Are Gambling Behaviour and Allais Paradox Two Sides of the Same Coin? Evidence from Horse Race Betting[♦]

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Abstract

This paper shows that the empirical observations from real-life gambling markets correspond with the Allais' experiments. Risk behaviour is modelled with an uncertainty function which is based on the dual theory model with probability weighting. We use a multinomial model with horse race betting data to estimate risk parameters. Our results imply that the assumption of risk aversion should not be rejected because probability weighting affects gambling decisions.

Keywords: Allais paradox, dual theory, probability weighting function, rank-dependent utility

JEL classification: D81, L83

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1. Introduction

The conventional approach to model decision making under risk and uncertainty has been the expected utility theory (EUT) by von Neumann and Morgenstern (1944). Despite its widespread use, EUT has received a fair share of criticism. The major source of criticism originated from the experiments in which decision-makers systematically violate the predicted behaviour under EUT (e.g. Allais 1953; Kahneman and Tversky 1979). In defence of EUT, it is not clear how these experiments reflect decision making in real-life situations.

Gambling markets have been a fertile ground for empirical research on decision making under risk and uncertainty. For instance, Jullien and Salanié (2000) and Snowberg and Wolfers 2010 tested the feasibility of decision making theories with data from real-life gambling markets. A well-known finding in gambling markets is the favourite-longshot bias (FLB), where bettors systematically overbet longshots and underbet favourites (Griffith 1949). FLB contradicts EUT if risk aversion is assumed. Subsequently, a general explanation for the observed FLB has been risk-love or misperception of probabilities (Snowberg and Wolfers 2010).

This paper investigates the correspondence of the empirical observations from real-life gambling markets with the experimental results of Allais (1953). Our novel approach is to model risk behaviour by appending Yaari’s (1987) dual theory model with probability weighting by using Prelec’s (1998) function. This approach is feasible because outcomes are inverses of subjective probabilities in pari-mutuel betting, and gamblers bet only a fraction of their wealth, which is a common observation in gambling markets. We use pari-mutuel horse race betting data to carry out empirical tests on gambling behaviour. The empirical testing procedure is closely related to Jullien and Salanié (2000).

Our key result is that decision makers' behaviour in gambling markets replicates the results of the famous Allais' (1953) experiments. Therefore, our findings suggest that the type of behaviour depicted in the Allais paradox can be found in real-life decisions as well. In particular, we find that while bettors are risk-averse, they are also prone to misperceive probabilities by overweighting low probabilities and underweighting high probabilities. This also confirms the findings of Snowberg and Wolfers (2010) with a different approach.

This paper proceeds as follows. Section 2 presents the literature review. In Section 3, we present the theoretical model. Section 4 describes the empirical procedure and data. Estimation results are reported in Section 5. Section 6 links these findings to the Allais experiments. Section 7 discusses the results, and Section 8 concludes the paper.

2. Literature Review

2.1 Allais Paradox

Empirical studies have revealed patterns in choice behavior that are inconsistent with EUT since the 1950s. The Allais common consequence and common ratio paradoxes are the primary departures from EUT. Consider the following hypothetical situations based on Allais' (1953) experiments:

1. Common Consequence (First Experiment):

A_1: Certainty of Receiving 1 million, $\Rightarrow A_1 = (1,\ 1M)$.

B_1: 1% chance of zero,
10% chance of 5 million,
89% chance of 1 million, $\Rightarrow B_1 = (0.01, 0M; 0.1, 5M; 0.89, 1M)$.

A_2: 11% chance of 1 million,
89% chance of zero, $\Rightarrow A_2 = (0.11, 1M;\ 0.89, 0M)$.

B_2: 10% chance of 5 million,
90% chance of zero, $\Rightarrow B_2 = (0.1, 5M;\ 0.90, 0M)$.

Under EUT, the consistent choice patterns are $A_1 > B_1$ and $A_2 > B_2$ or $A_1 < B_1$ and $A_2 < B_2$.

2. Common Ratio (Second Experiment):

A_3: Certainty of Receiving 3000, $\Rightarrow A_3 = (1, 3000)$.

B_3: 20% chance of zero, 80% chance of 4000, $\Rightarrow B_3 = (0.2, 0;\ 0.8, 4000)$.

A_4: 25% chance of 3000,
75% chance of zero, $\Rightarrow A_4 = (0.25, 3000;\ 0.75, 0)$.

B_4: 20% chance of 4000,
80% chance of zero, $\Rightarrow B_4 = (0.2, 4000;\ 0.8, 0)$.

With the same logic as in the first experiment, we expect choice patterns under EUT are $A_3 > B_3$ and $A_4 > B_4$, or $A_3 < B_3$ and $A_4 < B_4$, respectively. However, the majority of subjects choose $A_1 > B_1$ and $A_2 < B_2$ in the first experiment and $A_3 > B_3$ and $A_4 < B_4$ in the second experiment (e.g. Kahneman and Tversky 1979; MacCrimmon and Larsson 1979).

The observed choice patterns, known as the Allais paradoxes, contradict EUT. On the one hand, critics have pointed out that these experiments are not real-life decisions, because they do not provide feedback or a possibility of learning (Binmore 1999; Starmer 1999). On the other hand, using bookmaker data from British horse races, Jullien and Salanié (2000) show in their seminal article that bettors behave as the generalized Allais paradox predicts. In particular, they argue that decision makers display risk aversion towards losses in probability weighting and risk-love in local utility as Friedman and Savage (1948) suggest.

2.2 Dual Theory and Probability Weighting

The most popular explanation for deviations from EUT is probability weighting. The probability weighting function has been used to capture misperception of probabilities. A common assumption is a pattern in which small probabilities are overweighted and large probabilities are underweighted. In general, these theories are called rank-dependent expected utility theories (Quiggin 1982). Probability weighting is also the basis of prospect theory (Kahneman and Tversky 1979; Tversky and Kahneman 1992). Specific parametric forms for probability weighting functions have been proposed in the literature, the most well-known variant being Prelec (1998).

The other contender is Yaari's (1987) Dual Theory (DT), which models attitude towards risk in probability dimension. DT assumes nonlinear probabilities and linear utility of outcomes. This contrasts the assumption of linear probabilities and the nonlinear utility of outcomes in EUT. In particular, DT has a property that the utility is linear in wealth. However, the decision maker in DT does not perceive probabilities in a distorted manner as in probability weighting theories.

Yaari emphasizes that his essay deals with how the perceived risk is processed into choices, not how risk is processed into the perceived risk[1]. Thus his approach does not model misperceptions by using the probability weighting function.

2.3 Theories of Gambling

Gambling markets challenge the assumption that decision makers dislike uncertainty and they are risk-averse by offering an uncertain environment in which the expected game value is negative. There are many explanations to the paradox of gambling. Friedman and Savage (1948) suggest that people are risk-lovers at least at some wealth level. This translates into utility functions that are convex in the middle range of the wealth level and concave elsewhere. This means that bettors move from the convex segment to the concave segment of the utility function by betting a fraction of their incomes. However, this behavior is rarely observed in gambling markets. Most bettors gamble a small fraction of their wealth and therefore, neither winning nor losing takes the bettor far from the initial wealth level (Thaler and Ziemba 1988; Forrest and McHale 2007; Snowberg and Wolfers 2010)[2].

One perspective to betting is that it is motivated by pleasure of gambling rather than being a wealth-oriented exercise as described above.[3] Conlisk (1993) models the pleasure of gambling by appending an expected utility model for a risk-averse bettor with “a tiny utility of gambling”. Thus, gambling markets can be seen as entertainment or consumption, where bettors accept the negative expected value as a fee or price of the gambling enjoyment (Asch and Quandt 1990).

[1] Note also that Yaari’s (1987) purpose was not to model the attitude towards risk in the context of gambling where outcomes are determined by probabilities.
[2] See Sauer (1998) for discussion.
[3] For instance, Samuelson (1952) wrote: “When I go to a casino, I go to not alone for the dollar prizes but also for the pleasures of gaming – for the soft lights and the sweet music”.

Barbaris (2011) explains gambling by applying the probability weighting property of prospect theory in repeated casino games.

Although the decision to participate in a gamble could be explained by the utility of gambling as Conlisk (1993) suggests, bettors behave as if they were risk-lovers while they are gambling (Weitzman 1965; Ali 1977; Jullien and Salanié 2000). This means that they dislike favourites and prefer longshots which are the alternatives with a low winning probability. Since betting on a longshot constitutes a bet that has a greater variance than betting on the favorite, it indicates that bettors behave as risk-lovers. This phenomenon is known as the favourite-longshot bias (FLB), which contradicts the standard assumption of risk aversion. Risk-taking is an explanation for FLB in neoclassical theories at least at some wealth level (Weitzman 1965; Ali 1977). Another explanation suggested by non-expected utility theories is that bettors underweight the winning probabilities of favourites and overweight those of longshots (Jullien and Salanié 2000; Snowberg and Wolfers 2010). Since this systematic behaviour was first noted by Griffith (1949), many other explanations have surfaced (see e.g. Sauer 1998; Ottaviani and Sørensen 2008).

More recently, Gandhi (2008) and Chiappori et al. (2008, 2009) propose an empirical model where the heterogeneity of risk preference explains bettors' behaviour[4]. Their approach relies on the assumption that bettors have an unbiased view of probabilities, but their attitudes towards risk differ across the bettor population. Although most bettors are risk-averse or risk-neutral, a small fraction of bettors are risk-lovers. Consequently, FLB emerges in an empirical setting. These findings are important because they reject the conclusion that risk-loving explains the bettor behaviour in the racetrack betting.

[4] See also Chiappori et al (2009).

It is possible that bettor behaviour is conditional on different factors, such as time, place and race. In practice, this means that the bettors' risk behaviour is not invariant. Winter and Kukuk (2006) attempt to find the effects of last races, track, pool of prize (purse), and turnover of the race on the betting behaviour. Moreover, Sobel and Raines (2003) study the influence of the day of the week, last races, and track. None of these studies manage to find any major effects. These studies employ the same methods to determine the factors that influence bettor behaviour. Differences are detected by comparing the net rates of return in each race rank position between subsets of races.

3. Theory

This paper utilizes the general rank-dependent utility approach and dual theory (Quiggin 1982; Yaari 1987). We follow the formulation of the general rank-dependent utility taken by Diecidue et al (2009) and Zank (2010). We construct an uncertainty function that models the attitude towards risk in probability as suggested by Yaari (1987) and the misperception of probabilities suggested in rank-dependent utility models using Prelec's (1998) two-parameter probability weighting function. Next, we present the general rank-dependent utility model and the uncertainty function model, the differences between the two, and why we favour the latter in our approach.

Let $R=(r_0,...r_n)$ be the outcomes ordered from the worst (r_0) to the best (r_n). A gamble is a finite probability distribution over the set R. Let $P=(p*_0,r_0;...;p*_n,r_n)$ denote the true probability p_j* assigned to the outcome $r_j \in R$ for $j=0,...,n$. Alternatively, gambles can be represented in terms of decumulative probabilities as $P=(p_1,;...;p_n)$, where $p_j=\sum_{i=j}^{n} p_i*$

denotes the probability of getting at least r_j, $j = 1,...n$. To simplify notations, we suppress the outcomes and assign the decumulative probability of the worst outcome r_0 equal to 1.

Let L denote the set of all gambles, in which the preference relation $\succsim$ is assumed to hold over all gambles. As usual, $\succ$ denotes strict preference and $\sim$ indifference. The function V represents the preference relation $\succsim$ over L. That is, V is a mapping from L into the set of real numbers, $\mathbb{R}$, such that for all $P, Q \in L$,

$$P \succsim Q \Leftrightarrow V(P) \geq V(Q).$$

It can be shown that the general rank-dependent utility model holds when the preference relation over gambles satisfies the *weak order*, *monotonicity*, *Jensen-continuity*, and *comonotonic independence*, (see, e.g. Wakker et al 1994 and Diecidue et al 2009)[5]. The comonotonic independence is a weakened form of independence and means that preferences between gambles will be unaffected by the substitution of common outcome as long as these substitutions have no effect on the rank order of outcomes or gambles.

The general rank-dependent utility is represented by the function

$$V(P) = u(r_0) + \sum_{j=1}^{n} w(p_j)[u(r_j) - u(r_{j-1})], \qquad (1)$$

where u is the utility function and the weighting function w is a strictly increasing function from $[0,1]$ to $[0,1]$ with $w(0) = 0$ and $w(1) = 1$. Note that the general rank-dependent utility function becomes the expected utility model when the weighting function is linear and the dual

[5] Zank (2010) shows that the listed properties are necessary but not sufficient.

theory model when the utility function is linear[6]. Typically in the literature, a convex weighting function indicates probabilistic risk aversion (or "pessimism") and a concave function probabilistic risk-love (or "optimism") (e.g. Wakker 1994)[7].

Based on these models, we propose an alternative modelling strategy which is a variant of the general rank-dependent utility, *the uncertainty function model*. This can be written as

$$V(P) = r_0 + \sum_{j=1}^{n} U(p_j)[r_j - r_{j-1}], \qquad (2)$$

where the uncertainty function U is an increasing, continuous mapping $U:[0,1] \to [0,1]$ with $U(0) = 0$ and $U(1) = 1$. The uncertainty function combines both aspects of risk behaviour, i.e. the attitude towards risk and probability weighting. This is the main difference between the uncertainty function approach and the dual theory.

We consider that it is sufficient to model gambling behaviour without the specific utility function of risk u for two reasons. First, bettors' subjective probabilities of winning define returns *r* in pari-mutuel gambling markets (see Section 4.1). Second, most bettors gamble only a small fraction of their wealth. For this reason, one can assume that the utility is linear in wealth, $u(r) = r$ (Thaler and Ziemba 1988; Forrest and McHale 2007; Snowberg and Wolfers 2010). To be precise, the uncertainty function model has no specific measurements for the "utility of risk" and "probabilistic risk" as in the models that assume optimism and pessimism (e.g. Wakker 1994). It contains only a single measure $U(p)$ for the attitude towards risk in the probability

[6] The weighting function is either convex or concave in the dual theory.
[7] Additionally, Gonzalez & Wu (1999) suggest a rank-dependent utility model which contains a utility function for risk and a two-parameter probability weighting function where one parameter measures the discrimination of probabilities and the other measures the attractiveness of gambling.

dimension as Yaari (1987) suggests. The methodological advantage of this approach is that risk aversion and the diminishing marginal utility of the pay-off can be separated from each other, which are synonymous under EUT (Yaari 1987)[8]. Another advantage is that this model provides an opportunity to test risk behaviour empirically with a simple formulation that requires only estimation of a single parameter for the attitude towards risk and another parameter for probability weighting.

To capture risk behaviour $U(p)$, we use the function proposed by Prelec (1998), which is commonly used in the literature[9]:

$$U(p) = \exp\left\{-\beta(-\ln p)^{\alpha}\right\}, \beta, \alpha > 0 \qquad (3)$$

where α measures the (inverse) *S*-shape of the function and β measures its convexity. When $0 < \alpha < 1$, low probabilities are overweighted and high probabilities are underweighted as suggested in the empirical literature. Note, that the special case when $\alpha = 1$ yields the power function $U(p) = p^{\beta}, \beta > 0$ which implies only convexity, i.e. the attitude towards risk. In this case, the power function model is the dual theory model. If $\beta < 1$, $U(p)$ is concave, which implies risk-love. If $\beta > 1$, $U(p)$ is convex, which implies risk aversion. Furthermore, the uncertainty function model in Equation (2) nests the correct perception of probability and risk-neutrality when $\alpha = \beta = 1$.

[8] See also Wakker (1994).

[9] However, Prelec (1998) proposed also a single-parameter model $w(p) = \exp\left\{-(-\ln p)^{\alpha}\right\}$ which has an fixed inflection point at $p = 1/e = 0.37$, i.e. a function that is concave up to a fixed point and convex beyond.

4. Empirical Procedure

4.1 Model

Consider a horserace with $j = 1,...,n$ horses (outcomes) and a gamble in which gamblers are betting on the winner of a race with the pari-mutuel system. For a bet of one euro, a gambler wins return r with the probability p and loses the bet with the probability $(1-p)$. The pari-mutuel system is useful for empirical purposes, because it produces mechanically the market share of each horse (Chiappori et al. 2008). The return r is determined by the pari-mutuel rule, which says that all bets are collected in the pool and redistributed to the bettors who bet on the winning horse excluding the take-out rate τ which is the bookmaker's share to cover his expenses. Hence, the return can be written as

$$r_j = \frac{1-\tau}{w_j} - 1 \qquad (4)$$

where w_j is the proportion of bets b_j on the horse j, i.e., $w_j = b_j \big/ \sum b_j$. Thus, w_j is the bettors' subjective probability of winning.

Note that we observe only the odds $O_j = r_j + 1$, from which w_j can be calculated for each race and horse because probabilities sum to one. We take advantage of this property of the pari-mutuel gambling rule, because we know that returns are defined by the market shares of each outcome which are the subjective probabilities of the representative bettor. Therefore, probabilities and returns are directly "dual" to each other in the pari-mutuel gambling system[10]. Since they also implicitly define returns, all information about bettor behaviour is captured by w_j.

[10] This is exactly what Yaari (1987) meant by his comment: "*I should emphasize that playing games, with probabilities masquerading as payments and payments as probabilities, is not my object*" (pp. 95).

Assume a representative agent who is indifferent in betting between horses in a race[11]. Using the previous notation and the procedures proposed in Jullien and Salanié (2000), the menu offered to bettors in each race is $\mathrm{P} = \{(p_1, r_1), ..., (p_n, r_n)\}$. In equilibrium, $V(p_1, r_1) = \cdots = V(p_n, r_n)$, which means that the expected values are equal for all bets. Assuming a one euro bet, the uncertainty function model can be written as

$$V(p_j, r_j) = U(p_j) r_j + [1 - U(p_j)](-1) \qquad (5)$$

for $j = 1, ..., n.$ Following Chiappori et al. (2008), we normalize the value of a losing bet equal to zero and get

$$V(p_j, r_j) = U(p_j) r_j . \qquad (6)$$

Note that the empirical model is a two-outcome model, which is a special case of the general rank-dependent utility model. Our model was presented as a rank-dependent utility model in the theory section, but it is sufficient to treat it as a two-outcome model for empirical purposes[12]. We can use a dataset of winners to find an uncertainty function that provides the best fit to the empirical dataset. Due to a sufficiently large dataset at our disposal, an estimate for the objective probability p can be computed with the empirical frequency of the event "the horse valued j wins" across the races. The aim is to recover both objective probabilities and the parameters of function $U(.)$ which are in line with the observed w.

[11] Following Jullien and Salanié (2000), we do not model the decision whether or not the bettor should participate in the gamble.
[12] We present the model as a rank-dependent utility model because the Allais paradox needs to be considered as a general rank-dependent utility model as we will see in Section 7.

Let H be the inverse function of $U(p) = w$, which exists because $U(.)$ is continuous and increasing: $H(w) = U^{-1}(w) = p$. A multinomial model with the maximum likelihood method is used to estimate the parameters of $H(w)$. This follows the strategy proposed by Jullien and Salanié (2000). Furthermore, we use race-specific variables to explain the risk parameter of the power function. A related approach is introduced in Andersen et al. (2008) and Harrison and Rutstöm (2008). Compared to the procedures employed in the previous studies (e.g. Sobel and Raines 2003), which require categorizing each rank variable, calculating the net rate of return, and comparing it to all net rates of return, our method involves less calculations.

4.2 Estimation

Each race c has n_c horses. Since the winning horse is known, our data is constructed such that the dependent variable y_j takes the value 1 if the horse j wins and 0 otherwise. Different winning horses correspond to the probability that the horse j wins, which is drawn from the multinomial distribution by

$$p_j = \Pr[y_j = 1] = \frac{H(w_j)}{\sum_{l=1}^{n} H(w_l)}, \qquad j = 1, \dots, n. \qquad (7)$$

The multinomial density for a race can be conveniently written as

$$f(y) = p_1^{y_1} \times \cdots \times p_n^{y_n} = \prod_{j=1}^{n} p_j^{y_j}. \qquad (8)$$

The model for the probability that in each race c the horse j wins is

$$p_{cj} = \Pr[y_{cj} = 1] = F_j(\mathbf{x}_c, \beta), \quad j = 1, \dots, n, \quad c = 1, \dots, C, \qquad (9)$$

where β is a parameter and $\mathbf{x}_i$ are the regressors of the race-specific variables (e.g. turnover, last race).

Since we know the horse that won the race, and given that the races are independent from each other, the log-likelihood function for the sample is

$$\ln L(\beta) = \sum_{c=1}^{C} \ln p_{cj} \,, \tag{10}$$

where $p_{cj} = F_j(\mathbf{x}_c, \beta)$ is defined in (9) (for more details, see Jullien and Salanié 2000). Maximizing the likelihood function with respect to β gives us an estimator $\hat{\beta}$ that is consistent, asymptotically normal, and asymptotically efficient. Furthermore, we allow the parameter β to depend on race-specific variables (Andersen et al. 2008; Harrison and Rutström 2008).

4.3 Data Description

Our dataset was obtained from Suomen Hippos (the Finnish Trotting and Breeding Association). Suomen Hippos has a legal monopoly to organize pari-mutuel betting in horse (harness) racing in Finland. Every race hosts a gambling menu that covers the traditional win bet and exotic bets which are combinatorial bets that allow betting on multiple horses in a different order. We use win bet data in this study. In win bet, a bettor tries to pick a horse that wins the race. Our dataset indicates that the tote's take-out rate was 21% on average in the win bet.

Overall, the data set includes the win bet information on horse races in Finland from September 27 2004 to July 19 2007. There are 43 racetracks in Finland. Along on-track betting, off-track and Internet betting is possible on every race in Finland. The main track in the country, Vermo in Helsinki, hosts over 60 race events every year. The outliers were dropped out of the data. These were the races in which the take-out rate was obviously too high or low (mistakes in the records).

Altogether, the data consist of 15,973 harness races, and the total number of horses included is 190,457. Thus, our data contains odds for horses in every race, information on the exact finishing order of the horses in each race, and other race-specific variables, such as the number of horses, the race number in a day, track, turnover, date[13], purse, and the race distance.

5. Results

5.1 Bettors' Preferences

To make the estimation procedure clear, we describe the representative bettor's case as precisely as possible. Assume that the bettor's attitude towards risk is captured by a simple power function $U(p) = p^{\beta}$ introduced in Yaari's (1987) dual theory. A convex function $(U'' > 0)$ implies risk aversion, and a concave function $(U'' < 0)$ risk-love[14]. As noted earlier, we observe that $U(p) = p^{\beta} = w$, and we have

$$H(w) = w^{1/\beta} = p\,. \qquad (11)$$

Table 1 reports the results from maximizing the log-likelihood function (10) with (11). The estimate for β is 0.885 (with the standard error of 0.0072) which is statistically significant. Since the function is concave, bettors appear to be risk-lovers as expected. This finding implies FLB, and it is supported by the empirical evidence in the literature (Weitzman 1965; Ali 1977; Jullien and Salanié 2000).

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[13] On the basis of date, we can separate weekday races from weekend races.
[14] In general, if utility is convex in the outcome space, it corresponds to concave utility in the probability space and *vice versa* (Yaari 1987).

Next, we include the misperception of probabilities to the framework with the function proposed by Prelec (1998)

$$U(p) = \exp\left[-\beta(-\ln p)^{\alpha}\right], \qquad (12)$$

which nests the power function for $\alpha = 1$. When $0 < \alpha < 1$, it reflects non-linearity in bettors' probabilities such that low probabilities are overweighted and high probabilities are underweighted. If $\alpha = 1$, bettors' probability judgements are correct. This means that all aspects of behaviour are characterized by the attitude towards risk. On the other hand, if $0 < \alpha < 1$ it can be assumed that the probability weighting affects gamblers' behaviour. As a result, we write

$$H(w) = \exp\left[\left(\frac{-(-\ln w)^{1/\alpha}}{\beta}\right)\right]. \qquad (13)$$

The results of maximizing the log-likelihood function (10) with (13) are reported in Table 1 with the earlier findings from (10) with (11).

Table 1. Estimates of Weighting Function Parameters.

Parameter	Estimate	Std. Error	Log-likelihood
Dual Theory (Power Utility)			
β	0.885	0.0072	-29912.379
Uncertainty Function (Two Parameter Prelec)			
β	1.132	0.0545093	-29902.805
α	0.862	0.0275371	

The estimated $\hat{\alpha}$ is 0.862 and statistically significant. As the probability weighting parameter α is less than unity, it indicates that the representative bettor overestimates low probabilities and underestimates high probabilities. Moreover, the log-likelihood is higher for the two-parameter function as well. The value of the likelihood ratio test is 19.1 (with one degree of freedom) which favours the richer uncertainty function specification. The most interesting result, however, is that now the risk parameter β, which is larger than unity, implies risk aversion even in the representative bettor's case. More precisely, once the misperception of probabilities is included in the model, the representative bettor is risk-averse[15]. These results are in line with the current findings of Gandhi (2008) and Chiappori et al (2008), who argue that bettors are not risk-lovers, and Snowberg and Wolfers (2010), who argue that probability weighting explains bettors' behaviour.

5.2 Covariates

Next we investigate whether race-specific covariates in the data affect the risk behaviour. To find out the race-specific effects, we made several estimations with the variables mentioned in Section 4.3. If the risk parameter β is affected by these variables, this is captured by

$$\hat{\beta} = \hat{\beta}_0 + \hat{\beta}_1 x_{c1} + ... + \hat{\beta}_k x_{ck}. \qquad (14)$$

Estimates of the power function are reported in Table 2[16]. Note that we tested several other variables, e.g. the day of the week, two last races, number of horses, racing distance and so on. Some potential sources of heterogeneity are reported in Table 2.

The most important finding is that the race-specific covariate effects are marginal. First, the last race effect does not exist. This contradicts the earlier studies (e.g. Ali 1977) which argue that bettors take risk more aggressively in the last races because they want to recover their losses due

[15] As a robustness check, we also estimated a two-parametric function alternative suggested by Lattimore et al. (1992). The results were qualitatively same with the ones obtained with Prelec (1998).
[16] Using the probability weighting parameter α did not make any qualitative difference to the results.

to loss aversion suggested by Kahneman and Tversky (1979)[17]. Second, betting behaviour does not change between weekends and weekdays. So, even though the population of bettors may differ during weekdays (e.g. professionals or gambling addicts) and during weekends (e.g. amateurs or entertainment gamblers), their attitude towards risk remains unchanged.

Table 2. Results of Estimation with Race-specific Covariates.

Covariate	Estimate	Std. Error	*p*-value	Log-likelihood
	Power Function			
β_0	0.871633	0.018398	0.000	-29907.63
Turnover	0.000017	0.000012	0.130	
Weekend	0.009113	0.016185	0.573	
Last race	0.022889	0.025698	0.373	
Purse	-0.000002	0.000001	0.071	
Vermo track	-0.046202	0.022561	0.041	
Small track	0.039058	0.030931	0.207	
Time (trend)	0.000002	0.000025	0.947	

Next we tried to capture the level of market information by using variables such as turnover, purse and track size. We assume that a high purse in a race indicates that horses are familiar to bettors and more successful horses are likely to race in large tracks. In addition, the turnover of a race may represent the general level of interest towards a race. However, our data does not support these views. Betting behaviour is more aggressive in the largest track, Vermo, and the races that have higher purses. Furthermore, the turnover of a race is not significant. Finally, we attempt to approximate the influence of off-track betting. Since we know that the off-track betting has increased during the sampling period, we use a time trend to capture this effect. However, no effect was found for this.

[17] This conclusion has been challenged also by Snowberg & Wolfers (2010).

Overall, our results suggest that bettors' behaviour is invariant in time and the scale of betting. All that matters is the attitude towards risk and uncertainty. Furthermore, bettors' attitude towards risk has been observed to be universal with some exceptions. Another universally observed behavioural phenomenon is the Allais paradox. Therefore, we test whether these two deviations from the neoclassical theory are actually the same thing.

6. Allais Paradox Revisited

Recall, from Section 2.1, that the common consequence effect indicates the preference relation between gambles such that $A_1 = (1,\ 1M) \succ B_1 = (0.01, 0M;\ 0.89, 1M;\ 0.1, 5M)$, and $A_2 = (0.11, 1M;\ 0.89, 0M) \prec B_2 = (0.1, 5M;\ 0.90, 0M)$. If the previous gambles are written as (de)cumulative distributions over outcomes[18], then the choice between A_1 and B_1 is $1M$ or $U(0.99)1M + U(0.1)5M - U(0.1)1M$, and the choice between A_2 and B_2 is $U(0.11)1M$ or $U(0.10)5M$. Now, assume two inequalities $A_1 > B_1$ and $A_2 < B_2$ (Allais paradox). After some manipulation it follows that

$$1 - U(0.99) > U(0.11) - U(0.10). \qquad (15)$$

Next, we substitute the estimated parameters from Table 1 into the uncertainty function (12), and use the resulting weighted probabilities to compute (15). This yields

$$1 - 0.979 > 0.106 - 0.098 \Leftrightarrow 0.021 > 0.008,$$

which satisfies the inequality (15). So, the empirical findings from gambling markets with the uncertainty function approach satisfy the preference relation of the common consequence effect.

[18]This is, in general, the rank-dependent utility. For an illustrative generalisation of experiments of the Allais paradoxes and the rank-dependent utility, see Peel et al (2008).

The common ratio effect implies the preference relations

$$A_3 = (1, 3000) \succ B_3 = (0.2, 0; 0.8, 4000)\text{, and}$$

$$A_4 = (0.25, 3000; 0.75, 0) \prec B_4 = (0.2, 4000; 0.8, 0)\,.$$

Under the uncertainty function, the choice between A_3 and B_3 is 3000 or $U(0.8)4000$, and A_4, and B_4 is $U(0.25)3000$ or $U(0.2)4000$, respectively. The observed common ratio effect gives us the preference patterns $A_3 > B_3$ and $A_4 < B_4$, which can be written by the inequality

$$U(0.8) < \frac{3}{4} < \frac{U(0.2)}{U(0.25)}. \tag{16}$$

As before, we use the estimated parameters from Table 1 in equation (12) to compute (16). This yields $0.733 < 0.750 < 0.814$, which is consistent with (16). Again, the real-life decisions reflect the same preference relation patterns that were observed in the experimental literature over fifty years ago[19].

7. Discussion

In this paper, we followed closely the model proposed in Yaari's (1987) dual theory. The difference between our model and the dual theory is that we included the probability weighting in the model. This was done by using Prelec's (1998) two-parameter weighting function. In consequence, risk is modelled in probability as the dual theory suggests, but probabilities are weighted as prospect theory and non-expeceted utility theories suggest. Two reasons support this

[19] Note that the approach satisfies also other variants of these experiments. For instance, Kahneman and Tversky (1979): $A_1 = (1, 2400) \succ B_1 = (0.01, 0; 0.33, 2500; 0.6, 2400)$ and $A_2 = (0.34, 2400; 0.66, 0) \prec B_2 = (0.33, 2500; 0.67, 0)$, and $A_1 = (0.1, 0; 0.9, 3000) \succ B_1 = (0.45, 6000; 0.55, 0)$ and $A_2 = (0.002, 3000; 0.998, 0) \prec B_2 = (0.001, 6000; 0.999, 0)$

approach. First, subjective probabilities and returns are dual to each other in pari-mutuel betting markets. Second, there is no need to model the utility of wealth because bettors gamble only small fractions of their wealth and thus, the utility of wealth can be considered linear.

Our findings have a key difference to those obtained in Jullien and Salanié (2000). While their results suggest that bettors are risk-lovers at least locally, we find that bettors are risk-averse. In consequence, we argue that the misperception of probabilities leads to FLB, not the risk-loving attitude towards risk. This result is in line with the propositions laid out in prospect theory and the empirical findings of Snowberg and Wolfers (2010). The difference between our findings and Jullien and Salanié (2000) could lie in the applied functional forms[20]. In our approach, the utility of wealth is linear, whereas the utility of wealth is nonlinear in their approach.

It must be noted that we cannot reject the conclusion that heterogeneity explains FLB as Gandhi (2008) and Chiappori et al. (2008) suggest. However, Gandhi and Chiappori et al. cannot explain the Allais Paradox because they assume that the majority of bettors are risk-averse. In contrast, our model explains the Allais Paradox when the estimated results are substituted to the preference relations laid out in the Allais Paradox. Also Jullien and Salanié (2000) use their results to explain the generalized Allais Paradox. They assume that bettors are risk-lovers and use a separate probability weighting function for wins and losses to obtain the result. Our result, on the other hand, is based on risk-averse bettors, which is a general assumption in the standard economic decision making theory, and weighted probabilities which have been observed in behavioural economics.

[20] Also, our data differed from Jullien and Salanié (2000). They used bookmaker data, while our data is pari-mutuel betting data.

Finally, this paper's methodological contribution is to offer a simpler computational procedure to the one proposed by Jullien and Salanié (2000). The subjective probability summarizes the bettor's behaviour. When risk and probability weighting is estimated in the same function, this simplifies the computational burden considerably. Moreover, covariates can be included in the model, although in our case they were not statistically significant.

8. Concluding Remarks

Our model shows that decision-makers behave in gambling markets as proposed in the previous experimental literature. Our key finding is that gamblers are risk-averse, but at the same time, they are prone to misperceive probabilities by overweighting low probabilities and underweighting high probabilities. This explains both the favourite-longshot bias and Allais paradox. In addition, we proposed that individuals' behaviour is invariant in time and the scale of betting at the aggregate level. That is, decision-makers have the same attitude towards risk regardless of other factors.

In conclusion, we provide empirical support from real-life gambling markets to the findings of the experimental literature of decision making under risk and uncertainty. Our results indicate that it is not necessary to reject the assumption that decision makers are risk-averse in gambling markets. This contradicts the commonly used explanation of risk-love to rationalize gambling behaviour.

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