.11$ and $0.06 \leftrightarrow 0.09$, respectively. Again, the estimates of r , δ and η are virtually identical to those shown in Table 1 for the exponential specification.

Although our evaluation of alternatives to the exponential discounting model is not exhaustive, doing so would be exhausting. And the results from examining these two alternatives suggest that the effort would be unproductive.

D. Non-Expected Utility Theory

The concept of intertemporal correlation aversion does not depend on the use of the expected utility model, just as one can extend multiattribute utility theory to a wide range of non-EUT models (Fishburn [1984], Miyamoto and Wakker [1996]). To illustrate the generality of the results in this respect, we consider the effect of using a popular alternative model due to Quiggin [1982] which relaxes the independence axiom. The Rank-Dependant Utility (RDU) model posits probability weights based on some continuous function of the objective probabilities, and then infers decisions weights from these probability weights. The probability and decision weights depend on the rank of the outcome, in a familiar manner. If $\tilde{Z} > \tilde{z}$, then we can rewrite

$$EU = [p(\tilde{z}) \times U(\tilde{z})] + [p(\tilde{Z}) \times U(\tilde{Z})] \quad (14)$$

as

$$RDU = [(1-w(p(\tilde{Z})) \times U(\tilde{z}))] + [w(p(\tilde{Z})) \times U(\tilde{Z})] \quad (14')$$

for some probability weighting function $w(p)$. We use the simple power function

$$w(p) = p^\gamma \quad (34)$$

and recognize that there is a veritable menagerie of such functions in use. Of course, expected utility theory assumes the identity function $w(p)=p$, which is the case when $\gamma=1$.

When the outcome is simply an amount of money, as in our atemporal lottery tasks, there is no complication calculating the rank in order to apply the RDU model. When the outcome consists of two time-dated amounts of money, as in our temporal lottery tasks, one has to be a bit more careful. The natural quantity to base the rank on is then the present value of the atemporal utilities afforded by the two time-dated amounts of money. To see this explicitly, recall the expression for option A, referred to generically as lottery α in the definition of correlation aversion:

$$PEU_A = p(\tilde{X}, \tilde{y}) \times [D_t u(\tilde{X}) + D_{t+\tau} u(\tilde{y})]^{(1-\eta)/(1-\eta)} + p(\tilde{x}, \tilde{Y}) \times [D_t u(\tilde{x}) + D_{t+\tau} u(\tilde{Y})]^{(1-\eta)/(1-\eta)} \quad (25)$$

Define $v(\tilde{X}, \tilde{y}) = [D_t u(\tilde{X}) + D_{t+\tau} u(\tilde{y})]$ and $v(\tilde{x}, \tilde{Y}) = [D_t u(\tilde{x}) + D_{t+\tau} u(\tilde{Y})]$ for notational ease. The ranks of $v(\tilde{X}, \tilde{y})$ and $v(\tilde{x}, \tilde{Y})$ would be calculated, and of course these depend on the values of r and δ , as well as \tilde{x} , \tilde{X} , \tilde{y} and \tilde{Y} . It is not *a priori* possible to determine these ranks, but they are conceptually well defined given the values indicated. If the stream $\{\tilde{X}, \tilde{y}\}$ had a higher value than $\{\tilde{x}, \tilde{Y}\}$ then the rank-dependent utility would be

$$PRDU_A = w(p(\tilde{X}, \tilde{y})) \times v(\tilde{X}, \tilde{y})^{(1-\eta)/(1-\eta)} + (1-w(p(\tilde{X}, \tilde{y}))) \times v(\tilde{x}, \tilde{Y})^{(1-\eta)/(1-\eta)} \quad (25')$$

and if the stream $\{\tilde{X}, \tilde{y}\}$ had a *lower* value than $\{\tilde{x}, \tilde{Y}\}$ then the rank-dependent utility would be

$$PRDU_A = (1-w(p(\tilde{x}, \tilde{Y}))) \times v(\tilde{X}, \tilde{y})^{(1-\eta)/(1-\eta)} + w(p(\tilde{x}, \tilde{Y})) \times v(\tilde{x}, \tilde{Y})^{(1-\eta)/(1-\eta)} \quad (25'')$$

A similar construction applies for option B, although there one can trivially identify the ranks on an *a priori* basis.

After all that work, there is no real impact from allowing for this violation of expected utility theory. The estimated value of γ is 1.03, with a 95% confidence of 0.95 \leftrightarrow 1.10, so one cannot reject the hypothesis that $\gamma=1$. There are some slight changes in the other core structural parameters from Table 1: r is now estimated to be 0.53, δ is estimated to be 0.081, and η is estimated to be 0.47. None

of these are significantly different from their expected utility counterparts.

There are more flexible probability weighting functions, but they do not appear to add much to the characterization of risk attitudes for these subjects (see Andersen, Harrison, Lau and Rutström [2011a; §5A]).

5. Conclusions

We elicit intertemporal risk attitudes from a representative sample of the adult Danish population using real economic commitments and a theoretical framework derived from well-known multi-attribute theory. The results suggest that intertemporal risk aversion is a better characterization of the average Dane than intertemporal risk neutrality. This result implies that the convenient additive specification of the intertemporal utility function is not an appropriate representation of intertemporal preferences for the general Danish population. We also show that the characterization of intertemporal risk aversion leads to significantly different estimates of risk premia, compared to the estimates obtained when one assumes it away. Our findings have important implications for the characterization of intertemporal preferences in life-cycle modeling, labor supply over time, retirement planning, and policy applications with varying time profiles of costs and benefits.

Table 1: Maximum Likelihood Estimates Assuming CRRA Atemporal Utility Function

N=49,560 observations, based on 413 subjects

Parameter	Point Estimate	Standard Error	<i>p</i> -value	95% Confidence Interval	
<i>A. Atemporal Utility Function</i>					
r	0.55	0.031	<0.001	0.49	0.61
μ	0.18	0.011	<0.001	0.16	0.2
<i>B. Discounting Function</i>					
δ	0.077	0.0077	<0.001	0.062	0.093
μ'	0.13	0.009	<0.001	0.12	0.15
<i>C. Intertemporal Utility Function</i>					
η	0.44	0.082	<0.001	0.28	0.6
μ''	0.18	0.01	<0.001	0.16	0.2
<i>D. Implied Estimates</i>					
RRA	0.65	0.039	<0.001	0.57	0.73
1/RRA	1.54	0.093	<0.001	1.36	1.72
IES	1.81	0.102	<0.001	1.61	2.02
IES - 1/RRA	0.27	0.036	<0.001	0.2	0.34

Figure 1: Estimated Risk Premia

Prize sets in Option B of the intertemporal risk aversion task. Risk Premia for the correlation neutral (CN) case derive only from atemporal risk aversion; risk premia for the correlation averse (CA) case derive from both intertemporal and atemporal risk aversion.

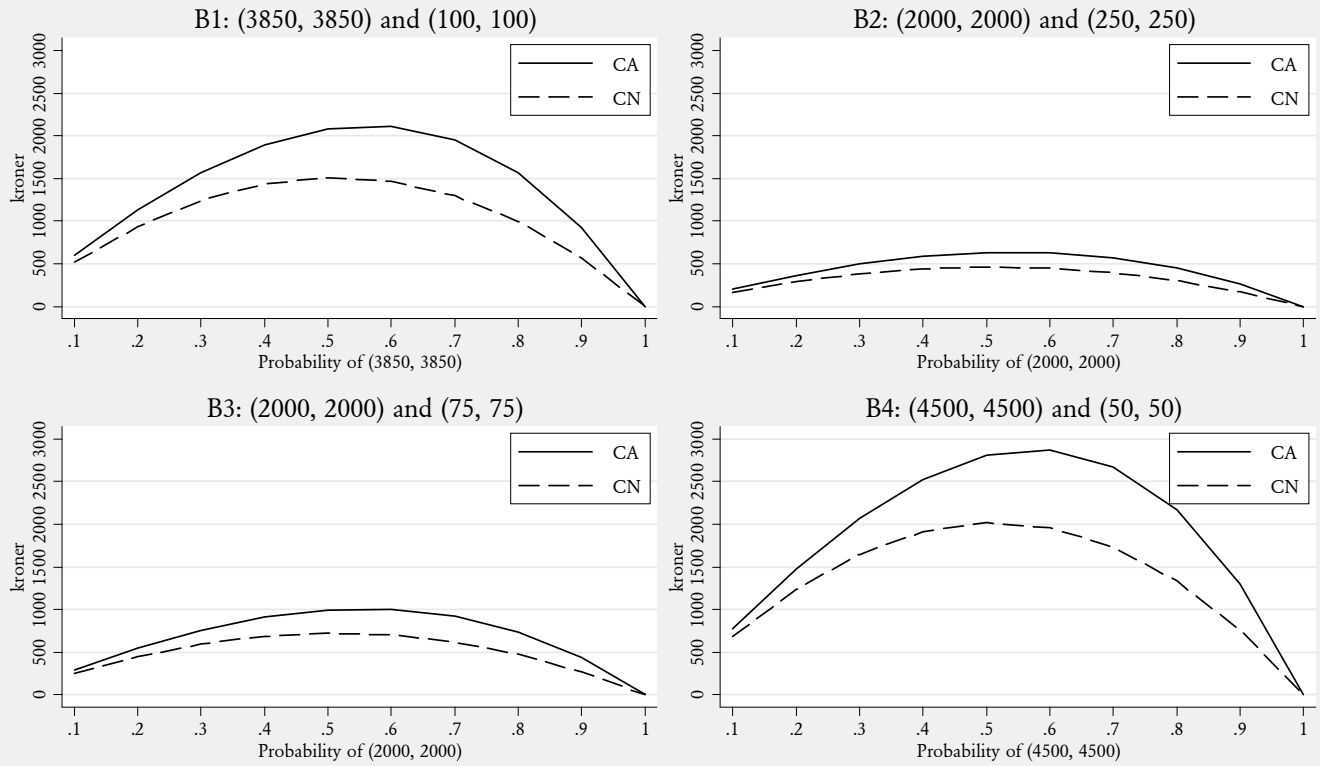


Figure 2: Distributions of the Intertemporal Elasticity of Substitution and the Inverse of Relative Risk Aversion
Predicted values based on ML estimates with observable demographics; N=413

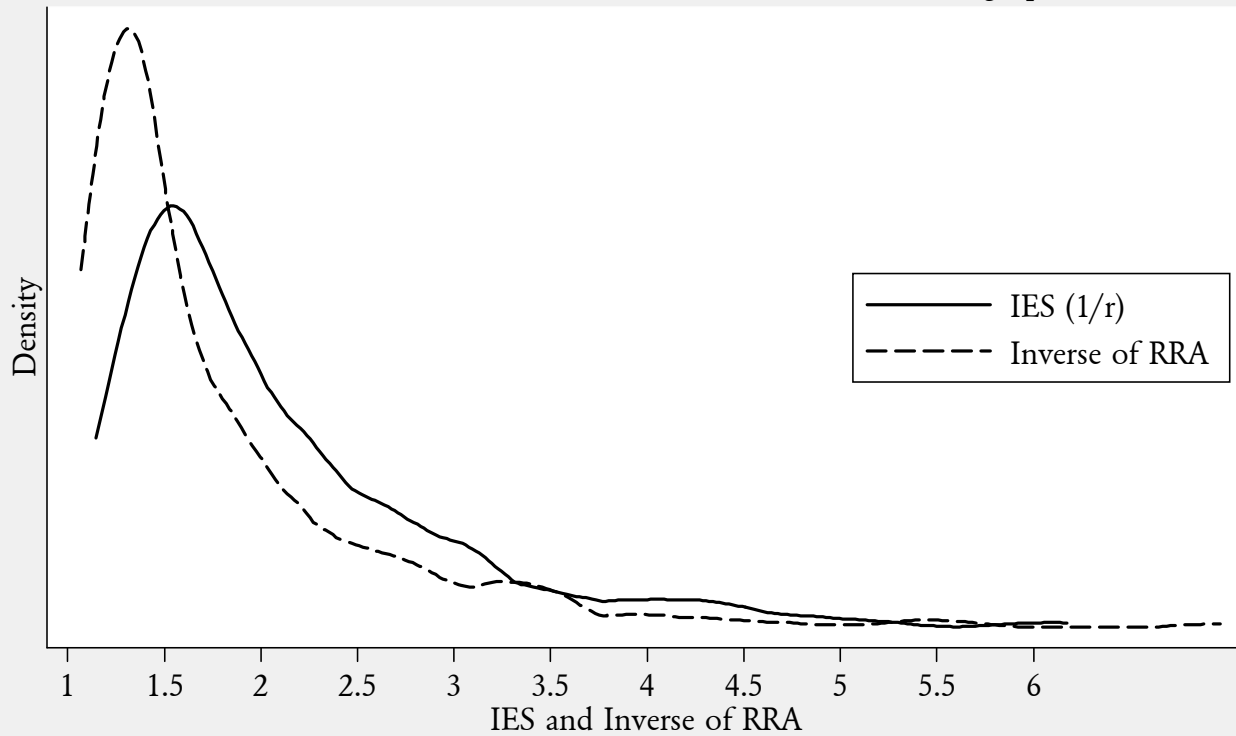


Figure 3: Difference in Distributions of the Intertemporal Elasticity of Substitution and the Inverse of Relative Risk Aversion

Predicted values based on ML estimates with observable demographics; N=413
Mean=0.16, std.dev.=1.20, 95% confidence interval=[-1.32, 1.38]

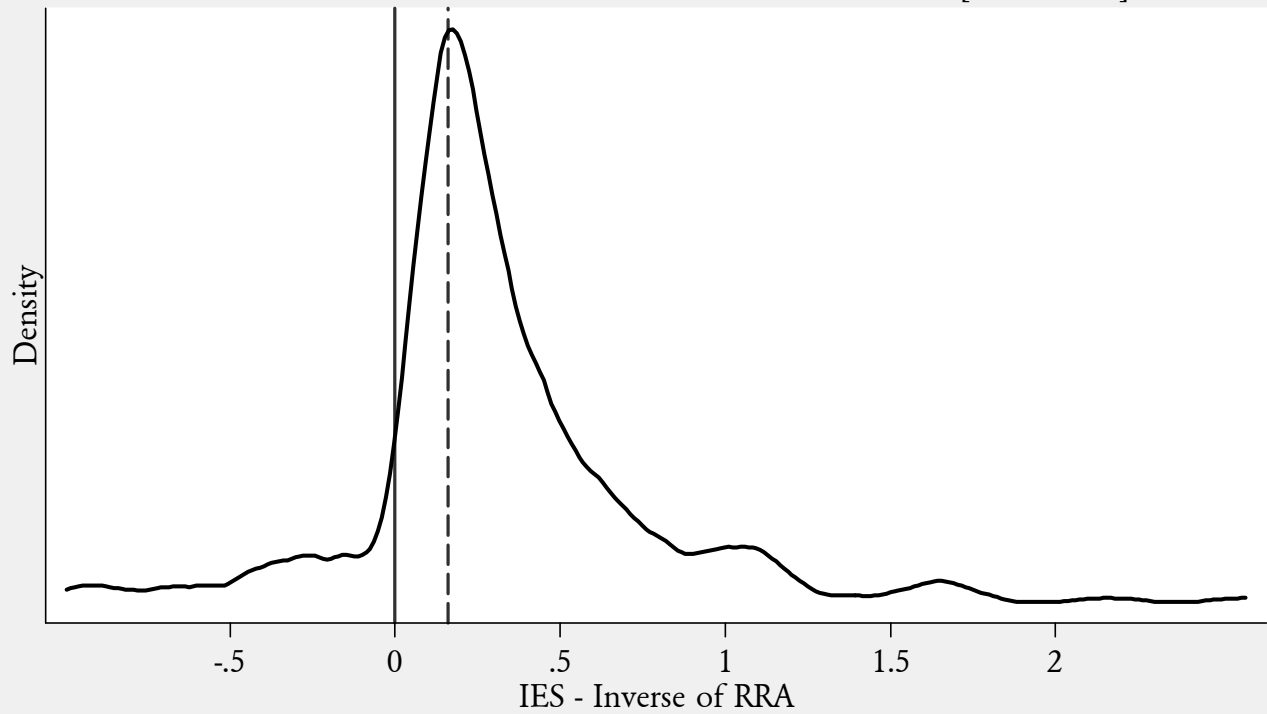
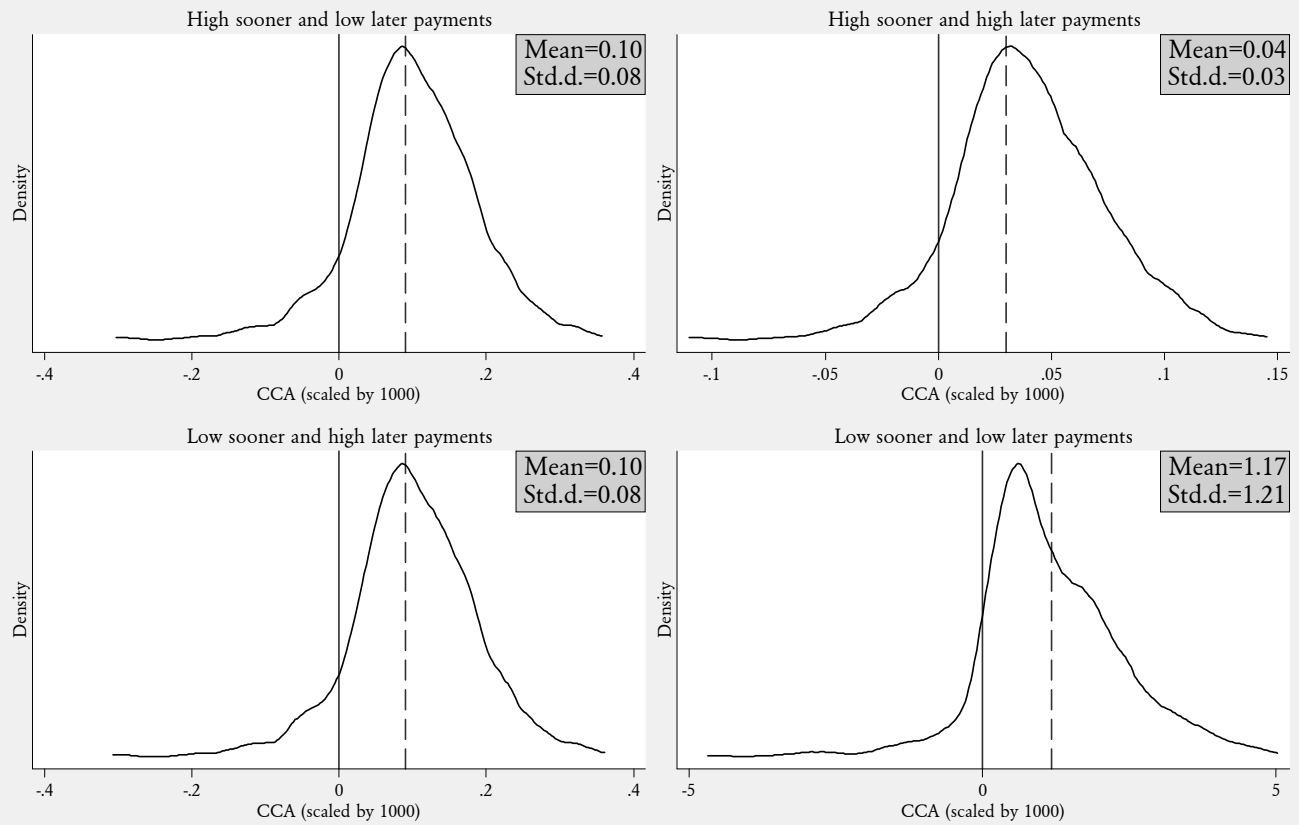


Figure 4: Distribution of the Coefficient of Correlation Aversion
Predicted values based on ML estimates with observable demographics; N=413



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Appendix A: Instructions (NOT FOR PUBLICATION)

We document the instructions by first listing the “manuscript” that shows what was given to subjects and read to them, and then we document some of the screen displays. The original Danish manuscript is available on request. The originals were in 14-point font, printed on A4 paper for nice page breaks (a horizontal line below indicates a page break), and given to subjects in laminated form. Any experimenter that would like to buy a used laminating machine should contact Steffen Andersen. The manuscript below was for the sessions in which the discount rate task was presented first. After these experimental tasks were completed there were additional tasks in the session that are not relevant here.

A. Experimental Manuscript

Welcome announcement

[Give informed consent form to subjects.]

Thank you for agreeing to participate in this survey. The survey is financed by the Social Science Research Council and the Carlsberg Foundation and concerns the economics of decision making.

Before we begin the survey, let me read out the informed consent form that is handed out to you. This form explains your rights as a participant in the survey, what the survey is about and how we make payments to you.

[Read the informed consent form.]

Is everyone able to stay for the full two hours of the meeting? Before we begin, I will ask each of you to pick an envelope from me. The envelope contains a card with an ID number that we will use to keep track of who answered which questions. All records and published results will be linked to anonymous ID numbers only, and not to your name. Please keep your ID numbers private and do not share the information with anyone else.

[Each subject picks an envelope.]

You will be given written instructions during the survey, but make all decisions on the computer in front of you. Please enter your ID number on the computer in front of you, but keep the card for later use.

You will now continue with the first task. The problem is not designed to test you. The only right answer is what you really would choose. That is why the task gives you the chance of winning money. I will now distribute the instructions and then read it out loud.

[Give IDR instructions to subjects.]

[Read the IDR instructions.]

Task D

In this task you will make a number of choices between two options labeled “A” and “B”. An example of your task is shown on the right. You will make all decisions on a computer.

All decisions have the same format. In the example on the right Option A pays 100 kroner today and Option B pays 105 kroner twelve months from now. By choosing option B you would get an annual return of 5% on the 100 kroner.

We will present you with 40 of these decisions. The only difference between them is that the amounts and payment dates in Option A and B will differ.

You will have a 1-in-10 chance of being paid for one of these decisions. The selection is made with a 10-sided die. If the roll of the die gives the number 1 you will be paid for one of the 40 decisions, but if the roll gives any other number you will not be paid. If you are paid for one of these 40 decisions, then we will further select one of these decisions by rolling a 4-sided and a 10-sided die. When you make your choices you will not know which decision is selected for payment. You should therefore treat each decision as if it might actually count for payment.

You will receive the money on the date stated in your preferred option. If you receive some money today, then it is paid out at the end of the experiment. If you receive some money to be paid in the future, then it is transferred to your personal bank account on the specified date. In that case you will receive a written confirmation from Copenhagen Business School which guarantees that the money is reserved on an account at Danske Bank. You can send this document to Danske Bank in a prepaid envelope, and the bank will transfer the money to your account on the specified date.

Before making your choices you will have a chance to practice so that you better understand the consequences of your choices. Please proceed on the computer to the practice task. You will be paid in caramels for this practice task, and they are being paid on the time stated in your preferred option.

[Subjects make decisions in the practice IDR task.]

I will now come around and pay you in caramels for your choice of A or B. Please proceed to the actual task after your earnings are recorded. You will have a 1-in-10 chance of being paid for one of the 40 decisions in the actual task.

Password 1: _____

[Subjects make decisions in the actual IDR task.]

I will now come around and ask you to roll a 10-sided die to determine if you are being paid for one of the decisions. If the roll of the die gives the number 1 you will be paid for one of the 40 decisions, but if the roll gives any other number you will not be paid. If you are paid for one of the 40 decisions, then I will ask you to roll a 4-sided and a 10-sided die to select one of the decisions for payment.

Password 2: _____

[Roll 10-sided die to determine if they are being paid.]

[Roll 4-sided and 10-sided dice to determine the decision for payment.]

You will now continue with the second task. I will distribute the instructions and then read it out loud.

[Give RA instructions to subjects.]

[Read the RA instructions.]

Task L

In this task you will make a number of choices between two options labeled “A” and “B”. An example of your task is shown on the right. You will make all decisions on a computer.

All decisions have the same format. In the example on the right Option A pays 60 kroner if the outcome of a roll of a ten-sided die is 1, and it pays 40 kroner if the outcome is 2-10. Option B pays 90 kroner if the outcome of the roll of the die is 1 and 10 kroner if the outcome is 2-10. All payments in this task are made today at the end of the experiment.

We will present you with 40 such decisions. The only difference between them is that the probabilities and amounts in Option A and B will differ.

You have a 1-in-10 chance of being paid for one of these decisions. The selection is made with a 10-sided die. If the roll of the die gives the number 1 you will be paid for one of the 40 decisions, but if the roll gives any other number you will not be paid. If you are paid for one of these 40 decisions, then we will further select one of these decisions by rolling a 4-sided and a 10-sided die. A third die roll with a 10-sided die determines the payment for your choice of Option A or B. When you make your choices you will not know which decision is selected for payment. You should therefore treat each decision as if it might actually count for payment.

If you are being paid for one of the decisions, we will pay you according to your choice in the selected decision. You will then receive the money at the end of the experiment.

Before making your choices you will have a chance to practice so that you better understand the consequences of your choices. Please proceed on the computer to the practice task. You will be paid in caramels for this practice task.

[Subjects make decisions in the practice RA task.]

I will now come around and pay you in caramels for your choice of A or B. I will ask you to roll a 10-sided die to determine the payment for your choice of A or B. Please proceed to the actual task after your earnings are recorded. You will have a 1-in-10 chance of being paid for one of the 40 decisions in the actual task.

Password 3: _____

[Subjects make decisions in the actual RA task.]

I will now come around and ask you to roll a 10-sided die to determine if you are being paid for one of the decisions. If the roll of the die gives the number 1 you will be paid for one of the 40 decisions, but if the roll gives any other number you will not be paid. If you are paid for one of the 40 decisions, then I will ask you to roll a 4-sided and a 10-sided die to select one of the decisions for payment. A third die roll with a 10-sided die determines the payment for your choice of Option A or B.

Password 4: _____

[Roll 10-sided die to determine if they are being paid.]

[Roll 4-sided and 10-sided dice to determine the decision for payment.]

[Roll 10-sided die to determine payment in Option A and B.]

You will now continue with the third task. I will distribute the instructions and then read it out loud.

[ADDITIONAL INSTRUCTIONS WERE PROVIDED HERE]

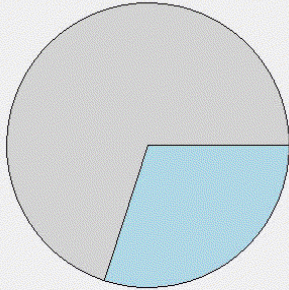
B. Typical Screen Shots for Lottery Choices

The first screen shot on the next page shows the full screen within which the text box is contained, so that one gets an impression of what the subject encountered in all screen shots. Then we display more detailed screen shots of the practice example and the first few lottery choices. Prior to each block of 10 lottery choices the subject was told that the lottery prizes for the next 10 choices would stay the same and the only thing that would vary would be the probabilities. We then show the sequence of the first two lotteries, and then lottery 11 which uses new prizes.

ID: 1234

Practice example

Option A

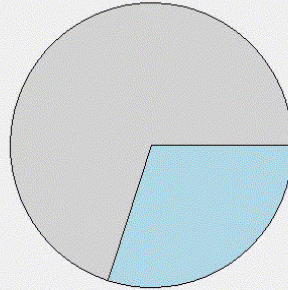


5 caramels if the number on the die is 1 to 3
4 caramels if the number on the die is 4 to 10

Select A

Continue

Option B



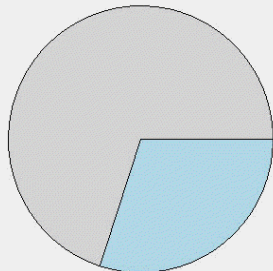
10 caramels if the number on the die is 1 to 3
1 caramel if the number on the die is 4 to 10

Select B

ID: 1234

Practice example

Option A

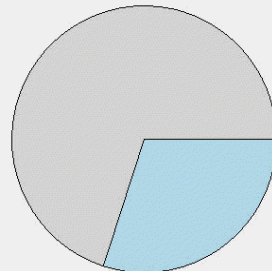


5 caramels if the number on the die is 1 to 3
4 caramels if the number on the die is 4 to 10

Select A

Continue

Option B



10 caramels if the number on the die is 1 to 3
1 caramel if the number on the die is 4 to 10

Select B

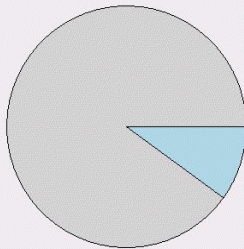
The amounts in the first 10 decisions are constant. The only difference between them is the varying probabilities in Options A and B.

Continue

ID: 1234

Decision number 1 out of 40

Option A

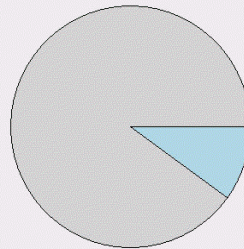


\$2000 if the number on the die is 1
\$1600 if the number on the die is 2 to 10

Select A

Continue

Option B



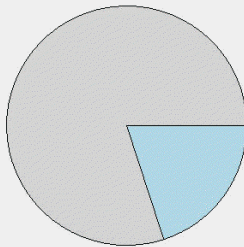
\$3850 if the number on the die is 1
\$100 if the number on the die is 2 to 10

Select B

ID: 1234

Decision number 2 out of 40

Option A

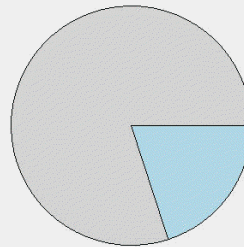


\$2000 if the number on the die is 1 to 2
\$1600 if the number on the die is 3 to 10

Select A

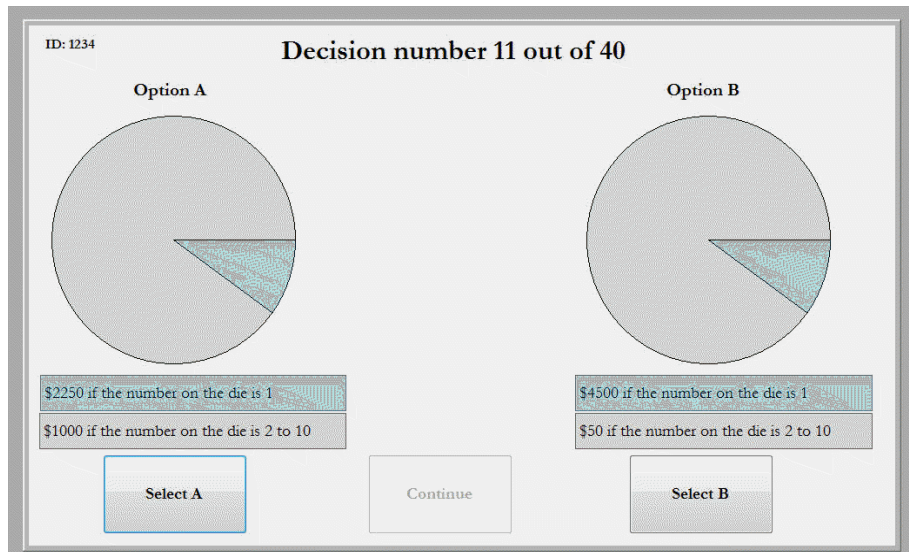
Continue

Option B



\$3850 if the number on the die is 1 to 2
\$100 if the number on the die is 3 to 10

Select B



C. Typical Screen Shots for Discounting Choices

The next page shows the practice example provided at the beginning of these tasks. The top panel shows the initial screen shot, and then the next two panels show how the selected option is highlighted to make it clear to the subject which option is being selected.

The following page shows the information that was given to each subject prior to each block of 10 choices. This information was that the principal and horizon would remain constant for the next 10 choices, but that the only thing that would change would be the amount in the “later” option. In these displays the implied interest rate is displayed.

Finally, after the first 10 choices a new horizon was selected for the next 10 choices.

ID: 1234

Practice example

Option A To be paid now 2 caramels	Option B To be paid at the end of the experiment 8 caramels
---	--

Select A Continue Select B

ID: 1234

Practice example

Option A To be paid now 2 caramels	Option B To be paid at the end of the experiment 8 caramels
---	--

Select A Continue Select B

ID: 1234

Practice example

Option A To be paid now 2 caramels	Option B To be paid at the end of the experiment 8 caramels
---	--

Select A Continue Select B

The dates of payment in the first 10 decisions are constant.
The only difference between them is the varying amounts
in Option B.

Continue

ID: 1234

Decision number 1 out of 40

Option A

To be paid today

\$1500

Option B

To be paid in 10 months

\$1562.24

Annual Interest Rate

5%

Select A

Continue

Select B

ID: 1234

Decision number 2 out of 40

Option A

To be paid today

\$1500

Option B

To be paid in 10 months

\$1624

Annual Interest Rate

10%

Select A

Continue

Select B

ID: 1234 **Decision number 11 out of 40**

Option A	Option B	Annual Interest Rate
To be paid today	To be paid in 7 months	
\$1500	\$1543.3	5%

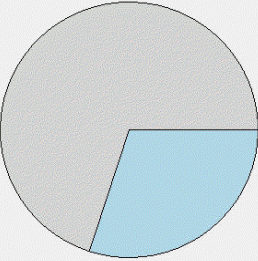
D. Typical Screen Shots for Intertemporal Risk Aversion Choices

The next page shows the practice example provided at the beginning of these tasks. The top panel shows the initial screen shot, and then the next two panels show how the selected option is highlighted to make it clear to the subject which option is being selected.

The following page shows two of the actual tasks for a subject with no front end delay. The lottery prizes were always the same. Option A always had a mixture of the higher and smaller amount, with the first option having the higher amount sooner and the smaller amount later, and the second option having the lower amount sooner and the higher amount later. Option B always had the all-high or all-lower amounts.

ID: 1234 **Practice example**

Option A

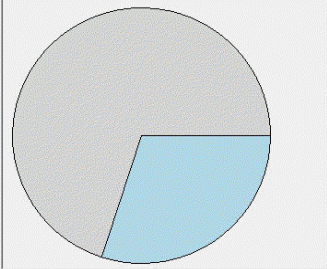


If the number on the die is 1 to 3:
8 caramels to be paid now, and
2 caramels to be paid at the end of the experiment

If the number on the die is 4 to 10:
2 caramels to be paid now, and
8 caramels to be paid at the end of the experiment

Select A

Option B



If the number on the die is 1 to 3:
2 caramels to be paid now, and
2 caramels to be paid at the end of the experiment

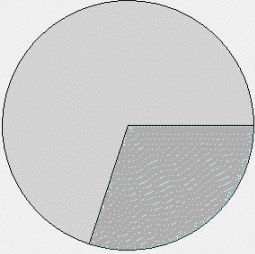
If the number on the die is 4 to 10:
8 caramels to be paid now, and
8 caramels to be paid at the end of the experiment

Select B

Outcome Show Payment Continue

ID: 1234 **Practice example**

Option A

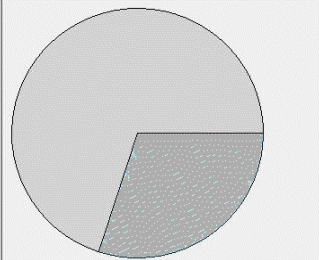


If the number on the die is 1 to 3:
8 caramels to be paid now, and
2 caramels to be paid at the end of the experiment

If the number on the die is 4 to 10:
2 caramels to be paid now, and
8 caramels to be paid at the end of the experiment

Select A

Option B



If the number on the die is 1 to 3:
2 caramels to be paid now, and
2 caramels to be paid at the end of the experiment

If the number on the die is 4 to 10:
8 caramels to be paid now, and
8 caramels to be paid at the end of the experiment

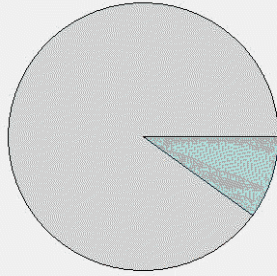
Select B

Outcome Show Payment Continue

ID: 1234

Decision number 1 out of 40

Option A



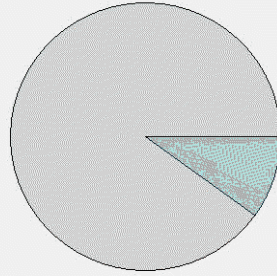
If the number on the die is 1:
\$3850 to be paid today, and
\$100 to be paid in 10 months

If the number on the die is 2 to 10:
\$100 to be paid today, and
\$3850 to be paid in 10 months

Select A

Continue

Option B



If the number on the die is 1:
\$3850 to be paid today, and
\$3850 to be paid in 10 months

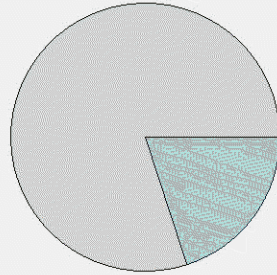
If the number on the die is 2 to 10:
\$100 to be paid today, and
\$100 to be paid in 10 months

Select B

ID: 1234

Decision number 2 out of 40

Option A



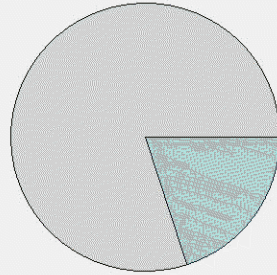
If the number on the die is 1 to 2:
\$3850 to be paid today, and
\$100 to be paid in 10 months

If the number on the die is 3 to 10:
\$100 to be paid today, and
\$3850 to be paid in 10 months

Select A

Continue

Option B



If the number on the die is 1 to 2:
\$3850 to be paid today, and
\$3850 to be paid in 10 months

If the number on the die is 3 to 10:
\$100 to be paid today, and
\$100 to be paid in 10 months

Select B

E. Parameter Values

Table A1 shows the parameters of the lottery choice tasks, Table A2 shows the parameters of the discounting choice tasks, and Table A3 shows the parameters of the intertemporal risk aversion choices.

In Table A1 the parameters are (1) the decision number, (2) the probability of the high prize in each lottery, (3) the high prize of lottery A, in kroner, (4) the low prize of lottery A, in kroner, (5) the high prize of lottery B, in kroner, (6) the low prize of lottery B, in kroner, (7) the expected value of lottery A, and (8) the expected value of lottery B. The information in columns (7) and (8) was not presented to subjects.

Table A1: Parameters for Lottery Choices

Decision	Probability of High Prize	Lottery A High Prize	Lottery A Low Prize	Lottery B High Prize	Lottery B Low Prize	EV of Lottery A	EV of Lottery B
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
1	0.1	1125	750	2000	250	787.5	425
2	0.2	1125	750	2000	250	825	600
3	0.3	1125	750	2000	250	862.5	775
4	0.4	1125	750	2000	250	900	950
5	0.5	1125	750	2000	250	937.5	1125
6	0.6	1125	750	2000	250	975	1300
7	0.7	1125	750	2000	250	1012.5	1475
8	0.8	1125	750	2000	250	1050	1650
9	0.9	1125	750	2000	250	1087.5	1825
10	1	1125	750	2000	250	1125	2000
11	0.1	1000	875	2000	75	887.5	267.5
12	0.2	1000	875	2000	75	900	460
13	0.3	1000	875	2000	75	912.5	652.5
14	0.4	1000	875	2000	75	925	845
15	0.5	1000	875	2000	75	937.5	1037.5
16	0.6	1000	875	2000	75	950	1230
17	0.7	1000	875	2000	75	962.5	1422.5
18	0.8	1000	875	2000	75	975	1615
19	0.9	1000	875	2000	75	987.5	1807.5
20	1	1000	875	2000	75	1000	2000
21	0.1	2000	1600	3850	100	1640	475
22	0.2	2000	1600	3850	100	1680	850
23	0.3	2000	1600	3850	100	1720	1225
24	0.4	2000	1600	3850	100	1760	1600
25	0.5	2000	1600	3850	100	1800	1975
26	0.6	2000	1600	3850	100	1840	2350
27	0.7	2000	1600	3850	100	1880	2725
28	0.8	2000	1600	3850	100	1920	3100
29	0.9	2000	1600	3850	100	1960	3475

30	1	2000	1600	3850	100	2000	3850
31	0.1	2250	1000	4500	50	1125	495
32	0.2	2250	1000	4500	50	1250	940
33	0.3	2250	1000	4500	50	1375	1385
34	0.4	2250	1000	4500	50	1500	1830
35	0.5	2250	1000	4500	50	1625	2275
36	0.6	2250	1000	4500	50	1750	2720
37	0.7	2250	1000	4500	50	1875	3165
38	0.8	2250	1000	4500	50	2000	3610
39	0.9	2250	1000	4500	50	2125	4055
40	1	2250	1000	4500	50	2250	4500

In Table A2 the parameters are (1) the horizon in months, (2) the task number in sequence if this horizon was selected for the subject to make choices over, (3) the principal of 3000 kroner if the subject had the “higher stakes” condition, (4) the principal of 1500 kroner if the subject had the “lower stakes” condition, (5) the annual interest rate presented to the subject if that treatment was applied (this is also the annual effective rate with annual compounding), (6) the delayed payment if the subject had the “higher stakes” condition, (7) the delayed payment if the subject had the “lower stakes” condition, (8) the implied annual effective rate with quarterly compounding, and (9) the implied annual effective rate with daily compounding. The values in columns (8) and (9) were not presented to subjects.

Table A2: Parameters for Discounting Choices

Horizon in months	Task	Principal in high stakes	Principal if low stakes	Annual Interest Rate	Delayed Payment if low stakes	Delayed Payment if high stakes	AER Quarterly	AER Daily
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
0.5	1	3000	1500	5%	3006.10	1503.05	5.1%	5.1%
0.5	2	3000	1500	10%	3011.94	1505.97	10.4%	10.5%
0.5	3	3000	1500	15%	3017.52	1508.76	15.9%	16.2%
0.5	4	3000	1500	20%	3022.88	1511.44	21.6%	22.1%
0.5	5	3000	1500	25%	3028.02	1514.01	27.4%	28.4%
0.5	6	3000	1500	30%	3032.98	1516.49	33.5%	35.0%
0.5	7	3000	1500	35%	3037.75	1518.87	39.9%	41.9%
0.5	8	3000	1500	40%	3042.36	1521.18	46.4%	49.1%
0.5	9	3000	1500	45%	3046.81	1523.40	53.2%	56.8%
0.5	10	3000	1500	50%	3051.11	1525.56	60.2%	64.8%
1	1	3000	1500	5%	3012.22	1506.11	5.1%	5.1%
1	2	3000	1500	10%	3023.92	1511.96	10.4%	10.5%
1	3	3000	1500	15%	3035.14	1517.57	15.9%	16.2%
1	4	3000	1500	20%	3045.93	1522.96	21.6%	22.1%
1	5	3000	1500	25%	3056.31	1528.15	27.4%	28.4%
1	6	3000	1500	30%	3066.31	1533.16	33.5%	35.0%
1	7	3000	1500	35%	3075.97	1537.99	39.9%	41.9%

1	8	3000	1500	40%	3085.31	1542.65	46.4%	49.1%
1	9	3000	1500	45%	3094.34	1547.17	53.2%	56.8%
1	10	3000	1500	50%	3103.10	1551.55	60.2%	64.8%
2	1	3000	1500	5%	3024.49	1512.25	5.1%	5.1%
2	2	3000	1500	10%	3048.04	1524.02	10.4%	10.5%
2	3	3000	1500	15%	3070.70	1535.35	15.9%	16.2%
2	4	3000	1500	20%	3092.56	1546.28	21.6%	22.1%
2	5	3000	1500	25%	3113.67	1556.84	27.4%	28.4%
2	6	3000	1500	30%	3134.09	1567.05	33.5%	35.0%
2	7	3000	1500	35%	3153.87	1576.93	39.9%	41.9%
2	8	3000	1500	40%	3173.04	1586.52	46.4%	49.1%
2	9	3000	1500	45%	3191.65	1595.83	53.2%	56.8%
2	10	3000	1500	50%	3209.74	1604.87	60.2%	64.8%
3	1	3000	1500	5%	3036.82	1518.41	5.1%	5.1%
3	2	3000	1500	10%	3072.34	1536.17	10.4%	10.5%
3	3	3000	1500	15%	3106.67	1553.34	15.9%	16.2%
3	4	3000	1500	20%	3139.91	1569.95	21.6%	22.1%
3	5	3000	1500	25%	3172.11	1586.06	27.4%	28.4%
3	6	3000	1500	30%	3203.37	1601.68	33.5%	35.0%
3	7	3000	1500	35%	3233.74	1616.87	39.9%	41.9%
3	8	3000	1500	40%	3263.27	1631.64	46.4%	49.1%
3	9	3000	1500	45%	3292.03	1646.01	53.2%	56.8%
3	10	3000	1500	50%	3320.05	1660.02	60.2%	64.8%
4	1	3000	1500	5%	3049.19	1524.59	5.1%	5.1%
4	2	3000	1500	10%	3096.84	1548.42	10.4%	10.5%
4	3	3000	1500	15%	3143.07	1571.53	15.9%	16.2%
4	4	3000	1500	20%	3187.98	1593.99	21.6%	22.1%
4	5	3000	1500	25%	3231.65	1615.83	27.4%	28.4%
4	6	3000	1500	30%	3274.18	1637.09	33.5%	35.0%
4	7	3000	1500	35%	3315.63	1657.81	39.9%	41.9%
4	8	3000	1500	40%	3356.07	1678.03	46.4%	49.1%
4	9	3000	1500	45%	3395.55	1697.78	53.2%	56.8%
4	10	3000	1500	50%	3434.14	1717.07	60.2%	64.8%
5	1	3000	1500	5%	3061.61	1530.81	5.1%	5.1%
5	2	3000	1500	10%	3121.53	1560.77	10.4%	10.5%
5	3	3000	1500	15%	3179.89	1589.94	15.9%	16.2%
5	4	3000	1500	20%	3236.78	1618.39	21.6%	22.1%
5	5	3000	1500	25%	3292.31	1646.15	27.4%	28.4%
5	6	3000	1500	30%	3346.55	1673.28	33.5%	35.0%
5	7	3000	1500	35%	3399.59	1699.80	39.9%	41.9%
5	8	3000	1500	40%	3451.50	1725.75	46.4%	49.1%
5	9	3000	1500	45%	3502.34	1751.17	53.2%	56.8%
5	10	3000	1500	50%	3552.16	1776.08	60.2%	64.8%
6	1	3000	1500	5%	3074.09	1537.04	5.1%	5.1%
6	2	3000	1500	10%	3146.43	1573.21	10.4%	10.5%
6	3	3000	1500	15%	3217.14	1608.57	15.9%	16.2%

6	4	3000	1500	20%	3286.34	1643.17	21.6%	22.1%
6	5	3000	1500	25%	3354.10	1677.05	27.4%	28.4%
6	6	3000	1500	30%	3420.53	1710.26	33.5%	35.0%
6	7	3000	1500	35%	3485.69	1742.84	39.9%	41.9%
6	8	3000	1500	40%	3549.65	1774.82	46.4%	49.1%
6	9	3000	1500	45%	3612.48	1806.24	53.2%	56.8%
6	10	3000	1500	50%	3674.23	1837.12	60.2%	64.8%
7	1	3000	1500	5%	3086.61	1543.30	5.1%	5.1%
7	2	3000	1500	10%	3171.52	1585.76	10.4%	10.5%
7	3	3000	1500	15%	3254.83	1627.42	15.9%	16.2%
7	4	3000	1500	20%	3336.65	1668.32	21.6%	22.1%
7	5	3000	1500	25%	3417.06	1708.53	27.4%	28.4%
7	6	3000	1500	30%	3496.14	1748.07	33.5%	35.0%
7	7	3000	1500	35%	3573.96	1786.98	39.9%	41.9%
7	8	3000	1500	40%	3650.59	1825.29	46.4%	49.1%
7	9	3000	1500	45%	3726.08	1863.04	53.2%	56.8%
7	10	3000	1500	50%	3800.50	1900.25	60.2%	64.8%
8	1	3000	1500	5%	3099.18	1549.59	5.1%	5.1%
8	2	3000	1500	10%	3196.81	1598.40	10.4%	10.5%
8	3	3000	1500	15%	3292.96	1646.48	15.9%	16.2%
8	4	3000	1500	20%	3387.73	1693.86	21.6%	22.1%
8	5	3000	1500	25%	3481.19	1740.60	27.4%	28.4%
8	6	3000	1500	30%	3573.42	1786.71	33.5%	35.0%
8	7	3000	1500	35%	3664.46	1832.23	39.9%	41.9%
8	8	3000	1500	40%	3754.39	1877.20	46.4%	49.1%
8	9	3000	1500	45%	3843.26	1921.63	53.2%	56.8%
8	10	3000	1500	50%	3931.11	1965.56	60.2%	64.8%
9	1	3000	1500	5%	3111.81	1555.91	5.1%	5.1%
9	2	3000	1500	10%	3222.30	1611.15	10.4%	10.5%
9	3	3000	1500	15%	3331.54	1665.77	15.9%	16.2%
9	4	3000	1500	20%	3439.59	1719.80	21.6%	22.1%
9	5	3000	1500	25%	3546.53	1773.27	27.4%	28.4%
9	6	3000	1500	30%	3652.40	1826.20	33.5%	35.0%
9	7	3000	1500	35%	3757.26	1878.63	39.9%	41.9%
9	8	3000	1500	40%	3861.16	1930.58	46.4%	49.1%
9	9	3000	1500	45%	3964.12	1982.06	53.2%	56.8%
9	10	3000	1500	50%	4066.21	2033.10	60.2%	64.8%
11	1	3000	1500	5%	3137.22	1568.61	5.1%	5.1%
11	2	3000	1500	10%	3273.89	1636.95	10.4%	10.5%
11	3	3000	1500	15%	3410.05	1705.03	15.9%	16.2%
11	4	3000	1500	20%	3545.72	1772.86	21.6%	22.1%
11	5	3000	1500	25%	3680.91	1840.46	27.4%	28.4%
11	6	3000	1500	30%	3815.66	1907.83	33.5%	35.0%
11	7	3000	1500	35%	3949.97	1974.99	39.9%	41.9%
11	8	3000	1500	40%	4083.87	2041.94	46.4%	49.1%
11	9	3000	1500	45%	4217.37	2108.69	53.2%	56.8%

11	10	3000	1500	50%	4350.49	2175.25	60.2%	64.8%
12	1	3000	1500	5%	3150	1575	5.1%	5.1%
12	2	3000	1500	10%	3300	1650	10.4%	10.5%
12	3	3000	1500	15%	3450	1725	15.9%	16.2%
12	4	3000	1500	20%	3600	1800	21.6%	22.1%
12	5	3000	1500	25%	3750	1875	27.4%	28.4%
12	6	3000	1500	30%	3900	1950	33.5%	35.0%
12	7	3000	1500	35%	4050	2025	39.9%	41.9%
12	8	3000	1500	40%	4200	2100	46.4%	49.1%
12	9	3000	1500	45%	4350	2175	53.2%	56.8%
12	10	3000	1500	50%	4500	2250	60.2%	64.8%

In Table A3 we present the parameters for one subject. Recall that we define the lottery α as a 50:50 mixture of $\{x, Y\}$ and $\{X, y\}$, and the lottery β as a 50:50 mixture of $\{x, y\}$ and $\{X, Y\}$. So α is a 50:50 mixture of bad and good outcomes in time t and $t+\tau$, and good and bad outcomes in the two time periods; and β is a 50:50 mixture of all-bad outcomes and all-good outcomes in the two time periods. In the screen image shown above lottery α is Option A, and lottery β is Option B. These parameters in Table A3 are (1) the decision number, (2) the probability for lottery α , (3) the low amount in kroner, (4) the high amount in kroner, (5) the front end delay in months for the sooner option, and (6) the horizon in months for the later option. The sooner option was for delivery in either 1 month, as shown here for this subject, or in the present. The later option was for delivery in the number of months shown in (6) *after* the front end delay. So for this subject decision #1 would have a later delivery time 9 months beyond the present. If this subject had *not* had a front end delay for the sooner option, the later option for decision #1 would have been 8 months from the present.

Table A3: Parameters for Intertemporal Lottery Choices

Decision	Probability for Lottery α	Low Prize (kroner)	High Prize (kroner)	Front End Delay (months)	Horizon (months)
(1)	(2)	(3)	(4)	(5)	(6)
1	0.1	50	4500	1	8
2	0.2	50	4500	1	8
3	0.3	50	4500	1	8
4	0.4	50	4500	1	8
5	0.5	50	4500	1	8
6	0.6	50	4500	1	8
7	0.7	50	4500	1	8
8	0.8	50	4500	1	8
9	0.9	50	4500	1	8
10	1	50	4500	1	8
11	0.1	50	4500	1	7
12	0.2	50	4500	1	7

13	0.3	50	4500	1	7
14	0.4	50	4500	1	7
15	0.5	50	4500	1	7
16	0.6	50	4500	1	7
17	0.7	50	4500	1	7
18	0.8	50	4500	1	7
19	0.9	50	4500	1	7
20	1	50	4500	1	7
21	0.1	50	4500	1	4
22	0.2	50	4500	1	4
23	0.3	50	4500	1	4
24	0.4	50	4500	1	4
25	0.5	50	4500	1	4
26	0.6	50	4500	1	4
27	0.7	50	4500	1	4
28	0.8	50	4500	1	4
29	0.9	50	4500	1	4
30	1	50	4500	1	4
31	0.1	50	4500	1	1
32	0.2	50	4500	1	1
33	0.3	50	4500	1	1
34	0.4	50	4500	1	1
35	0.5	50	4500	1	1
36	0.6	50	4500	1	1
37	0.7	50	4500	1	1
38	0.8	50	4500	1	1
39	0.9	50	4500	1	1
40	1	50	4500	1	1

Appendix B: Estimates with Covariates (NOT FOR PUBLICATION)

Table B1 extends the maximum likelihood estimates by including covariates for each of the core structural parameters to reflect observable heterogeneity in responses. We include covariates for individual demographic characteristics as well as task characteristics. Unless otherwise noted, all variables are binary.

Variable FEMALE indicates a female; YOUNG is someone aged less than 30; MIDDLE is someone aged between 40 and 50; OLD is someone aged over 50 (so the omitted age category are those aged between 30 and 39); SINGLE is someone who lives without a spouse or partner; KIDS is someone who has children; OWNER is someone who owns their apartment or house; RETIRED is someone who is retired; STUDENT is someone who is a student; SKILLED is someone with some post-secondary education³⁰; LONGEDU is someone who has substantial higher education³¹; INCLOW is someone with household income in 2009 below 300,000 kroner; and INCHIGH is someone with household income in 2009 of 500,000 kroner or more.

Turning to the task treatments, variable RA_FIRST indicates if the risk aversion task was presented before the discounting task; and FEE_HIGH indicates if the higher show-up fee of 500 kroner was used to recruit the subject (rather than 300 kroner); RAHIGH indicates if the two highest prize sets in the atemporal risk aversion tasks were used; FED indicates if a 30-day front end delay was employed for the “sooner” option; IDRORDER indicates if the subject was presented the horizons in increasing order (rather than decreasing order); IDRHIGH indicates if the higher principal of 3000 kroner was used (rather than 1500 kroner); INFO indicates if information on implied interest rates was provided, and IRAHIGH indicates if the two highest prize sets in the inter-temporal risk aversion tasks were used.

The results in Table B1 display considerable homogeneity in the elicited parameters across the subjects in the sample. Implied values of RRA and IES are also reported in Table B2. We find that only one of the demographic characteristics is significantly correlated with variations in the core parameters across subjects. Women appear to have a higher estimated atemporal risk attitudes (τ) than men, with an estimated marginal effect of 0.22 that is statistically significant with a p -value of 0.025. There is no significant effect from sex on the elicited intertemporal risk attitudes (η) or on individual discount rates (δ).

The results also show that there are no significant effects of task characteristics on the estimated core parameters in the model. The absence of treatment effects on the curvature of the atemporal utility function and individual discount rates are noteworthy, since several of our treatments have been found to have significant effects on behavior in studies that used related experimental and survey methods. In particular, we do not find a significant effect

³⁰ Specifically, if the individual has completed vocational education and training or “short-cycle” higher education. Danes commonly refer to the cycle of education in this manner: most short-cycle higher education programs last for less than 2 years; medium-cycle higher education lasts for 3 to 4 years, and includes training for occupations such as a journalist, primary or lower secondary school teacher, nursery and kindergarten teacher, and ordinary nurse; long-cycle higher education typically lasts 5 years and is offered at Denmark’s five ordinary universities, at the business schools and various other advanced institutions.

³¹ Specifically, the completion of medium-cycle or longer-cycle higher education.

from varying the stakes in the atemporal risk aversion task or in the discount rate task, and individual discount rates do not vary significantly with the delay to the sooner payment option or information on implied interest rates by choosing the later payment option.

We can also evaluate the total effects of several of the demographic characteristics on the estimated RRA and IES, by estimating marginal effects without controls for other characteristics. We calculate total effects since many demographic characteristics co-vary in the population and therefore also in our sample. For example, the men in our sample have a number of characteristics that differ from the women apart from sex. By not controlling for these other characteristics of men, we can estimate the difference in RRA and IES between men and women that jointly reflects all of these differences. To consider the total effects, we simply repeat the statistical analysis shown in Table B1 but with only one demographic characteristic included at a time. In this manner our estimates include all of the demographic characteristics correlated with the characteristic of interest. The maximum likelihood estimates of RRA and IES for a selection of demographic characteristics are displayed in Table B3.

We find that women are more risk averse than men with an estimated RRA of 0.75 for women and 0.50 for men. This difference in RRA between men and women is statistically significant with a p -value of 0.005. The results also show that men have a higher IES than women. The estimated coefficient for men is 2.56 and is 1.71 for women, and the difference of 0.85 is statistically significant with a p -value of 0.006. We do not find any significant variation in the estimated RRA and IES coefficients for the other individual characteristics that are included in Table B3. Hence we can not reject the hypothesis that the coefficients of RRA and IES are similar across age groups. Subjects between 30 and 40 years of age appear to be more risk averse than other age groups, but the estimated RRA coefficients for the four age groups are jointly insignificant with a p -value of 0.94. The results also point to a lower IES coefficient for subjects older than 50 years of age compared to younger age groups, however the variation in estimated IES coefficients is statistically insignificant (p -value of 0.51). Finally, the results show no significant variation in estimated RRA and IES coefficients across educational levels and income groups.

Table B1: Maximum Likelihood Estimates with Covariates

N=49,560 observations, based on 413 subjects

Parameter	Point Estimate	Standard Error	p-value	95% Confidence Interval	
<i>Atemporal Utility Function (r)</i>					
Constant	0.43	0.243	0.074	-0.04	0.91
RAfirst	-0.01	0.102	0.895	-0.21	0.19
FEEhigh	0.09	0.093	0.325	-0.09	0.27
RAhigh	0.06	0.073	0.444	-0.09	0.20
Female	0.22	0.010	0.025	0.03	0.42
Young	-0.09	0.213	0.678	-0.51	0.33
Middle	-0.03	0.113	0.764	-0.26	0.19
Old	0.04	0.156	0.818	-0.27	0.34
Single	-0.04	0.099	0.669	-0.24	0.15
Kids	-0.10	0.135	0.455	-0.36	0.16
Owner	0.08	0.089	0.387	-0.10	0.25
Retired	-0.05	0.114	0.629	-0.28	0.17
Student	-0.25	0.133	0.063	-0.51	0.01
Skilled	-0.10	0.193	0.603	-0.48	0.28
Longedu	-0.01	0.194	0.941	-0.39	0.37
IncLow	0.13	0.137	0.346	-0.14	0.40
IncHigh	-0.05	0.081	0.518	-0.21	0.11
μ	0.18	0.013	<0.001	0.15	0.2
<i>Discounting Function (δ)</i>					
Constant	0.113	0.080	0.155	-0.04	0.27
RAfirst	0.015	0.035	0.674	-0.05	0.08
FEEhigh	-0.026	0.028	0.361	-0.08	0.03
FED	0.018	0.022	0.416	-0.03	0.06
IDRorder	-0.044	0.027	0.193	-0.09	0.02
IDRhigh	0.012	0.022	0.590	-0.03	0.06
INFO	-0.035	0.026	0.168	-0.09	0.01
Female	-0.051	0.038	0.185	-0.13	0.02
Young	0.032	0.060	0.597	-0.09	0.15
Middle	-0.001	0.039	0.973	-0.08	0.08
Old	-0.016	0.051	0.751	-0.12	0.08
Single	0.027	0.055	0.622	-0.08	0.14
Kids	0.052	0.048	0.270	-0.04	0.15
Owner	-0.037	0.032	0.254	-0.10	0.03
Retired	0.021	0.059	0.716	-0.09	0.14
Student	0.112	0.091	0.220	-0.07	0.29
Skilled	0.032	0.063	0.608	-0.09	0.16
Longedu	0.003	0.060	0.963	-0.11	0.12
IncLow	-0.025	0.053	0.632	-0.13	0.08
IncHigh	0.016	0.034	0.651	-0.05	0.08
μ'	0.12	0.011	<0.001	0.1	0.14

Intertemporal Utility Function (η)

Constant	0.81	0.563	0.150	-0.29	1.92
RAfirst	-0.03	0.164	0.856	-0.35	0.29
FEEhigh	0.05	0.123	0.708	-0.19	0.29
FED	-0.01	0.106	0.929	-0.22	0.20
IDRorder	0.03	0.083	0.682	-0.13	0.20
IRAhigh	-0.10	0.267	0.707	-0.62	0.42
Female	0.15	0.431	0.721	-0.69	1.00
Young	-0.32	0.407	0.431	-1.12	0.48
Middle	-0.18	0.569	0.751	-1.29	0.93
Old	-0.49	0.380	0.201	-1.23	0.26
Single	-0.28	0.800	0.730	-1.84	1.29
Kids	-0.23	0.326	0.490	-0.86	0.41
Owner	0.02	0.138	0.856	-0.24	0.29
Retired	-0.23	0.629	0.709	-1.47	1.00
Student	-0.18	0.606	0.766	-1.37	1.01
Skilled	-0.19	0.463	0.688	-1.09	0.72
Longedu	-0.07	0.283	0.794	-0.63	0.48
IncLow	0.15	0.462	0.742	-0.75	1.06
IncHigh	0.09	0.205	0.665	-0.31	0.49
μ''	0.18	0.01	<0.001	0.16	0.2

Table B2: Implied Maximum Likelihood Estimates

N=49,560 observations, based on 413 subjects

Parameter	Point Estimate	Standard Error	<i>p</i> -value	95% Confidence Interval	
<i>Relative Risk Aversion</i>					
Constant	0.66	0.290	0.022	0.09	1.23
RAfirst	-0.02	0.102	0.870	-0.22	0.18
FEEhigh	0.07	0.072	0.364	-0.08	0.21
RAhigh	0.03	0.036	0.365	-0.04	0.10
FED	0.00	0.030	0.929	-0.06	0.06
IDRorder	0.01	0.025	0.700	-0.04	0.06
IRAhhigh	-0.03	0.086	0.742	-0.20	0.14
Female	0.16	0.174	0.360	-0.18	0.50
Young	-0.16	0.227	0.488	-0.60	0.29
Middle	-0.07	0.234	0.751	-0.53	0.38
Old	-0.11	0.135	0.424	-0.37	0.16
Single	-0.11	0.333	0.743	-0.76	0.54
Kids	-0.13	0.154	0.383	-0.44	0.17
Owner	0.05	0.077	0.500	-0.10	0.20
Retired	-0.11	0.267	0.692	-0.63	0.42
Student	-0.22	0.284	0.439	-0.78	0.34
Skilled	-0.12	0.239	0.612	-0.59	0.35
Longedu	-0.03	0.189	0.874	-0.40	0.34
IncLow	0.11	0.208	0.597	-0.30	0.52
IncHigh	0.00	0.071	0.959	-0.14	0.14
<i>Intertemporal Elasticity of Substitution</i>					
Constant	2.30	1.289	0.074	-0.22	4.83
RAfirst	0.07	0.546	0.892	-1.00	1.14
FEEhigh	-0.40	0.479	0.405	-1.34	0.54
RAhigh	-0.26	0.362	0.470	-0.97	0.45
Female	-0.78	0.863	0.365	-2.47	0.91
Young	0.59	1.212	0.627	-1.79	2.96
Middle	0.20	0.745	0.792	-1.26	1.66
Old	-0.18	0.859	0.838	-1.86	1.51
Single	0.25	0.824	0.763	-1.37	1.86
Kids	0.69	0.838	0.408	-0.95	2.34
Owner	-0.35	0.456	0.445	-1.24	0.55
Retired	0.33	0.858	0.697	-1.35	2.01
Student	3.04	4.053	0.453	-4.90	10.99
Skilled	0.69	1.190	0.561	-1.64	3.02
Longedu	0.08	1.037	0.940	-1.95	2.11
IncLow	-0.53	0.869	0.543	-2.23	1.17
IncHigh	0.31	0.707	0.656	-1.07	1.70

Table B3: Maximum Likelihood Estimates of Total Effects

N=49,560 observations, based on 413 subjects

Variable	Estimate	Standard Error	<i>p</i> -value	95% Confidence Interval	
<i>Relative Risk Aversion</i>					
Female	0.75	0.069	<0.001	0.61	0.89
Male	0.50	0.100	<0.001	0.29	0.70
Young	0.58	0.098	<0.001	0.39	0.78
Adult	0.66	0.114	<0.001	0.44	0.88
Middle	0.57	0.164	0.001	0.25	0.89
Old	0.58	0.106	<0.001	0.37	0.79
Unskilled	0.60	0.131	<0.001	0.34	0.85
Skilled	0.57	0.120	<0.001	0.34	0.81
Longedu	0.63	0.082	<0.001	0.47	0.79
IncLow	0.66	0.097	<0.001	0.47	0.85
IncMiddle	0.58	0.115	<0.001	0.35	0.81
IncHigh	0.56	0.138	<0.001	0.29	0.83
<i>Intertemporal Elasticity of Substitution</i>					
Female	1.71	0.188	<0.001	1.34	2.08
Male	2.56	0.360	<0.001	1.85	3.26
Young	2.47	0.409	<0.001	1.67	3.27
Adult	2.20	0.411	<0.001	1.40	3.01
Middle	2.39	0.602	<0.001	1.21	3.57
Old	1.97	0.260	<0.001	1.46	2.48
Unskilled	2.17	0.415	<0.001	1.36	2.99
Skilled	2.26	0.395	<0.001	1.49	3.04
Longedu	2.01	0.247	<0.001	1.52	2.49
IncLow	1.93	0.283	<0.001	1.37	2.48
IncMiddle	2.01	0.301	<0.001	1.42	2.61
IncHigh	2.37	0.446	<0.001	1.50	3.25