Bayesian model comparison of quantum versus traditional models of decision making for explaining violations of the dynamic consistency principle

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Abstract

Recently, quantum decision theory has achieved considerable success as a new theory for providing a coherent account of a variety of different empirical findings that appear paradoxical for traditional decision theory. But critics argue that this success may simply mean that quantum theories can "fit" better because they are more complex. To examine this issue, we compared quantum models with traditional models using a Bayes factor, which provides one of the most rigorous methods for evaluating models with respect to accuracy and parsimony. For this comparison, we used a large data set with a large number of conditions and subjects that examined a puzzling phenomenon called dynamic inconsistency – the failure of decision makers to carry out their planned decisions. The results of this model comparison supports the quantum model as compared to the traditional model of decision making.

Keywords: Quantum probability, prospect theory, dynamic consistency, interference effects, Bayes factor.

Decision theorists are becoming increasingly concerned with the stochastic nature of choice for decisions under risk and uncertainty (Busemeyer & Townsend, 1993; Harless & Camerer, 1994; Hey & Orme, 1994; Loomes & Sugden, 1995; Regenwetter, Dana, & Davis-Stober, 2010; Wilcox, 2011). What is the fundamental source of this stochasticity? The most common explanation, described early on by Becker, DeGroot, and Marschak (1963), is that either the utilities or the subjective weights assigned to outcomes randomly vary across choice occasions. These stochastic mechanisms are "classic" in nature – they assume that at any moment of choice, specific utilities and specific weights are realized, and the choice is determined by the realizations experienced at that moment. An alternative idea, described early by Luce (1959), is that choice is intrinsically probabilistic. The latter idea has not been pursued very intensively until recently, where several decision theorists have begun exploring "non classical" quantum probabilistic models of choice (Busemeyer, Wang, Lambert-Mogiliansky, 2009; Lambert-Mogiliansky, Zamir, Zwirn, 2009; Khrennikov and Haven, 2009; Yukalov & Sornette, 2010). .

Quantum choice models were introduced to account for decision making paradoxes that have resisted explanations by "classical" type of stochastic models (Pothos & Busemeyer, 2009). Perhaps quantum models succeed where classic models fail simply because quantum models are more complex and have greater model fitting flexibility (after all they are based on complex numbers). The purpose of this paper is to examine this issue by comparing a classic type of random utility model with a quantum model using Bayesian model comparison methods (Berger, 1985). The model comparison is based on a large experiment designed to examine dynamic inconsistency in choice among two stage gambles (Barkan & Busemeyer, 2003). Dynamic consistency is a principle of decision making required for backward induction when applied to decision trees. Dynamic consistency requires that a planned course of action for a future decision is implemented as planned when that decision is finally realized. Barkan and Busemeyer (2003) observed systematic violations of dynamic consistency, and they used a random utility version of prospect theory to account for these findings. But more recently, Yukalov and Sornette (2010) argued that a quantum theory can also account for these findings (Yulalov & Sornette, 2009). Therefore, in this paper, two different types of models are proposed to explain these findings: a random utility model based on prospect theory (Kahneman & Tversky, 1979) and a quantum decision model (Pothos and Busemeyer, 2009).

The paper is organized as follows. First we review the Barkan and Busemeyer (2003) experimental methods and results. Second, we describe the two models that are being compared. Third, we present fits to the mean data for each model to get a rough idea about how well each model accounts for the findings (but this is not our main concern). Fourth, we present the results of the Bayesian model comparison (which is our main concern). Finally, we draw some preliminary conclusions from this model comparison analysis.

1 Barkan and Busemeyer (2003)

A two stage gambling paradigm was used to study dynamic consistency, which was a modification of the paradigm used by Tversky and Shafir (1992) to study the disjunction effect. A total of 100 people participated and each person played the 17 gambles involving real money shown in Table 1 twice. Each gamble had an equal chance of producing a win or a loss. The columns labeled 'win' and 'loss' indicate the money that could be won or lost for each gamble (one unit was worth one cent) and the column labeled EV shows the expected value of each gamble. For each gamble in Table 1, the person was forced to play the first round, and then contingent on the outcome of the first round, they were given a choice whether or not to play the second round with the same gamble. On each trial the person was first asked to make a plan for the second play contingent on each possible outcome of the first play. In other words, during the planning stage they were asked two questions: "if you win the first play, do you plan to play the second gamble? and "if you lose the first play, do you plan to play the second gamble?" Following the plan, the outcome of the first gamble was revealed, and then the person was given a final choice: decide again whether or not to play the second gamble after observing the first play outcome. To incentivize both plan and final choices, the computer randomly selected either the planned choice or the final choice to determine the real monetary payoff for each trial. The final payment for the trial was then shown to the person at the end of each trial. Participants were paid by randomly selecting four problems from the entire set, randomly selecting either their plan or final choice, and randomly selecting an outcome for each gamble to determine the actual payment.

Table 1 displays the results obtained after averaging across the two replications for each person, and after averaging across all 100 participants. The probability of planning to take the gamble is shown under the column labeled "Plan." There was little or no difference between the probabilities of taking the gamble, contingent each planned outcome of the first gamble, and so the results shown here are averaged across the two hypothetical outcomes during the plan. See Barkan and Busemeyer (2003) for the complete results listed separately for each contingent outcome. The probability of taking the gamble during the final stage is shown under the column labeled 'Final.' The columns under the label "Gamble" display the amount to win and lose for each gamble. Changes in probabilities down the rows of the Table show the effect of the gamble payoffs on the probability of taking the gamble. The difference between the planned and final columns indicates a dynamic inconsistency effect. Notice that following a win (the first 4 columns), the probability of taking the gamble at the final stage was always smaller than the probability of taking the gamble at the planning stage. In other words, participants changed their minds and became more risk averse after experiencing a win as compared to planning for a win. Notice that following a loss (the last 4 columns), the probability of taking the gamble at the final stage was always smaller than the probability of taking the gamble at the planning stage. In other words, participants changed their minds and became more risk seeking after experiencing a loss as compared to planning for a loss.

Table 1: Barkan and Busemeyer (2003) Experiment							
Gamble		Win First Play		Gamble		Lose First Play	
Win	Loss	Plan	Final	Win	Loss	Plan	Final
200	220	0.46	0.34	80	100	0.36	0.44
180	200	0.45	0.35	100	120	0.47	0.63
200	200	0.59	0.51	100	100	0.63	0.64
120	100	0.70	0.62	200	180	0.57	0.69
140	100	0.62	0.54	160	140	0.68	0.69
200	140	0.63	0.53	200	160	0.67	0.72
200	120	0.74	0.68	160	100	0.65	0.73
200	100	0.79	0.70	180	100	0.68	0.80
				200	100	.85	.82

2 Choice Models

2.1 Reference point change model

Barkan and Busemeyer (2003) accounted for these results by using a model based on prospect theory originally proposed by Tversky and Shafir (1992) to account for the disjunction effect. The essential idea is that the decision maker ignores the planned wins or losses, but the decision maker is later affected by the experienced wins or losses. Ignoring the first stage outcome during the plan but then later incorporating these outcome during the final decision causes a change in the reference point of the utility function used for planned versus final decisions.

Consider a generic gamble G that produces a win equal to x_W or a loss equal in magnitude to x_L with equal probability. (The payoff amounts shown in Table 1 were first rescaled by dividing each payoff by 100 to convert to dollars.) In prospect theory, the utility function for an outcome x is often represented by a power function: $u(x) = x^a$ for $x \geq 0$, and $u(x) = -b \cdot |x|^a$ for $x < 0$. The parameter a is used to model risk aversion and it usually has a value between 0 and 1; the parameter b is used to model loss aversion and usually it has a value greater than one.

During the planning stage, it is assumed that people ignore the planned outcome of the first gamble and simply compute a utility for playing the second gamble based solely on the payoffs for the second gamble:

$$u(G|Plan) = (.50) \cdot x_W^a - (.50) \cdot b \cdot x_L^a. \tag{1}$$

The choice for the plan is based on the comparison of the utility of gambling on the second play to status quo (a zero outcome), $D_P = u(G|Plan) - 0$.

Following the experience of a win, the person includes the win from the first gamble into the evaluation of the payoffs for the second gamble and uses the following utility function

$$\begin{aligned} u(G|Win) &= (.50) \cdot (x_W + x_W)^a + (.50) \cdot (x_W - x_L)^a, \text{ if } (x_W - x_L) > 0 \quad (2) \\ u(G|Win) &= (.50) \cdot (x_W + x_W)^a - (.50) \cdot b \cdot |(x_W - x_L)|^a, \text{ if } (x_W - x_L) < 0. \end{aligned}$$

The choice after experiencing a win is based on the comparison of the utility of gambling again on the second play to the utility of keeping the amount of the win from the first gamble, $D_W = u(G|Win) - x_W^a$.

Following the experience of a loss, the person includes the loss from the first gamble into the evaluation of the payoffs for the second gamble and uses the following utility function

$$\begin{aligned} u(G|Loss) &= (.50) \cdot (x_W - x_L)^a - (.50) \cdot b \cdot (x_L + x_L)^a, \text{ if } (x_W - x_L) > 0 \quad (3) \\ u(G|Loss) &= -(.50) \cdot b \cdot |(x_W - x_L)|^a - (.50) \cdot b \cdot (x_L + x_L)^a, \text{ if } (x_W - x_L) < 0 \end{aligned}$$

The choice after experiencing a loss is based on the comparison of the utility of gambling again on the second play to the utility of keeping the amount of the loss from the first gamble, $D_L = u(G|Loss) - (-b \cdot x_L^a)$.

Essentially the reference point for evaluating gains and losses changes in this model. For example, during the plan, the possibility of losing 100 on the second gamble is evaluated as a loss (because any payoff below zero is considered a loss). But after finding out that \$200 was won on the first play, then the possibility of losing 100 on the second play is evaluated as a reduced gain (any payoff below -200 is now considered a loss). In short, dynamic inconsistency arises from the use of different utility functions, defined by different reference points, for plans versus final decisions.

So far, this model is deterministic and cannot produce choice probabilities. To convert these utilities into probabilities, it is common to assume an extreme value random utility type of model (McFadden, 1981). Under this assumption, the choice probabilities are determined by a logistic probability distribution function, which produces the following probabilities to play the second gamble for each of the three conditions:

$$\begin{aligned} p(T|Plan) &= \frac{1}{1 + e^{-\gamma \cdot D_P}} \\ p(T|Win) &= \frac{1}{1 + e^{-\gamma \cdot D_W}} \\ p(T|Loss) &= \frac{1}{1 + e^{-\gamma \cdot D_L}}. \end{aligned}$$

where γ is a parameter that adjusts the sensitivity of choice probability to the utility of the gamble.

This model has three free parameters (a, b, γ) that were fit to the 34 mean data points in Table 1 (17 plan probabilities and 17 final probabilities). The best fitting parameters (minimizing sum of squared error) are $a = .8683$, $b = .9223$, and $\gamma = 2.6980$. The loss aversion parameter b is less than one (less sensitivity to losses), even though it is theoretically expected to be greater than one (greater sensitivity to losses). The model produced an $R^2 = .7745$ and an *adjusted* $R^2 = .7599$.

We also fit a model that allowed the decision weight to change for gains and losses. In this case, we replaced the .50 probability of a win with a decision weight parameter w, and the decision weight for the probability of a loss equaled

$(1-w)$. The best fitting four parameters were $a = .8533$, $b = .9960$, $\gamma = 2.4681$ and $w = .5227$, and this model produced $R^2 = .7982$ and an *adjusted* $R^2 = .7780$.

2.2 Quantum decision model

The quantum model used to account for the dynamic inconsistency effect is the same model that was previously developed by Pothos and Busemeyer (2009) to account for the disjunction effect. The essential idea is that the decision maker uses a consistent utility function for plans and final decisions and always incorporates the outcomes from the first stage into the decision for the second stage. The planned decision differs from the final decision, because the plan is based on a superposition over possible first stage outcomes that will be faced during the final stage.

The two stage game involves a set of four mutually exclusive and exhaustive outcomes $\{WT, WR, LT, LR\}$ where for example WT symbolizes the event 'win the first stage' and 'take the second stage gamble,' and LR represents the event 'lose the first stage' and 'reject the second stage gamble.' These four events correspond to four mutually exclusive and exhaustive basis states $\{|WT\rangle, |WR\rangle, |LT\rangle, |LR\rangle\}$. The four basis states are represented in the quantum model as four orthonormal basis vectors that span a four dimensional vector space. The state of the decision maker is a superposition over these four orthonormal basis states.

$$\begin{aligned} |\psi\rangle &= \psi_{WT} \cdot |WT\rangle + \psi_{WR} \cdot |WR\rangle + \psi_{LT} \cdot |LT\rangle + \psi_{LR} \cdot |LR\rangle, \\ \|\, |\psi\rangle \,\|^2 &= 1. \end{aligned}$$

The initial state is represented by a 4×1 matrix ψ_I containing elements ψ_{ij} $i = W, L$ and $j = T, R$ which is the amplitude distribution over the four basis states. Initially, during the planning stage, an equal distribution is assumed so that ψ_I has elements $\psi_{ij} = 1/2$ for all four entries. The state following experience of a win is updated to ψ_W which has $1/\sqrt{2}$ in the first two entries and zeros in the second two. The state following experience of a loss is updated to ψ_L which has $1/\sqrt{2}$ in the last two entries and zeros in the first two entries.

Evaluation of the payoffs causes the initial state ψ_I to be "rotated" by a unitary operator U into final states used to make a choice about taking or rejecting the second stage gamble:

$$\begin{aligned} \psi_F &= U \cdot \psi_I \\ U &= \exp\left(-i \cdot \frac{\pi}{2} \cdot (H_1 + H_2)\right) \end{aligned}$$

where

$$H_1 = \begin{bmatrix} \frac{h_W}{\sqrt{1+h_W^2}} & \frac{1}{\sqrt{1+h_W^2}} & 0 & 0 \\ \frac{1}{\sqrt{1+h_W^2}} & \frac{-h_W}{\sqrt{1+h_W^2}} & 0 & 0 \\ 0 & 0 & \frac{h_L}{\sqrt{1+h_L^2}} & \frac{1}{\sqrt{1+h_L^2}} \\ 0 & 0 & \frac{1}{\sqrt{1+h_L^2}} & \frac{-h_L}{\sqrt{1+h_L^2}} \end{bmatrix}, \; H_2 = \frac{-\gamma}{\sqrt{2}} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}.$$

The upper right corner of H_1 is defined by the payoffs given a win; and the bottom right corner of H_1 is defined by the payoffs given a loss (this is described in more detail below). The matrix H_2 aligns beliefs and actions by amplifying the potentials for states WT, LR and and attenuating potentials for states WR, LT. The parameter γ is a free parameter that allow changes in beliefs during the decision process.

The utilities for taking the gamble are mapped into the parameters h_W and h_L in H_1, and the latter must be scaled between -1 to $+1$. To accomplish this, the parameter h_W used to define H_1 is defined as

$$h_W = \frac{2}{1+e^{-D_W}} - 1,$$

where D_W is defined by Equation 2. The parameter h_L used to define H_2 is defined as

$$h_L = \frac{2}{1+e^{-D_L}} - 1,$$

where D_L is defined by Equation 3.

The projection matrix $M = diag\,[1,0,1,0]$ is used to map states into the response for taking the gamble on the second stage. The probability of planning to take the second stage gamble equals

$$p(T|Plan) = ||M \cdot U \cdot \psi_I||^2. \tag{4}$$

The probability of taking the second stage game following the experience of a win equals

$$p(T|Win) = ||M \cdot U \cdot \psi_W||^2. \tag{5}$$

The probability of taking the second stage game following the experience of a loss equals

$$p(T|Loss) = ||M \cdot U \cdot \psi_L||^2. \tag{6}$$

In sum, this quantum model has only three parameters: a and b are used to determine the utilities in Equations 2 and 3 in the same way as used in the reference point change model; the third is the parameter γ for changing beliefs to align with actions. These three parameters were fit to the $17 \times 2 = 34$ data points in Table 1, and the best fitting parameters (minimizing sum of squared error) are $a = .7101$, $b = 2.5424$, and $\gamma = -4.4034$. The risk aversion parameter is a bit below one as expected, and the loss parameter b exceeds one, as it should be. The model produced an $R^2 = .8234$ and an adjusted $R^2 = .8120$.

If we force $\gamma = 0$, then the quantum model is no longer produces 'quantum like' interference effects. Instead, the choice probability for the plan is an equal weight average of the two choice probabilities produced after either winning or losing the first stage: $p(T|plan) = (.50) \cdot p(T|Win) + (.50) \cdot p(T|loss)$, where $p(T|Win)$ is defined by Equation 5 with $\gamma = 0$ and $p(T|loss)$ is defined by Equation 6 with $c = 0$. This model was fit to the results in Table 1 by using only two parameters a and b for the quantum model (with $\gamma = 0$), and it produced an $R^2 = .7854$ and an adjusted $R^2 = .7787$ which still falls below the adjusted R^2 for the three parameter quantum model, and so the γ parameter is making a useful contribution in this application.

Finally we also fit a model that allowed the decision weight to change for gains and losses. In this case, we replaced the .50 probability of a win with a parameter w, and the probability of a loss equaled $(1 - w)$. The best fitting four parameters were $a = .8205$, $b = 2.5280$, $\gamma = -4.3739$, $w = .5141$, and the model produced and $R^2 = .8328$ and an adjusted $R^2 = .8160$.

In summary, comparing the two key models on the basis of fitting the means, we find that the quantum model produced a .05 increase in R-square over the traditional reference point model when the two models were fit using the same number of parameters. However, controlling number of parameters does not guarantee equal model complexity. Therefore a Bayesian analysis was performed to provide a more rigorous model comparison.

There is one other curious finding concerning the R-square as a function of the γ parameter obtained from the quantum model. Figure 1 plots the R-square as a function of γ.As can be seen in this figure, the R-square function

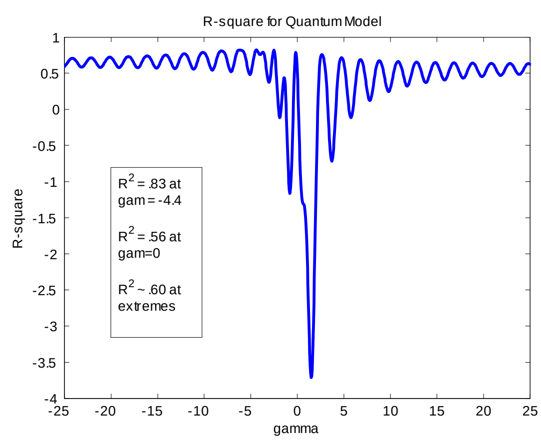

Figure 1: R-square fit to means for the quantum model plotted as a function of the gamma parameter

has a damped oscillation pattern. It deeps deep down around (but not exactly) at zero, and then rises and oscillates around a reasonably high value as the parameter goes to extreme values.

3 Bayesian model comparison

3.0.1 Log likelihood for each person

The Bayesian model comparison was computed separately for each participant using the 33 choice trials observed by that person. On each trial, a gamble was presented and the person made both a plan for an outcome and a final choice after observing that same outcome. For person i on trial t we observe a data pattern $X_i(t) = [x_{TT}(t), x_{TR}(t), x_{RT}(t), x_{RR}(t)]$ defined by $x_{ij}(t) = 1$ if event (i, j) occurs and otherwise zero, where TT is the event "planned to take gamble and finally did take the gamble," TR is the event "planned to take gamble but changed and finally rejected gamble." RT is the event "planned to reject the gamble but changed and finally did take the gamble'" and RR is the event "planned to reject gamble and finally did reject the gamble."

Two allow for possible dependencies between a pair of choices within a single trial, an additional memory recall parameter was included in each model. For both models, it was assumed that there is some probability m, $0 \leq m \leq 1$ that the person simply recalls and repeats the planned choice for the final choice, and there is some probability $1 - m$ that the person forgets or ignores the planned choice when making the final choice. After including this memory parameter, the prediction for each event becomes

$$\begin{aligned}
p_{TT} &= p(T|plan) \cdot (m \cdot 1 + (1-m) \cdot p(T|final)) \\
p_{TR} &= p(T|plan) \cdot (1-m) \cdot p(R|final) \\
p_{RT} &= p(R|plan) \cdot (1-m) \cdot p(T|final) \\
p_{RR} &= p(R|plan) \cdot (m \cdot 1 + (1-m) \cdot p(R|final))
\end{aligned}$$

Using these definitions for each model, the log likelihood function for the 33 trials from a single person can be expressed as

$$\begin{aligned}
\ln L(X_i(t)) &= \sum x_{jk}(t) \cdot \ln(p_{jk}) \\
\ln L(X_i) &= \sum_{i=1}^{33} \ln L(X_i(t)).
\end{aligned}$$

3.1 Grid analysis of log likelihood function

Each model has four parameters $\theta = (a, b, m, \gamma)$, a risk aversion parameter, a loss aversion parameter, a memory parameter, and a choice model parameter. The first three parameters were common across both models and they only differ with respect to the fourth parameter. We used a fine grid of 41 points per parameter. We compared this grid of 41 points with a less fine grid using only

21 points per parameter, and there was no meaningful difference between the final Bayes factors using these two grids, and so we concluded that 41 points was more than sufficient.

$$\begin{aligned}
a &\in [.400, .425, ..., .875, .90, .925, ..., 1.375, 1.40]\,, \\
b &\in [.50, .55, ..., 1.45, 1.50, 1.55, ...2.45, 2.50]\,, \\
m &\in [.000, .025, ..., .475, .500, .525, ..., .975, 1.00]\,, \\
\gamma &\in [0, .10, ..., 1.90, 2.0, 2.10, ...3.90, 4.0]\,,\ \text{(reference point)} \\
\gamma &\in [-5.00, -4.75, ..., -.25, 0.00, .25, ..., 4.75, 5.00]\,.\ \text{(quantum)}
\end{aligned}$$

This grid generated $41^4 = 2,825,761$ combinations, and we evaluated the log likelihood function for each model at each combination. These ranges were chosen on the basis of past fits of these models. The risk aversion parameter ranges from risk aversion to risk seeking; the loss aversion parameter ranges across loss insensitivity to loss sensitivity; and the memory parameter ranges from no recall to perfect recall. These ranges were used for both models and so they do not differ on these three parameters. The only parameter for which the models differ is the choice parameter γ : it ranges across random choice to almost deterministic choice for the reference point model; and it ranges from positive to negative values for the quantum model.

Once again, a surprising feature was noted with regard to the log likelihood function of the quantum model that is important to point out. Figure 2 plots the log likelihood for the quantum model when the parameter γ varies across a wide range of values. As can be seen in the figure, the log likelihood function

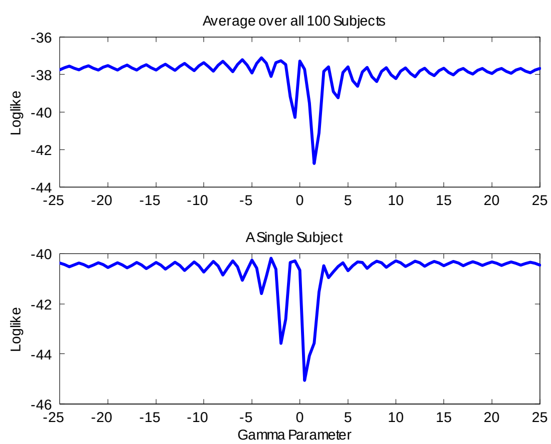

Figure 2: Log likelihood plotted as a function of gamma. Top panel is averaged across all participants and bottom panel shows one example person.

once again has the form of a damped oscillation, and this is true both for the average across participants as well as for individual participants.

3.2 Prior distributions

Two different prior distributions were examined: a uniform and a normal distribution. The uniform distribution assigned equal probability to each grid combination point (see Figure 3). For the normal prior, we assumed independent normal distributions for each parameter (see Figure 4). The prior for the risk aversion parameter was normally distributed around a mean of .90 (slight risk aversion) and a standard deviation of .25. The prior for the loss aversion parameter was normally distributed around a mean of 1.25 (slight loss aversion) and a standard deviation equal to .25. The memory parameter was normally distributed around a mean of .50 and a standard deviation equal to .25. The prior for the gamma parameter for the reference point model was normally distributed around a mean of 1.5 (producing a reasonable S-shape function) and a standard deviation equal to .75. The prior for the gamma parameter for the quantum model was normally distributed around a mean of zero with a standard deviation equal to 5.

3.3 Bayes factor

The Bayes factor was computed for each model by first computing the expected likelihood for each model, which is a weighted average of all the likelihoods with a weight for each likelihood, $w(\theta_i)$, determined by the prior probability assigned to that likelihood. The Bayes factor for each person equals the ratio of the expected likelihoods.

$$\begin{aligned}
p_m(X_i|\theta_i) &= \exp\left(\ln L(X_i|\theta_i)\right) \\
p_m(X_i) &= E\left[p_m(X_i|\theta_i)\right] = \sum_{\theta} w(\theta_i) \cdot p_m(X_i|\theta_i) \\
BF_i &= \frac{p_Q(X_i)}{p_P(X_i)}.
\end{aligned}$$

Figures 3 and 4 display the relative frequency distributions for the log of the Bayes factor produced by the uniform and normal priors, respectively. The sum of the log Bayes factors for the uniform distribution equals 243 and 90% of the participants produced log Bayes factors greater than zero favoring the quantum model. The sum of the log Bayes factors for the normal distribution equals 208 and 83% of the participants produced log Bayes factors greater than zero favoring the quantum model. In sum, using a Bayes factor to compare models, the conclusion from both a uniform and a normal prior distribution is that the Bayes factor favors the quantum model.

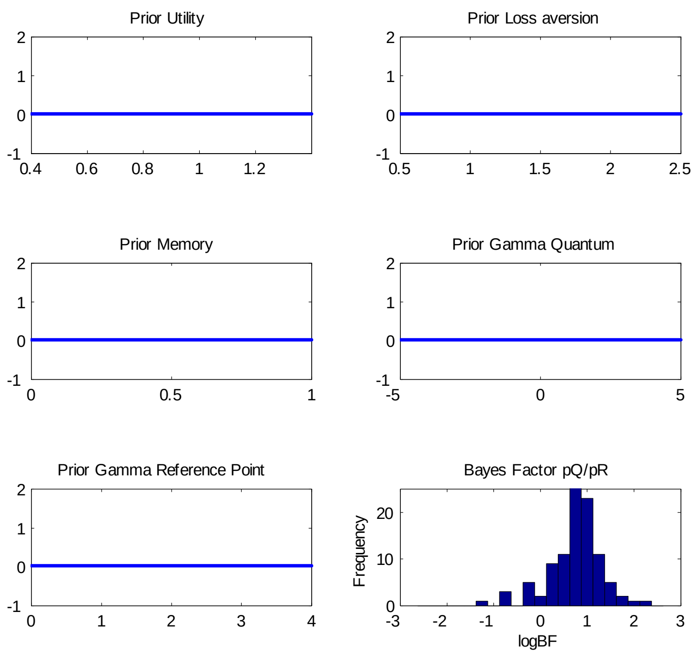

Figure 3: Uniform prior and Bayes factor distribution produced by this prior.

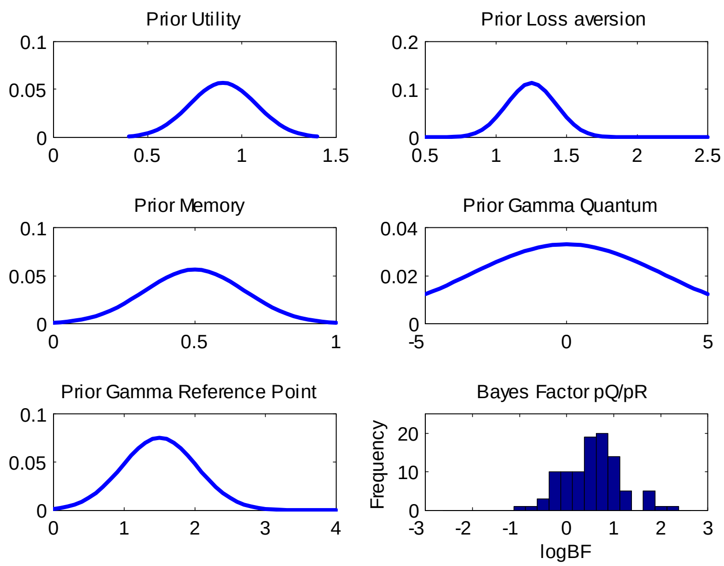

Figure 4: Normal prior and distribution of the Bayes factor produced by this prior.

4 Conclusions

This article began by introducing two different views for understanding the stochastic nature of human choice behavior under risk and uncertainty. One is a "classical" view that choices for the same pair of risky options vary across occasions because either the utility function or the probability weighting function changes across occasions. From this view, choice on each occasion is a deterministic function of the values sampled on that occasion for either the utility function or the probability weighting function. If we new the realized values of these functions at a particular occasion, we could perfectly predict the choice at that moment. Another viewpoint is that choice is intrinsically probabilistic, and it is impossible to perfectly predict the choice at any moment. The latter view is consistent with a quantum view of probabilistic choice. Quantum models have been proposed recently to account for some paradoxical findings that have resisted explanations by "classic" models. But perhaps this success simply results from extra flexibility obtained by using quantum probabilities? The main purpose of this paper is to evaluate this question using rigorous Bayesian model comparison methods to compare a traditional reference point change model to a quantum decision model using data from a large experiment investigating dynamic inconsistency by Barkan and Busemeyer (2003).

Both the reference point model and the quantum model share the same number of free parameters. Both models were initially fit to the mean data using simple least square methods (3 parameters were used to fit 34 means). Both models fit the means reasonably well, but the quantum model produced a 5% improvement in R-square over the reference point model. However this might reflect more flexibility of the quantum model. To perform the Bayesian comparison, a Bayes factor was computed separately for each participant using two different prior distributions over the parameters: a uniform and a normal prior. The Bayes factor overwhelmingly favored the quantum model over the reference point model.

Of course, it is much too soon to conclude that the quantum model is superior to the reference point model. The models need to be compared using other data sets from various other experiments. Even within the same data set, various other prior distributions need to be examined. But the surprising lesson learned from this model comparison exercise was that contrary to expectations, the quantum model did so well, and one cannot jump to the conclusion that it succeeds over traditional models only because it is more flexible.

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