Reference Dependence and Loss Aversion in Probabilities: Theory and Experiment of Ambiguity Attitudes

Jianying Qiu* Utz Weitzel†

Abstract

In standard models of ambiguity, the evaluation of an ambiguous asset, as of a risky asset, is considered as an independent process. In this process only information directly pertaining to the ambiguous asset is used. These models face significant challenges from the finding that ambiguity aversion is more pronounced when an ambiguous asset is evaluated alongside a risky asset than in isolation. To explain this phenomenon, we developed a theoretical model based on reference dependence in probabilities. According to this model, individuals (1) form subjective beliefs on the potential winning probability of the ambiguous asset; (2) use the winning probability of the (simultaneously presented) risky asset as a reference point to evaluate the potential winning probabilities of the ambiguous asset; (3) code potential winning probabilities of the ambiguous asset that are greater than the reference point as gains and those that are smaller than the reference point as losses; (4) weight losses in probability heavier than gains in probability. We tested the crucial assumption, reference dependence in probabilities, in an experiment and found supporting evidence.

Keywords: Ambiguity Aversion, Reference Point, Comparison, Experiment

JEL classification: C91, D03, D81

---

*Department of Economics, Radboud University Nijmegen, Thomas van Aquinostraat 5.12, 6525 GD Nijmegen, the Netherlands, Email: j.qiu@fm.ru.nl

†Department of Economics, Radboud University Nijmegen, Thomas van Aquinostraat 5.12, 6525 GD Nijmegen, the Netherlands, Email: u.weitzel@fm.ru.nl

1 Introduction

The evaluation of ambiguous assets is much more complicated than risky assets due to the absence of objective probabilities. Savage (1954) suggests that individuals could form subjective additive beliefs. The subjective beliefs could then be used to evaluate the ambiguous assets according to the expected utility theory. Ellsberg (1961), however, presented compelling examples in which people prefer to bet on known rather than on unknown probabilities. One version of the Ellsberg paradox is as follows: There are two urns: a known urn and an unknown urn. The known urn contains 50 white and 50 black balls. The unknown urn also contains 100 black and white balls, but the proportion of black and white balls is unknown. For either urn, an individual first chooses a color and then draws a ball from the urn. She receives 100 if the color of chosen ball matches her chosen color. Typically, an individual is willing to pay less for the gamble on the unknown urn than on the known urn. Several variations of Ellsberg's (1961) original designs have been implemented in laboratory experiments, and a similar discount on the unknown urn was observed (see Wakker, 2010, for a review of the literature). This phenomenon, termed ambiguity aversion, is inconsistent with Savage's (1954) subjective expected utility model.

Ambiguity aversion has received tremendous research interest and numerous models have been put forward to explain it: Schmeidler's (1989) Choquet expected utility, Gilboa and Schmeidler's (1989) maxmin expected utility, Klibanoff et al.'s (2005) smooth ambiguity model; Nau's (2006) multistage recursive model, and Abdellaoui et al.'s (2011) non-expected utility model of source functions, to name just a few leading examples.

These approaches face serious descriptive difficulties since the work of Fox and Tversky (1995). They showed that the willingness to pay for the ambiguous asset was not systematically different from the risky asset when each of them was evaluated in isolation. Only when the ambiguous asset was evaluated alongside the risky asset, ambiguity aversion became more pronounced. Fox and Tversky (1995) refer to this effect as 'comparative ignorance'. Chow and Sarin (2002) and Fox and Weber (2002) extended Fox and Tversky's (1995) experiment and provided further evidence that a significant portion of ambiguous aversion results from the fact that the ambiguous and the risky asset were evaluated together. This is a fundamentally different perspective of the evaluation of an ambiguous asset. As stated by Wakker (2000): "Fox and Tversky's finding seems to place the Ellsberg paradox in an entirely new light".

In standard models of ambiguity, the evaluation of an ambiguous asset, as of a risky asset, is considered to be an independent process. In this process only information directly pertaining to the ambiguous asset is used. We believe, while this assumption seems appropriate for the evaluation of a risky asset with clearly defined objective probabilities, it is too strong for the evaluation of an ambiguous asset. Given that there is little information about the ambiguous asset per se, it seems natural that individuals attempt to 'improve' their evaluation of the ambiguous asset by using other information. One reasonable way would be to find a risky asset with a similar structure and use it as reference. As a result, the evaluation of the ambiguous asset becomes simpler and clearer. We therefore suggest that individuals are inclined to use of the information pertaining to the risky asset in the evaluation of the ambiguous asset when the two types of assets are presented together. There are a number of ways to use the information of the risky asset. One interesting way is suggested by Trautmann et al. (2011). They argue that subjects take the value – the Willingness To Pay (WTP) – of the risky prospect as a reference point for their valuation of the ambiguous prospect, and loss aversion in the payoffs could result in ambiguity aversion.

Trautmann et al.'s (2011) approach can explain the findings in Fox and Tversky (1995) without deviating much from prospect theory. However, it seems a rather complicated process to first complete the valuation process of the risky asset and then use that value as a reference point to evaluate the payoffs of the ambiguous prospect. Note that the payoffs are the same between the risky and ambiguous prospects in their study; it is the information on probability dimension that differs the ambiguous prospect from the risky prospect. In a recent paper, Arieli et al. (2011) found – via tracking subjects' eyeball movements – that most subjects, when comparing two risky prospects, compare payoffs and probabilities separately. Rubinstein (1988) suggested a procedure for preference determination, where the different attributes are decisive in determining preferences for prospects. Thus it seems counter-intuitive that subjects facing ambiguous prospect and risky asset would quickly resolve their difficulty in the probability dimension – which are different between the two – and instead focus on the payoff domain – which are the same for the two and thus deserves less attention.

For this reason, we shall use a slightly different approach. Empirical studies on ambiguity aversion typically used the same monetary payoff structure between the risky asset and the ambiguous asset. Also, in most valuation models of ambiguous assets payoffs and probabilities are treated separately.[1] Thus, with the same payoff structure, the manipulation of probabilities seems to remain as the most likely explanation for Fox and Tversky's (1995) findings without introducing additional effects. Based on this observation, we hypothesize that, when the risky asset and the ambiguous asset are presented together, (1) individuals form subjective beliefs on the potential winning probability of the ambiguous asset; (2) they use the winning probability of the risky asset as the reference point to evaluate the potential winning probabilities of the ambiguous asset; (3) analogous to prospect theory where payoffs are coded as gains or losses relative to a reference point, they code potential winning probabilities of the ambiguous asset that are greater than the reference point as gains and probabilities smaller than the reference point as losses; (4) finally and again in analogy to prospect theory, individuals exhibit loss aversion in probability by assigning a larger weight to losses in probability than to the same amount of gains in probability. Henceforth we shall refer this set of assumptions as 'reference-dependence hypothesis'. Further, in order to distinguish between ambiguity aversion in isolation and ambiguity aversion in combination with a risky asset, we shall call the latter 'reference-dependent ambiguity aversion'.

Our approach differs substantially from the prevailing explanation of comparative ignorance, which is typically interpreted as a companion of the "competence hypothesis" (Heath and Tversky, 1991). According to this hypothesis, people try to avoid ambiguity in areas of incompetence, while they exhibit less ambiguity aversion if they feel more competent in the decision domain. Referring to the competence hypothesis, Fox and Tversky (1995) and Heath and Tversky (1991) argue that the joint presentation of the

[1] Let $(x_i, p_i)_{i\in\{1,2,\ldots,n\}}$ denote a risky or ambiguous prospect, where x_i is the outcome i and p_i its associated (subjective) probability. Then most valuation models of ambiguity can be written as $\sum_{i\in\{1,2,\ldots,n\}} v(x_i)w(p_i, x_i)$, where $v(x_i)$ evaluates outcome x_i and $w(p_i, x_i)$ evaluates probability.

two assets highlights the relative incompetence about the ambiguous urn, resulting in a lower value for it.

The aim of this paper is to accommodate Fox and Tversky's (1995) findings without introducing additional elements such as individuals' perceptions of competence. The central idea of our approach is the use of the risky asset as a reference point in probability for the evaluation of the ambiguous asset. In doing this, we are more in line with (cumulative) prospect theory and not the competence hypothesis. We tested our crucial assumption, the reference-dependence of ambiguity aversion, in an experiment and found supporting evidence.

In many studies, ambiguity aversion is also found without an explicit comparable risky asset. Chow and Sarin (2002), for example, while finding evidence consistent with Fox and Tversky (1995), also report sizable ambiguity aversion without a directly comparable risky asset. This finding is not inconsistent with our model. Even when the ambiguous asset is valued in isolation, individuals might still try to simplify the evaluation process by finding a reference that is not explicitly presented. Such a reference is often rather salient. For example, in the two-color Ellsberg urn mentioned above it seems natural to think of the probability 0.5 as the reference. Fox and Weber's (2002) study on, inter alia, order effects also indicates that reference-dependent ambiguity aversion can extend beyond a directly comparative evaluation context. Of course, individuals might be less loss averse in probabilities when the reference is not explicitly mentioned.

The paper proceeds as follows. Section 2 demonstrates the theoretical model, Section 3 presents the experimental design. Results are reported in Section 4. Finally, Section 5 concludes.

2 The Model

There are two assets, one is risky (denoted by R) and the other is ambiguous (denoted by U). To simplify the derivation, let's assume that there are only two states of world:

a good state and a bad state. For the risky asset there is an objective probability p_0 for the good state (and thus $1 - p_0$ for the bad state). Individuals receive some monetary payment G if the good state realizes, and zero otherwise. We let $v(G)$ denote the utility or value that individuals attach to a monetary payoff of G, where $v(0) = 0$. Without loss of generality, we normalize $v(G) \equiv 1$. Since our main purpose is a descriptive one, we followed prospect theory (Kahneman and Tversky, 1979; Tversky and Kahneman, 1992). We assume that individuals evaluate the risky asset in the following way:

$$V(R) = w(p_0)v(G) = w(p_0), \tag{1}$$

where $w(p_0)$ is the probability weighting function.

For the ambiguous asset, there is no clear probability one can assign to the good state. One can, however, form a subjective belief for the good state. Let p denote the subjective probability an individual assigns to the good state, and let $F(p)$ denote the cumulative subjective belief distribution that the individual has about the probability p. Then, if an individual behaves according to the subjective expected utility model, she should evaluate the ambiguous asset as follows:

$$V(U) = v(G)\int_0^1 p dF(p), \tag{2}$$

which is independent of p_0 (and therefore the value of the risky asset).

But the individual might use the objective probability of the risky asset as a reference to evaluate her subjective probability of the good state under ambiguity. Analogous to prospect theory, where payoffs are coded as gains or losses relative to a reference point, we assume that the individual codes p larger than p_0 as gains in probability and p smaller than p_0 as losses in probability. Moreover, the individual evaluates gains and losses in probability differently. Let $l(p - p_0)$, $p < p_0$, denote the way the individual evaluates losses in probability, and $g(p - p_0)$, $p \geq p_0$ denote the way the individual evaluates gains in probability. Let Δ denote the absolute difference between p and p_0. In the spirit of loss aversion in prospect theory, we assume that $-l(-\Delta) > g(\Delta), \forall \Delta \in [0, 1]$, i.e. for any equal amount of difference in probability losses in probability are always weighted more heavily than gains in probability. With above assumptions, the evaluation process becomes

$$V(U) = w(p_0) + \overbrace{\int_0^{p_0} l(p - p_0)dF(p) + \int_{p_0}^1 g(p - p_0)dF(p)}^{\text{ambiguity component}}. \tag{3}$$

In equation (3) there are essentially two components. The first component captures an individual's evaluation of the risky asset within the framework of prospect theory. The second component, the sum of the two integrals, captures an individual's evaluation of the ambiguous asset. We refer to the latter as "ambiguity component". When the ambiguity component is zero, individuals are ambiguity neutral, because their values for the ambiguous and for the risky asset are the same. This would happen when individuals have a degenerated point belief at p_0. When the ambiguity component is negative, the ambiguous asset is valued lower than the comparable risky asset and there is ambiguity aversion. When the ambiguity component is positive, the ambiguous asset would be evaluated higher than the risky asset and individuals are ambiguity seeking.

In our model the subjective perception of the levels of ambiguity and individuals' attitudes toward ambiguity are separated from each other. The former is captured by the subjective beliefs $F(p)$. The higher the standard deviation of $F(p)$, the more ambiguous the individual perceives the situation. The maximum ambiguity is obtained when $F(p)$ is bimodal distribution with peaks at 0 and 1. The latter is captured by the functions $l(-\Delta)$ and $g(\Delta)$.

A distinction between the evaluating functions of gains and losses in probability we used here and the probability weighting functions used in (cumulative) prospect theory (Kahneman and Tversky, 1979; Tversky and Kahneman, 1992) must be made. In (cumulative) prospect theory, w^+ (w^-) stands for the probability weighting function for gains (losses) in the *payoff domain*. Although the two probability weighting functions are allowed to be different in gains and losses of payoffs, this difference is not important for any of the central results in prospect theory. Also, empirically there is no strong evidence suggesting differences between the two functions. The $g(\Delta)$ and $l(-\Delta)$ we used here represent the evaluation functions of gains and of losses in the *probability domain*. We assume that individuals weight losses in probability heavier than gains in probability. This assumption is the main driving force of our model. In prospect theory a closer companion to $g(\Delta)$ or $l(-\Delta)$ would be the value functions of payoffs in gains or losses.

Apart from this assumption, the parametric form of the two functions can be rather general. It is not even required that the two functions are continuous or differentiable. Yet, it could be useful to think of some simple and intuitive ways of capturing loss aversion in probability. Analogous to the definition of loss aversion in prospect theory we could choose the following specification: $g(\Delta) = \Delta^\alpha$ and $l(-\Delta) = -\lambda(\Delta)^\alpha$, where $\lambda > 1$ is the loss aversion coefficient and α is some constant. Another specification could be $g(\Delta) = \Delta^\alpha$ and $l(-\Delta) = -(\Delta)^\beta$, where $\alpha > \beta$ captures loss aversion in probability. Since our theory is mainly motivated by loss aversion in prospect theory, we shall use the former to illustrate the main features of the model. Using the latter does not change the result qualitatively.

2.1 Some Examples

In the following we will use a few examples to provide an intuitive illustration of the implications of the ambiguity component in equation (3). Let's assume that the subjective belief of p is uniformly distributed between 0 and 1, i.e. $F(p) = p$. This assumption is strictly for illustrative purposes. In general, $F(p)$ should be context specific and could be different for different individuals. Figure 1 illustrates the calculation of the ambiguity component. Notice first that, when $F(p) = p$, the integral $\int_0^{p_0} l(p - p_0)dF(p)$ simply becomes the area above the line $l(-\Delta)$, and the integral $\int_{p_0}^1 g(p - p_0)dF(p)$ becomes the area below the line $g(\Delta)$.

Consider first the Ellsberg paradox, where the objective probability of the risky asset is 0.5. In this case, $p_0 = 0.5$, and thus the ambiguity component is

$$\text{Ambiguity component} = \int_0^{0.5} l(p - 0.5)dF(p) + \int_{0.5}^1 g(p - 0.5)dF(p). \tag{4}$$

This is illustrated in the top graph of Figure 1. Since $-l(-\Delta) > g(\Delta), \forall\Delta \in [0, 1]$, it can be immediately seen that the negative area (bottom left) is larger than the positive area

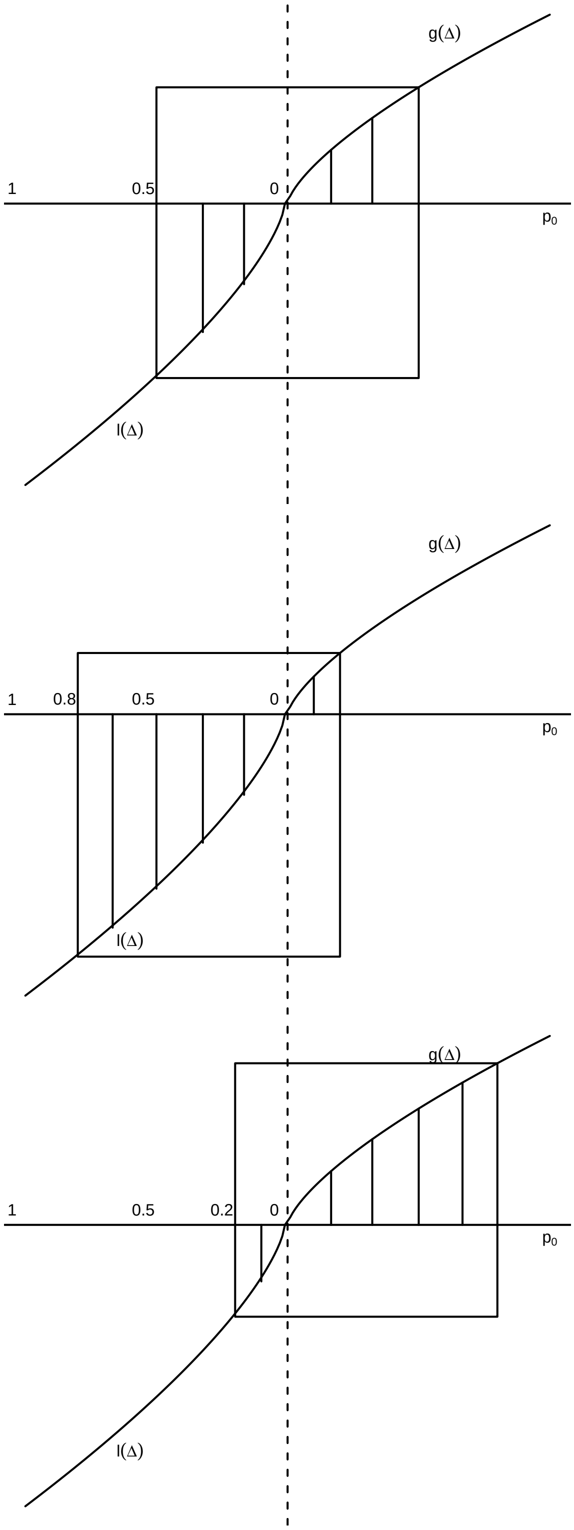


Figure 1: The graph is produced by assuming $F(p) = p$, $g(\Delta) = \Delta^{0.7}$, and $l(\Delta) = -1.5(\Delta)^{0.7}$.

(top right). Hence, the ambiguity component is negative,

$$\begin{aligned}\text{Ambiguity component} &= \int_0^{0.5} l(p-0.5)dF(p) + \int_{0.5}^1 g(p-0.5)dF(p) \\ &< -\int_{0.5}^1 g(p-0.5)dF(p) + \int_{0.5}^1 g(p-0.5)dF(p) = 0, \end{aligned} \quad (5)$$

which represents ambiguity aversion in the Ellsberg paradox. Note that this result can be obtained for any $F(p)$ that gives a symmetric density mass around p_0, as long as the assumption of loss aversion in probability is satisfied.

In some ambiguous situations we could have p_0 larger than 0.5. For example, let's consider two urns, each with 100 balls of five colors: black, white, yellow, green, and blue. In the risky urn there are 20 balls for each color. In the ambiguous urn the proportion of colored balls is unknown. For each urn, an individual can choose four colors and wins if the drawn ball matches any of the four colors. In this scenario, $p_0 = 0.8$, and the ambiguity component is:[2]

$$\text{Ambiguity component} = \int_0^{0.8} l(p-0.8)dF(p) + \int_{0.8}^1 g(p-0.8)dF(p). \quad (6)$$

The calculation is illustrated in the middle graph of Figure 1. As the negative area is larger than the positive area, the ambiguity component is negative and, consequently, there is ambiguity aversion.

Finally, in some ambiguous situations we could have p_0 much smaller than 0.5. Consider the two five-color-urns mentioned above. For each urn, an individual can now choose only one color and wins if the drawn ball matches this color. In this scenario, $p_0 = 0.2$, and

[2]In this scenario the assumption of $F(p) = p$ is questionable. As discussed later, a more natural assumption of $F(p)$ would perhaps be a distribution which assigns density mass symmetric to $p_0 = 0.8$. In this example, we use the assumption nevertheless, because it provides a more intuitive calculation of the ambiguous component.

the ambiguity component is:

$$\text{Ambiguity component} = \int_0^{0.2} l(p-0.2)dF(p) + \int_{0.2}^{1} g(p-0.2)dF(p). \tag{7}$$

As illustrated in the bottom graph of Figure 1, the negative area is now smaller than the positive area, and, consequently, the ambiguity component is positive. That is, in situations where the objective reference probability is small, individuals are ambiguity seeking. This result is not as unusual as it may first appear. In fact, several studies provided evidence of ambiguity seeking when the objective probability of winning an ambiguous gamble was small (see Camerer and Weber, 1992, Table 3, for an overview).

2.2 Links to Other Models

So far, purely for illustrative purposes, we have assumed $F(p) = p$ when p_0 is small ($p_0 = 0.2$), medium ($p_0 = 0.5$), and large ($p_0 = 0.8$). This is, of course, an ad hoc assumption, which becomes questionable when p_0 moves away from 0.5. An arguably more reasonable assumption of $F(p)$ would be that $F(p)$ exhibits more mass around p_0, and the further p moves away from p_0, the less mass individuals attach to these probabilities. One such form of subjective belief, for example, is a normal distribution with a mean p_0 and values that are restricted to $[-p_0, 1-p_0]$.[3]

As we normalized $V(G) \equiv 1$, the value of the risky asset is simply $w(p_0)$. We can take a similar perspective for the value of the ambiguous asset. Let $V(U) \equiv \Pi(p_0)$, we could regard the value of the ambiguous asset as its weighting function in an ambiguous situation. A comparison between $w(p_0)$ and $\Pi(p_0)$ then reflects the difference between decision making under risk and under ambiguity. Figure 2 provides such a comparison.

[3] Since the normal distribution has values extending outside of the range $[-p_0, 1-p_0]$, the restriction of $p \in [-p_0, 1-p_0]$ makes the cumulative mass in the range of $[-p_0, 1-p_0]$ smaller than 1. For a proper distribution function, one could adjust the normal distribution function to

$$\frac{1}{N(-p_0, 1-p_0)} \times dN(p),$$

where $N(-p_0, 1-p_0)$ denotes the cumulative mass between $-p_0$ and $1-p_0$.

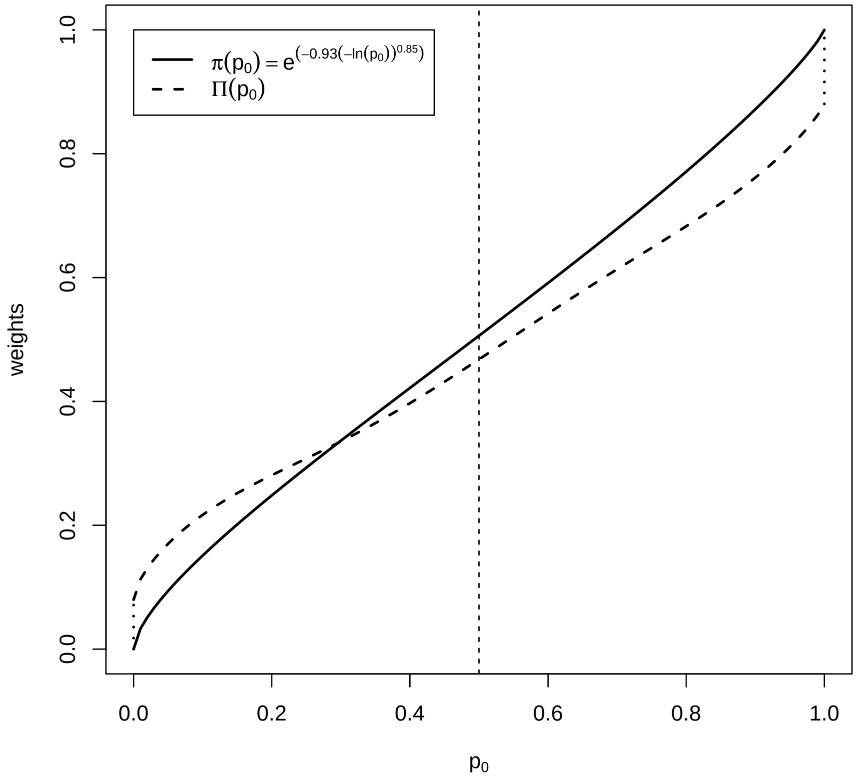


Figure 2: The parametric form of $\pi(p_0) = e^{-0.93(-ln(p_0))^{0.85}}$ is taken from the empirical estimate of Abdellaoui et al. (2011). The functional form of $\Pi(p_0)$ is produced by assuming $g(\Delta) = \Delta$ and $l(-\Delta) = -1.5\Delta$. $F(p)$ is assumed to be normally distributed around p_0 with a standard deviation of 0.20.

Figure 2 bears a remarkable similarity to Figure 3 in Abdellaoui et al. (2011). In particular, our theory suggests, similar to their empirical findings, that: (1) individuals are ambiguity seeking with small p_0 and ambiguity averse with large p_0; (2) decision weights in ambiguity, $\Pi(p_0)$, are flatter than decisions weights in risk, $w(p_0)$, suggesting less sensitivity to changes in probability under ambiguity than under risk. Note that Abdellaoui et al.'s (2011) results were obtained without explicit comparison between the ambiguous and the risky asset.[4] However, in their experimental setting, there often exists a salient probability that may be used as reference.[5] Hence, although the subjects in Abdellaoui et al.'s (2011) study did not explicitly face a risky asset as comparison, they might nevertheless have formed an implicit reference when evaluating the ambiguous asset. Of course, based on the finding of Fox and Tversky (1995), ambiguity aversion should be weaker when the reference is not explicit. This may explain why our model suggests a stronger certainty effect than the empirical findings in Abdellaoui et al. (2011), as revealed by a larger gap in our curve when p_0 approaches 0 or 1.[6]

3 Experimental Design

The crucial assumption in our model is the reference-dependence hypothesis, which states that the objective probability of the risky asset is used as a reference to evaluate the subjective probability of the underlying state of the ambiguous asset. Thus, unlike other models of ambiguity, our model suggests that the probability of the risky asset has an impact on the value of the ambiguous asset.

We decided to directly employ a relatively strong test condition: we held the ambiguous

[4] Abdellaoui et al. (2011) used the certainty equivalence method to obtain the value of the ambiguous asset and the risky asset, but they never presented the two assets simultaneously to subjects.

[5] For example, for the two five-color-urns (with 100 balls) mentioned earlier, it seems reasonable to assume that $p_0 = 0.2$ and $p_0 = 0.8$ are the most prominent comparable probabilities in the minds of the subjects, even if they are not displayed simultaneously when they take their decisions.

[6] The shape of $\Pi(p_0)$ certainly depends on the assumption of $F(p)$ and the particular parametric forms of $l(-\Delta)$ and $g(\Delta)$. The general properties of $\Pi(p)$ are rather robust. We simulated $\Pi(p_0)$ with a few other specifications of $F(p)$ and a number of parametric forms of $l(-\Delta)$ and $g(\Delta)$. All properties discussed above remain intact.

asset constant while we varied the winning probabilities of the risky asset. As the ambiguous asset remains the same, its value should, according to standard ambiguity models, remain constant no matter what risky asset is presented with it. If, however, the value of the ambiguous gamble changes with the variations in the probabilities of the risky asset, this direct effect can be interpreted as strong support for the reference-dependence hypothesis.

Another, more elaborate way of testing the our model is to vary the winning probabilities of the risky assets alongside several values of fixed underlying probabilities of the ambiguous asset. The willingness to pay (WTP) for the risky and for the ambiguous asset could then be used to produce a graph similar to the one in Figure 2. However, as this test would primarily focus on the extended model, it is unnecessarily complicated for a basic test of the reference-dependence hypothesis.

Given that the ambiguous asset is fixed, a between-subjects design becomes a natural choice. We administered to each subject a questionnaire that described two gambles: one ambiguous and one risky. The ambiguous gamble was presented as a bag with 10 black and white chips of unknown proportion. The subject could first choose a color (white or black) and then draw a chip out of the bag. If the color of the chip matched the chosen color, she won 10 euro, and zero otherwise.[7] Similarly, the risky gamble was presented as a bag with 10 black and white chips of known proportion. The subject drew a chip out of the bag and if the chip was black, she won 10 euro, and zero otherwise. An example of the the questionnaire is shown in Figure 4 and Figure 5 in the Appendix. In all questionnaires, the ambiguous gamble was held constant and thus all subjects faced the same ambiguous gamble. Different subjects, however, faced different risky gambles. There were seven risky gambles: a bag with n black chips of a total 10 chips, where n=1, 3, 4, 5, 6, 7, or 9. These seven bags corresponded to seven risky gambles with a winning probability of 0.1, 0.3, 0.4, 0.5, 0.6, 0.7, or 0.9 respectively, constituting seven treatments.

[7]The fact that subjects were able to choose a color does not mean that the chosen color is governed by a belief distribution that always lies above a winning probability of 0.5. What is needed is that the belief distribution of the chosen color gives less mass on low winning probabilities and/or more mass to high winning probabilities than the unchosen color. For example, it would be sufficient if the subjective belief of the chosen color first order degree stochastically dominates the unchosen color.

Individuals typically read questionnaires from the left to the right. If individuals evaluate the gamble on the left hand side first, they might be more inclined to use the probability of the risky asset as the reference when it is presented on the left hand side. To test for such an order effect, we counter balanced the sides on which the risky and the ambiguous gamble were presented.

We elicited subjects' WTP for the risky gamble and the ambiguous gamble by using the BDM mechanism (Becker et al., 1964). For the real payment of a subject, we generated a random number between 0 and 10 (by drawing a chip from a bag with 11 respectively numbered chips). The drawn number would then be compared with her WTP for the gamble on either the left or the right hand side (determined by a coin flip) of the questionnaire. If her WTP was smaller than the randomly generated number, then she would not get to play the gamble and her experiment ended. If the subject's WTP was equal to or larger than the randomly generated number, then she played the gamble, but payed only the price of the randomly generated number.

Constructing the ambiguous gamble properly is critical to the experiment. Although the construction procedure must be perfectly clear, the gamble itself should be regarded by students as ambiguous. We constructed the ambiguous gamble according to the following procedure, which was announced publicly before we implemented it. At the start of the experiment we asked ten randomly selected subjects, who did not participate in rest of the experiment, to pick an integer number between 0 to 10. We did not tell them how are we going to use those numbers. Each of them left the experimental room, secretely wrote the number on a small piece of paper, which they then folded, to hide the number. They then brought the folded pieces of paper into the experimental room and we asked another randomly chosen subject, who did not participate in rest of the experiment, to help us with the construction of the ambiguous gamble. This helper randomly picked one of the ten folded papers. We then gave the helper two bags: one with 10 black chips and one with 10 white chips. We asked the helper to leave the experimental room and use the two bags to produce an ambiguous bag of 10 chips, in which the number of black chips should be the number on the paper she has drawn. When finished, the helper returned with two closed bags, placed the ambiguous bag on the desk in the front and stashed the other away. Neither the experimenters nor the subjects knew the contents of any of the two bags throughout the experiment.

The experiment was done on 7th September, 2011 in the Radboud University Nijmegen. The subjects were 210 second year business and economics students in the bachelor course "Corporate Finance" (third lecture). Before the lecture started, we placed the questionnaires on the desks, with the back side facing up. To reduce the influence of peer students on one's decisions, we always left one empty seat between any two seats. Furthermore, we distributed the questionnaire in such a way that every student was surrounded by different treatments. When the lecture started, we guided students to their seats in an orderly manner. We asked them not to touch the questionnaire on the desk until we told them so. They were also told that they could read the side that was facing up (which explained the payment procedure). Once the classroom was filled, we did not allow the rest of students to enter the classroom. They waited outside the classroom where we gave them some questionnaires to fill in to avoid making them feel excluded. Those questionnaires are, however, excluded from data analysis. After all students were seated, we asked them to flip the questionnaire around and to read the front side of questionnaire, but not to fill in the questionnaire yet. We gave students five minutes to read questionnaires. After the five minutes, we asked students to once again turn to the back side of the questionnaire. We then read through the payment procedure aloud. We told the students that 20 of them would be chosen for real payment after the lecture. Subjects that were chosen for real payment would receive an endowment of 10 euro. In our explanations of the payment procedure, special attention was given to the BDM mechanism.[8] We asked students to raise their hands if they had any questions. Questions were answered individually.

We then carefully explained the procedure of constructing the ambiguous gamble (as described above). After the ambiguous gamble was produced, we placed the ambiguous bag on a desk in front of all students. We told them explicitly that this was the ambiguous bag described in their questionnaires. Questionnaires were then filled in and collected. Each student retained an ID number that was also printed on the questionnaire. Then

[8] To give the students an intuitive feeling for the BDM mechanism, we asked them to imagine that, in order to play the gamble, their WTP would have to compete against a computer that produces a random price between 0 and 10.

the lecture started. In a 15-minute break, after the first half of the lecture, we asked the student helper to randomly pick 20 questionnaires out of the pile. The ID numbers of these questionnaires were announced and shown on a screen. The students with these ID numbers were asked to stay after the lecture. The rest was allowed to also stay and watch the experiment, but they needed to be quiet. After the lecture was over, we asked the 20 students to come to the front of the classroom and show their ID number. For each student, we then determined via the BDM mechanism whether they could really play the gamble, as described above. Those who could not play the gamble received 10 euro and stayed outside of the experimental area. The others played the gamble.[9]

In total the experiment (exluding lecture time) lasted about 45 minutes and the average payoff of the selected students was 9.88 Euro.

4 Experimental results

In total 210 students participated the experiment. Two students did not give their WTPs for the ambiguous gamble, and were therefore excluded from analysis.

Table 1 summarizes the WTPs of the risk gambles with 1, 3, 4, 5, 6, 7, 9 black chips (corresponding a winning probability of 0.1, 0.3, 0.4, 0.5, 0.6, 0.7, 0.9), and WTPs of the ambiguous gamble that were presented along with them. We further split the WTPs by the positioning of the risky gamble on the questionnaire, that is, whether the *risky* gamble was shown on the left or on the right side of the questionnaire. As can be seen by comparing the WTPs of the ambiguous and the risky gamble on the row of 0.5 winning probability, there is significant ambiguity aversion. The mean and median WTPs of the risky gamble with 0.5 winning probability are 4.28 and 4.5, whereas the mean and median of the ambiguous gamble are 3.35 and 3.0, respectively. A two-sided wilcoxon test shows that the difference is significant ($p < 0.01$).

[9]The payoff therefore was determined as 10 Euro minus the randomly drawn number between 0-10 plus the outcome of the gamble (0 or 10).

probability of the risky gamble	WTP of the risky gamble				the WTP of the ambiguous gamble			
	Overall		the *risky* gamble appeared on		Overall		the *risky* gamble appeared on	
	mean	median	left (mean)	right (mean)	mean	median	left (mean)	right (mean)
0.1	1.82	1.00	1.00	2.05	3.95	4.00	2.00	4.48
0.3	3.02	3.00	2.66	3.50	3.85	4.70	3.32	4.57
0.4	3.45	4.00	3.84	3.06	3.37	4.00	3.61	3.13
0.5	4.28	4.50	4.05	4.53	3.35	3.00	3.20	3.50
0.6	5.14	6.00	4.75	5.52	3.67	4.00	2.92	4.42
0.7	5.24	5.00	5.03	5.58	3.69	4.00	3.55	3.92
0.9	7.31	8.00	8.00	7.25	3.85	4.00	4.00	3.83

Table 1: WTPs of the risky gamble and the ambiguous gamble. "Left" ("right") means the risky gamble was on the left (right) hand side of the questionnaire.

If students behaved according to standard models of ambiguity, we should not observe much variation of the WTPs of the ambiguous gamble when the probability of the risky gamble changes. This is clearly not what we observed in the data. As we can see from Table 1, the ambiguity aversion is most pronounced when the risky gamble has a winning probability of 0.5: the mean difference of WTPs between the risky and the ambiguous gamble is 0.93 and the median difference is 1.5. The WTPs of the ambiguous gamble increases (hence ambiguity aversion, defined as the difference of WTPs between the risky gamble with $p_0 = 0.5$ and the ambiguous gamble, decreases) as the winning probability of the risky gamble moves away from $p_0 = 0.5$: mean differences of ambiguity aversion are 0.91, 0.43, and 0.33 respectively for the risky gamble with winning probability of 0.4, 0.3, and 0.1, and are 0.61, 0.59, 0.43 respectively for the risky gamble with winning probability of 0.6, 0.7, and 0.9; median difference of -0.2 for the risky gamble with winning probability of 0.3 and median difference of 0.5 for others. An graphic illustration is reported in Figure 3.[10] In this graph, the x-axis is the winning probability of the risky gambles: p_0 =0.1, 0.3, 0.4, 0.5, 0.6, 0.7, 0.9, the y-axis is a boxplot of the values of the ambiguous gamble. The difference between the WTPs of the ambiguous gamble at $p_0 = 0.5$ and all other values of p_0 is statistically significant at a 90 percent confidence interval (two-sided Wilcoxon test, $p = 0.0802$). We also tested the difference between the WTPs of the ambiguous gamble at $p_0 = 0.5$ and (jointly) at $p_0 = 0.1$ and 0.3, and between $p_0 = 0.5$ and (jointly) at $p_0 = 0.7$ and 0.9. Again, both differences are significant at a 90 percent confidence interval (two-

[10]One should not compare this figure with Figure 2. Figure 2 is produced by varying the ambiguous gamble along with p_0, in such a way that the ambiguous gamble has an "ambiguity neutral" probability of p_0. But here the ambiguous gamble is fixed.

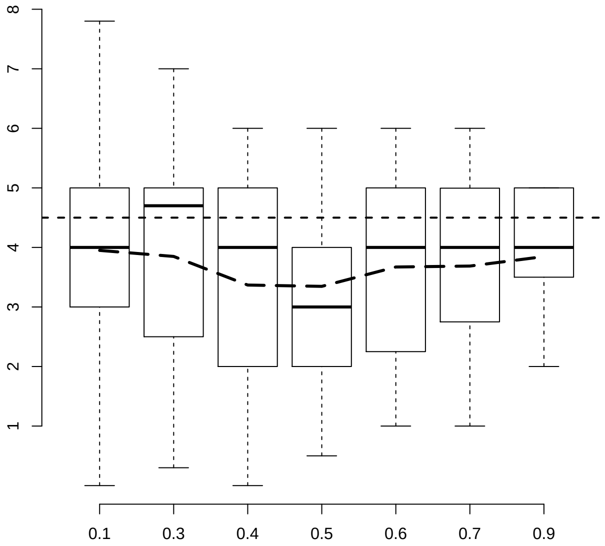


Figure 3: A boxplot of the values of the same ambiguous gamble when it was presented with risky gambles with different probabilities. The bold black lines in the boxes are the median WTPs of the ambiguous gamble in the corresponding scenario. The long dashed curve connects the mean WTPs of the ambiguous gamble in the corresponding scenario. The dashed line is the median value (4.5) of the risky gamble with winning probability of 0.5.

sided wilcoxon tests, $p = 0.0595$ and $p = 0.0925$, respectively). Among the ambiguous gambles with $p_0 \neq 0.5$ there is no single pair with statistically significant differences in their WTPs (two-sided wilcoxon tests, $p > 0.1$ for each pair).

The students typically read from the left to the right. This implies that they could have read and evaluated the left hand side gamble first. This could have an important implication for our model: if the risky gamble was positioned on the left hand side, subjects might have been more inclined to use its winning probability as reference for the ambiguous gamble on the right hand side than the other way around. Consequently, reference-dependent ambiguity aversion could be more pronounced when the risky gamble was shown on the left hand side. To check this possibility we compared the WTPs of the ambiguous gamble when the risky gamble was presented on the left and on the right. We find statistically significant differences in WTPs for ambiguous gambles that were presented with the risky gambles of 0.1, 0.3, and 0.6 winning probabilities (one-sided wilcoxon test, $p < 0.05$). In contrast, when the risky gambles of 0.4 and 0.9 were on the left hand side the average ambiguous WTPs were even lower, although not statistically significant (two-sided wilcoxon test, $p > 0.10$). All together, there seems to be a tendency for a more pronounced ambiguity aversion on the right hand side of the questionnaire, but the evidence is sparse.[11] This result is consistent with the finding in Alevy (2011), where they found that subjects priced ambiguous assets lower than risky assets only when they were previously exposed to risky assets.

We calculated each subject's risk aversion as the difference of the risk neutral evaluation $10 \times p$ and her WTP for the risky gamble. As a measure of ambiguity aversion, we subtracted each subject's WTP for the ambiguous gamble from the mean of the risky gamble with a winning probability of 0.5 (4.28 on average). We then computed the non-parametric Spearman correlation of the risk aversion and ambiguity aversion and found a statistically significant positive correlation of 0.49 ($p < 0.01$). This result is consistent with the study of Bossaerts et al. (2010), where the authors suggest that a positive risk-ambiguity correlation may be able to explain the 'value effect' in historical financial market data.

5 Discussion

In this paper we propose a novel approach to explain ambiguity aversion. The central idea is the use of a more or less salient reference point in probability for the evaluation of the ambiguous asset. We argue that individuals form subjective beliefs on the potential winning probability of the ambiguous asset and that the winning probability of risky asset in the classic setting of Fox and Tversky (1995) is used as a reference point to evaluate the potential winning probabilities of the ambiguous asset. Analogous to prospect theory where payoffs are coded as gains or losses relative to a reference point, potential winning

[11] For the risky gambles, the WTP differences between the left and the right hand side are comparatively weak (two-sided wilcoxon test, $p < 0.10$ for the risky gambles of 0.3 and 0.4, $p > 0.10$ for all other risky gambles).

probabilities of the ambiguous asset that are greater than the reference point are coded as gains and probabilities smaller than the reference point are coded as losses, and individuals exhibit loss aversion in probability by assigning a larger weight to losses in probability than to the same amount of gains in probability. We tested the crucial assumption of the model, reference-dependence, in an experiment and found supporting evidence.

Our approach and explanation of ambiguity aversion is fundamentally different from standard models, which attempt to explain ambiguity aversion as an isolated phenomenon. Also, our approach differs from the predominant explanation of comparative ignorance, first presented by Fox and Tversky (1995). As a companion of the competence hypothesis (Heath and Tversky, 1991), the comparative ignorance hypothesis per se does not depend on a comparison or even a reference point, but on the perception of (in)competence as a state of mind. Although the original experimental design presentated the risky and the ambiguous urn together, (in)competence about an ambiguous situation can also be created in settings without direct comparisons where the ambiguous urn is effectively evaluated in isolation (see e.g. Camerer and Weber, 1992), but under different perceptions of competence.

In contrast, our approach does not assume that ambiguity is evaluated in isolation. We suggest that people try to evaluate ambiguity by comparing it to a similar, but less ambiguous situation of, ideally, pure risk. If such a salient risky comparison exists, we argue that this is readily used as a reference point in probability for the ambiguous situation.

References

Abdellaoui, M., Baillon, A., Placido, L., and Wakker, P. P. (2011). The rich domain of uncertainty: Source functions and their experimental implementation. *American Economic Review*, 101(2):695–723.

Alevy, J. E. (2011). Ambiguity in individual choice and market environments: On the importance of comparative ignorance. Technical report.

Arieli, A., Ben-Ami, Y., and Rubinstein, A. (2011). Tracking decision makers under uncertainty. *American Economic Journal: Microeconomics*, 3(4):68–76.

Becker, G., DeGroot, M., and Marschak, J. (1964). Measuring utility by a single-response sequential method. *Behavioral Science*, 9(3):226–232.

Bossaerts, P., Ghirardato, P., Guarnaschelli, S., and Zame, W. R. (2010). Ambiguity in asset markets: Theory and experiment. *Review of Financial Studies*, 23(4):1325–1359.

Camerer, C. and Weber, M. (1992). Recent developments in modeling preferences: Uncertainty and ambiguity. *Journal of Risk and Uncertainty*, 5(4):325–70.

Chow, C. C. and Sarin, R. K. (2002). Known, unknown, and unknowable uncertainties. *Theory and Decision*, 52(2):127–138.

Ellsberg, D. (1961). Risk, ambiguity, and the savage axioms. *The Quarterly Journal of Economics*, 75(4):pp. 643–669.

Fox, C. R. and Tversky, A. (1995). Ambiguity aversion and comparative ignorance. *The Quarterly Journal of Economics*, 110(3):pp. 585–603.

Fox, C. R. and Weber, M. (2002). Ambiguity aversion, comparative ignorance, and decision context. *Organizational Behavior and Human Decision Processes*, 88(1):476–498.

Gilboa, I. and Schmeidler, D. (1989). Maxmin expected utility with non-unique prior. *Journal of Mathematical Economics*, 18(2):141–153.

Heath, C. and Tversky, A. (1991). Preference and belief: Ambiguity and competence in choice under uncertainty. *Journal of Risk and Uncertainty*, 4(1):5–28.

Kahneman, D. and Tversky, A. (1979). Prospect theory: An analysis of decision under risk. *Econometrica*, 47(2):pp. 263–292.

Klibanoff, P., Marinacci, M., and Mukerji, S. (2005). A smooth model of decision making under ambiguity. *Econometrica*, 73(6):1849–1892.

Nau, R. F. (2006). Uncertainty aversion with second-order utilities and probabilities. *Management Science*, 52(1):pp. 136–145.

Rubinstein, A. (1988). Similarity and decision-making under risk (is there a utility theory resolution to the allais paradox?). *Journal of Economic Theory*, 46(1):145–153.

Savage, L. J. (1954). *The Foundations of Statistics.* Courier Dover Publications.

Schmeidler, D. (1989). Subjective probability and expected utility without additivity. *Econometrica*, 57(3):571–87.

Trautmann, S. T., Vieider, F. M., and Wakker, P. P. (2011). Preference reversals for ambiguity aversion. *Management Science*, 57(7):1320–1333.

Tversky, A. and Kahneman, D. (1992). Advances in prospect theory: Cumulative representation of uncertainty. *Journal of Risk and Uncertainty*, 5(4):297–323.

Wakker, P. (2010). *Prospect Theory: for Risk and Ambiguity.* Cambridge University Press, Cambridge, UK.

Wakker, P. P. (2000). Uncertainty aversion: a discussion of critical issues in health economics. *Health Economics*, 9(3):261–263.

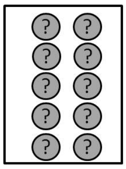
Appendix: Experimental questionnaire

ID number: Sept. 11-AA-01 (Please take this ID number with you. It is important for your potential payment.)

There is

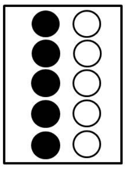
- a bag (below left) filled with 10 chips that are black or white, but you do not know their relative proportion.
- and a second bag on the table (below right) filled with exactly 5 black chips and 5 white chips,

Suppose that you are offered a ticket to the game that is to be played as follows:
First, you choose a color (black or white).
Next, *without looking*, you are to draw a ball out of the bag below.
If the color that you draw is the same as the one you predicted, then you will win 10 Euro; otherwise you win nothing.
What is the most that you would pay to play such a game for the bag below? (0-10 Euro)



The most that I would be willing to pay for a ticket to play the game with this bag **(? black; ? white)** is: ____ , ____ Euro

Suppose that you are offered a ticket to the game that is to be played as follows:
Without looking, you are to draw a ball out of the below bag. If the color that you draw is ***black***, then you will win 10 Euro; otherwise you win nothing.
What is the most that you would pay to play such a game for the bag below? (0-10 Euro)



The most that I would be willing to pay for a ticket to play the game with this bag **(5 black; 5 white)** is: ____ , ____ Euro

20 of you will be chosen for payment. The payment procedure is explained on the back of this paper.

ID number: Sept. 11-AA-01

Figure 4: An example of the Questionnaire used in the experiment: the front

Payment procedure

- At the end of the lecture 20 questionnaires will be chosen randomly. The ID numbers on the questionnaires will be announced.
- If your ID number is not announced, you won't be able to play the game. You can leave the classroom for the break or stay to watch the payout procedure, but if you stay in the room, please be quiet.
- If your ID number is announced, then you are chosen for playing the game. Please come to the front.
- You will receive 10 Euro.
- You will be put into a group of 10 students, either into Group 1 or Group 2. Then a coin will be flipped. If it is heads, all students in Group 1 will have the chance to play the left side game (? black; ? white). All students in Group 2 will have the chance to play the right hand side game (5 black; 5 white). If it is tails, the game–group-combination is reversed (Group 1 plays right game; Group 2 plays left game).
- Then a number between 1 to 10 will be randomly drawn for each student. If the drawn number is larger than your stated ticket price for the game, then you keep the 10 Euro and you don't play the game. You may then leave if you wish. If the drawn number is smaller or equal to your stated ticket price for the game, then you play the game. Your final earning will then be: the 10 Euro you received at the beginning, minus the price you pay for the game, plus the reward from the game (10 Euro or 0 Euro). The price you pay for the ticket to play this game will only be the price of the randomly drawn number. This is to your advantage as the drawn number is lower or equal than your stated ticket price! Note that, given this mechanism, it is your own best interest to state your true price.
- Please raise your hand if you have any questions.

Your decisions in this questionnaire are irrelevant for your course grade!

There is not right or wrong answer, so feel free to take the decision you are most comfortable with. Good Luck!

Figure 5: An example of the Questionnaire used in the experiment: the back