Alternation Bias and Reduction in St. Petersburg Gambles: An Experimental Investigation*

Kim Kaivanto†

Lancaster University, Lancaster LA1 4YX, UK

Eike B. Kroll

Otto-von-Guericke University Magdeburg, 39104 Magdeburg, Germany

this version: April 30, 2012

Abstract

The Reduction of compound lotteries is an implicit assumption both in the statement of the St. Petersburg Paradox as well as in its resolution by Expected Utility (EU). Yet despite the pivotal role of this assumption, to date there has been no empirical substantiation of its validity. Here we report three real-money experiments in which the standard compound-lottery form of the (truncated) St. Petersburg Gamble is explicitly juxtaposed with its reduced form. In the first experiment, we elicit Subjects' Certainty Equivalents for each form of the gamble. In the second experiment, Subjects choose between reduced and compound forms within a multiple price list format, where a different sure payment (in €1 increments), is added either to the reduced or the compound form. With this instrument, we can test for both 'weak-form' and 'strong-form' violations of Reduction. The third experiment replicates the second and then checks for robustness against range and increment manipulation. In the first experiment we find that the Certainty Equivalent of the compound form is stochastically dominated by, and significantly smaller than, the objectively equivalent reduced form. This bias toward the reduced form is borne out in the second and third experiments, where 90%–100% display weak-form violation and 48%–87.5% display strong-form violation. These results are consistent with the operation of *alternation bias*, which may be understood as a subjective distortion of conditional probability. Together these experiments offer evidence that the Reduction assumption may have limited descriptive validity in St. Petersburg Gambles.

Keywords: St. Petersburg paradox, reduction of compound lotteries axiom, alternation bias, law of small numbers, test for indifference

JEL classification: D81, C91

†tel +44(0)1524594030; fax +44(0)1524594244; e-mail k.kaivanto@lancaster.ac.uk

1 Introduction

For 299 years, the St. Petersburg Paradox has hinged on an implicit assumption that has become so deeply imbedded in the mathematics and economics professions that it is commonly deployed without any perceived need for separate justification. The assumption is that, for the purpose of modeling choice, compound lotteries may be reduced to their probabilistically equivalent simple lotteries "whose prizes are all the possible prizes of the compound lottery ticket, each evaluated with the compound probabilities that the classical algebra of probability defines" (Samuelson, 1952, p. 671). Within the program to formalize Expected Utility (EU), this assumption was made explicit and instated as an axiom of rational preferences: the Reduction of Compound Lotteries Axiom,[1] by which rational decision makers are required to be indifferent between a multi-stage *compound lottery* and its probabilistically equivalent 'collapsed' single-stage *simple lottery*.

Reduction is implicit both in the statement of the St. Petersburg Paradox as well as in its resolution. The statement of the paradox hinges on powers i of the payoff kernel \$2 precisely off-setting corresponding degrees i of compounding the probability of Heads $\frac{1}{2}$ for all $i \in \mathbb{Z}_{++}$, yielding an infinite sum. Daniel Bernoulli's resolution, just as the modern resolution by EU, also implements Reduction[2] in specifying the probability of heads on the i^{th} toss as $P_i(H) = \left(\frac{1}{2}\right)^i$ $\forall\, i \in \mathbb{Z}_{++}$.

Nevertheless the experimental literature on probability perception suggests that *alternation bias* – a subjective distortion of conditional probability in binary sequences – should be empirically relevant for St. Petersburg Gamble (StPG) coin-toss sequences. Indeed specifically coin-toss sequences have been investigated in many 'perception of randomness' experiments (e.g. Rapoport and Budescu, 1997; Kareev, 1995; Budescu, 1987; for a review see Bar-Hillel and Wagenaar, 1991). Purely from a theoretical standpoint, augmentation of mathematical expectation with alternation bias is sufficient *by itself* to ensure that Willingness To Pay (WTP) for the StPG is finite and within the generally accepted empirical range (Kaivanto, 2008). Moreover, insofar as alternation bias is manifest as the subjective attribution of negative autocorrelation to objectively memoryless and unbiased Bernoulli processes, it suggests that Reduction may not hold empirically for StPGs.

[1] von Neumann and Morgenstern (1947) refer to it as Axiom 3:C:b (p. 26).

[2] implicit in the former, explicit in the latter

The present paper addresses the as-yet untested empirical question, *Is Reduction violated in StPGs?* The experimental design developed for this purpose incorporates several novel features. Firstly, as is pertinent given the emphasis on sequential effects, the lotteries in this experiment are *truncated* real-money StPGs.[3] Secondly, we introduce reduced-form StPGs based on a single draw from an Urn. And thirdly, we introduce a new multiple price list instrument. By design, this instrument permits simultaneous investigation of (i) weak-form violation of Reduction, (ii) strong-form violation of Reduction, and (iii) heterogeneity in the magnitude of subjects' alternation bias.

In total we report three real-money experiments in which the standard compound-lottery form of the (truncated) StPG is explicitly juxtaposed with its reduced form. The former is implemented with a coin toss sequence consistent with convention, while the latter is implemented with a single random draw from a probabilistically equivalent urn. In the first experiment, we elicit Subjects' certainty (cash) equivalents for each form of the gamble. In the second experiment, Subjects face a multiple price list consisting of 11 choices between the compound and the reduced form of the gamble, where each choice task has a different distinct sure fixed payment added to one of the alternatives. The list starts with €5 added to the reduced form (urn) and ends with €5 added to the compound form (coin), changing in increments of €1. This configuration offers a test of 'strong-form' violation of Reduction, in which Subjects reveal with real-money choices whether they violate Reduction, and if so, how much they explicitly forgo in doing so. In the third experiment, we (a) replicate the results of the second experiment with a larger sample, and then (b) investigate possible range and increment effects with an 11-item price list that ranges from €1 added to the reduced form to €1 added to the compound form in increments of 20 Euro cents.

In the first experiment we find that the distribution of certainty equivalents for the reduced form stochastically dominates the distribution of certainty equivalents for the compound form. Within subjects, the reduced form's certainty equivalent is statistically significantly larger than that of the objectively identical reduced form.[4] Therefore we conclude that Reduction is violated in this 'judged valuation' task, revealing a bias toward the reduced form. This bias is borne out in the second experiment. When choice is costless – i.e. in a straight choice between the reduced form and the compound form – 90% of the Subjects choose the reduced form

[3]Previous experiments have employed finite but not *truncated* StPGs; see Section 5.

[4]The effect size – i.e. mean difference – is yyyyyy.

over the compound form. This constitutes a weak-form violation of Reduction in which the null hypothesis is premised on strict adherence to Expected Utility and thus the absence of 'secondary criteria' influencing choice (Bernasconi and Loomes, 1992). Furthermore, 45% of the Subjects forgo a sure €1 added to the compound form in order to obtain the reduced form.[5] These 45% violate Reduction in the strong-form sense, whereby a distinct preference for the reduced form is expressed through choice which involves giving up a certain €1. Part (a) of the third experiment replicates this result with twice the number of Subjects.[6] Again 90% choose the reduced form when choice is costless, while 47.5% give up a certain €1 to obtain the reduced form instead of the compound form. In part (b), a full 100% choose the reduced form over the compound form when it is objectively costless to do so (weak-form violation of Reduction), and 87.5% choose to forgo 20 Euro cents to obtain the reduced form rather than the objectively equivalent compound form (strong-form violation of Reduction).

In each of these experiments, both the rejection of the EU-based null hypothesis as well as the direction of this departure are consistent with the operation of *alternation bias*. This is a distortion of conditional probability, distinct from distortion of outcomes[7] and distortion of unconditional probabilities.[8] By design, the present experiments preclude the possibility that the observed choice behavior may be due to distortion of outcomes (e.g. risk aversion) or distortion of unconditional probabilities (e.g. probability weighting). Altogether, these experiments provide evidence that the Reduction assumption (Axiom) may have limited descriptive validity in St. Petersburg Gambles. These results carry implications for both the demonstration of the St. Petersburg Paradox as well as for its resolution, each of which invokes Reduction without separate justification.

2 Reduction of compound lotteries

The Reduction of compound lotteries – sometimes stated as an assumption, sometimes as an axiom – is present both in classical EU as well as in modern behavioral theories such as Cumulative Prospect Theory (CPT). Some formalizations stipulate that simple one-stage lotteries are the basic objects of choice to which theory applies, and that multi-stage compound lotteries

[5]Conversely, none of the subjects is willing to forgo a sure €1 added to the reduced form in order to obtain the compound form.

[6]40 Subjects rather than 20.

[7]Bernoulli's 'moral worth' and concave utility of money

[8]Yaari's dual theory and Prospect Theory's probability weighting

are ‘reduced’ into such simple lotteries through algebra alone (Samuelson, 1952; Hauser, 1978). Other formalizations require the decision maker to be indifferent between a multi-stage compound lottery and its probabilistically equivalent simple one-stage lottery (e.g. Harrison et al., 2012). von Neumann and Morgenstern (1947) denote it as Axiom 3:C:b and describe it as an expression of the ‘algebra of combining’ (p. 26). In parts of the EU literature the axiom is known by this latter label (Aumann, 1962; Fishburn, 1978). Luce and Raiffa (1957) instead invoke it as an assumption rather than as an axiom (Assumption 2, p. 26).

The modern Prospect Theory literature has adopted a variety of different measures for compound lotteries (prospects). All but one particular functional form of the probability weighting function leads to violation of Reduction; different implementations of Prospect Theory have finessed this in different ways. Original Prospect Theory simply excluded compound prospects from consideration by restricting the domain of representable preferences to simple prospects (Kahneman and Tversky, 1979). In a subsequent exploratory investigation by the same authors, Reduction was found to be violated by the certainty effect and the pseudo-certainty effect focusing attention on the second-stage prospect (Tversky and Kahneman, 1981). Alternatively, the Reduction of Compound Prospects Assumption can be invoked as a theoretical requirement, much as it is within EU (Wakker, 2010, p. 60). Finally, the reduction of compound prospects can be facilitated by restricting the form of the probability weighting function to that particular parameterization of the Prelec two-parameter class (Prelec, 1998) which admits the reduction of compound prospects (Prelec, 2000; Luce, 2001).[9]

Numerous theoreticians, past and present, view Reduction as a very strong assumption. Hauser (1978) expresses the view that it is “perhaps the stongest assumption in the utility axioms.” Fishburn (1978) notes that “...many people exhibit systematic and persistent violations of... ...the reduction or invariance principle which says that preference or choice between acts depends only on their separate probability distributions over outcomes” (p. 492).

Numerous experimental studies have investigated various aspects of Reduction (Tversky and Kahneman, 1981; Keller, 1985; Bernasconi and Loomes, 1992; Halevy, 2007). Recently, Kaivanto and Kroll (2012) find *weak-form* violation of Reduction in real-money choices between probabilistically equivalent compound (two-stage) and simple (single-state) lotteries offering a

[9]Interestingly, Blavatskyy (2005) employs a limit approximation of the Tversky-Kahneman weighting function that satisfies Reduction. It is not clear from the context where this is an intended objective of employing the limit approximation or whether it is a fortuitous coincidence.

1-in-10 chance of €100 ($136). This experiment, which includes control treatment for ratio bias and computation costs, finds that 80% of subjects violate Reduction, consistent with 'negative recency', which in the binary sequence context is known as 'alternation bias'. Nevertheless, according to the most recent study of Reduction by Harrison et al. (2012), such violations of Reduction should be mere artifacts of the Random Lottery Incentive scheme. Harrison et al. (2012) find that in choices between compound and reduced forms that are 'played out and paid out' for real money in 1-in-1 cases – i.e. in each case individually – the violation of Reduction disappears. However, Kaivanto and Kroll's (2012) experiments find substantial (weak-form) violation of Reduction even though they play out and pay out 1-in-1 cases.

3 Alternation bias

Alternation bias is a binary sequence manifestation of the local representativeness effect, whereby the population (or infinite limit) properties of a stochastic process are attributed, erroneously, to small finite samples. The local representativeness effect was introduced into the economics literature by Rabin (2002) as the *law of small numbers*, whereby people misjudge and "exaggerate how likely it is that a *small* sample resembles the parent population from which it is drawn" (p. 775).

For finite Bernoulli sequences generated by an objectively fair and memoryless coin, the local representativeness effect leads a Subject to expect, within finite sequences, (i) close to a 50%–50% balance between Heads and Tails, and (ii) excessive local irregularity, i.e. too many reversals between Heads and Tails. It is this latter subjective predisposition to expect too many reversals that we call *alternation bias.* This may be understood more formally as a negatively distorted conditional subjective probability belief, or alternatively as an alternation rate that is subjectively upward-distorted $P_S(H|T) = P_S(T|H) > 0.5$.

Alternation bias was first hypothesized by Reichenbach (1934), and it has been amply documented and replicated in the 'perception of randomness' experimental literature (see summary in Bar-Hillel and Wagenaar, 1991). Experimental studies place the magnitude of first-order alternation bias at $P_S(H|T) = P_S(T|H) = 0.6$ (Budescu, 1987; Bar-Hillel and Wagenaar, 1991; Kareev, 1995). This forms a lower bound, as alternation bias effects have been estimated up to sixth order (Budescu, 1987). Within economics there are multiple studies, using both observational and laboratory data, that substantiate and replicate local representativeness, alternation bias and their manifestations the Gambler's Fallacy and the Hot Hand effect (Asparouhava et al., 2009; Clotfelter and Cook, 1993; Terrell, 1994, 1998; Croson and Sundali, 2005).

Alternation bias holds clear implications for preferences between compound lotteries and their probabilistically equivalent reduced-form lotteries. *Ceteris paribus*, Subjects whose perception of randomness is characterized by alternation bias will not be indifferent between the compound form of a lottery (where alternation bias is operative) and its reduced-form equivalent (where there is no sequential structure to trigger alternation bias). In other words, the *indifference* between reduced- and compound-form lottery variants stipulated by Reduction is predicted to be violated under alternation bias.

Moreover, specifically for StPGs, alternation bias carries implications for mathematical expectation embodying this subjective distortion.[10] Let $\tilde{n} \in \mathbb{Z}_{++}$ be the (random) index of the first toss on which a fair coin first turns up 'Heads'. As $\tilde{n}$ is characterized by the geometric distribution with parameter $p = \frac{1}{2}$, the $n = 1, 2, ...$ stage probabilities are $p_n = \frac{1}{2}(1 - \frac{1}{2})^{n-1} = 2^{-n}$. Based on first-order alternation bias alone, $P_S(H|T) = 0.6$ and $P_S(T|T) = 0.4$, so the subjective (distorted) probability of the coin landing 'Heads' for the first time on toss n becomes

$$p_n^{fo} = \begin{cases} P_S(H) = \frac{1}{2} & \text{for } n = 1 \\ \frac{1}{2} P_S(H|T) P_S(T|T)^{n-2} = 0.3 \cdot 0.4^{n-2} & \text{for } n \geq 2 \end{cases} \tag{3.1}$$

giving a subjectively distorted mathematical expectation of

$$E_S^{fo}(G_{StP}) = \sum_{n=1}^{\infty} p_n^{fo} 2^n = 7.0 \tag{3.2}$$

without any need to invoke risk aversion or unconditional probability weighting.

4 Hypothesis development: weak-form and strong-form violation

For the purpose of formal testing, the Reduction assumption taken in isolation is insufficient for deriving a null hypothesis concerning choice behavior.[11] Reduction may only be tested as part of a joint hypothesis, ideally derived from an axiomatic theory of choice. In the present context, the natural candidate is EU, which explicitly incorporates Reduction as an axiom and constitutes the modern counterpart to Bernoulli's 'moral worth' solution of the St. Petersburg Paradox.

[10] First pointed out in Kaivanto (2008).

[11] Additional assumptions, such as completeness and monotonicity, are required.

The Reduction Axiom of EU stipulates that compound lotteries are evaluated as their equivalent single-stage reduced forms, and thus that indifference holds between compound lotteries and their reduced-form simple-lottery counterparts. However EU does not provide explicit guidance for choice when indifference holds. Any choice from among lotteries judged to be indifferent is consistent with EU.[12] Where indifference holds between two lotteries, the decision maker loses no utility regardless of which lottery he chooses; choice is 'costless'.

Nevertheless it is not the case that EU places *no* restrictions on choice between lotteries for which the indifference relation holds. EU restricts attention to lottery payoffs and probabilities. Under EU, no other characteristics are admitted as being choice-relevant. Moreover, under the Reduction Axiom, the only attributes to be legitimately (rationally) consulted in making a choice are the probabilities and payoffs of the reduced-form, single-stage, simple lottery. Under EU, preference is independent of all distinctions[13] between the compound form and its probabilistically equivalent reduced form.

This extends even to the difference in 'complexity' between compound and reduced forms. Hence augmentation of EU with a further lexicographic criterion – albeit potentially an intuitively appealing rationalization of any revealed bias toward the reduced form – is in fact formally incompatible with the strictly theoretical formulation of EU. In view of this strict and pure theoretical interpretation, lexicographic biasing of choice toward either the reduced form or the compound form would constitute a violation of EU. Given the stringency of the theoretical assumptions being maintained, we refer to this as a *weak-form* violation of Reduction. Null hypotheses for tests of weak-form violation of Reduction stipulate symmetry of empirical choice frequencies between compound-form and reduced-form lotteries.

Following Vernon Smith's precepts for valid microeconomic experiments (induced value theory), we must nevertheless recognize that Subjects face numerous costs in supplying the null hypothesis response instead of the alternative hypothesis response (Smith, 1982; Harrison, 1994). These costs, the aggregate of which we denote with the symbol δ, include the cost of cognitive effort, concentration, fighting distraction or boredom, and the effect of other components of the Subject's utility that are higher under the alternative hypothesis than under the null hypothesis. Moreover, specifically when a Subject is indifferent between alternatives, "*secondary criteria* may be quite important and apparently small or seemingly irrelevant changes to the framing

[12]As an example, note that Nash equilibrium in mixed strategies exploits this property.

[13]including e.g. complexity and compoundness itself

of decisions (e.g. positioning of words) may have a marked effect" (italics added, Bernasconi and Loomes, 1992). To overcome these costs and secondary criteria, the *dominance precept* requires that "the rewards corresponding to the null hypothesis are perceptively and motivationally greater [by at least δ] than the rewards corresponding to the alternative hypothesis" in order to "overcome any costs (e.g. the psychic cost of effort or of concentration) or components of the Subject's utility that might induce a response that is not in accordance with the null hypothesis" (Harrison, 1994).

As observed by Chernoff (1954), Bernasconi and Loomes (1992), and Mandler (2005), EU in fact offers a crisp prediction when a fixed sure bonus $x \in \mathbb{R}_{++}$ is added to one of the lotteries. Due to the monotonicity axiom of EU, such a sure bonus x, no matter how small, causes the indifference relation to be replaced by strict preference for the bonus-augmented lottery. Hence adding a fixed bonus x to either the compound form or the reduced form causes EU decision makers to choose the bonus-augmented option. The formal experimental design property of *dominance* is satisfied when $x > \delta$. But as the value of δ is unknown and Subject-specific, a range of values of x may be employed. Where larger values of x lead to lower rates of deviation from the EU prediction – both where x augments the compound-form lottery as well as where x augments the reduced-form lottery – any related *weak-form* violation of Reduction may be ascribed to the failure to satisfy *dominance*. However, where choice is consistent with EU in bonus-augmented reduced forms but inconsistent with EU in bonus-augmented compound forms – or vice versa – this asymmetric configuration constitutes a *strong-form* violation of Reduction. Here larger values of x no longer offset larger values of δ,[14] but instead offset the effects of more extreme degrees of alternation bias.

5 Materials and methods

Truncated StPGs The experiments reported here employ truncated, as opposed to merely finite, StPGs. Cox et al. (2009), for instance, employ finite StPGs that pay nothing in the outcome where all coin tosses in the sequence land 'Tails'. Since, in the unrestricted StPG, the player's payoff increases the *longer* the run of Tails, it is important to recognize that if the i^{th} toss lands Tails, extending a run of $i-1$ Tails by one, the player is entitled to a *minimum payout*

[14] If δ were being offset, there would be no reason for the cost of cognitive effort, concentration, and fighting distraction or boredom to be different for bonus-augmented reduced forms than for bonus-augmented compound forms.

of €2^{i+1}. We employ StPGs that are truncated at $k = \{2, 6\}$ tosses. In the $k = 2$ case a run of 2 Tails pays off €2^3, while in the $k = 6$ case a run of 6 Tails pays off €2^7. In this sense the StPGs employed here are proper *truncations*.

Reduced form StPG Denote the 'probability of landing Heads for the first time on toss n' in an StPG truncated to k tosses ($n \leq k$) as $p_{n(k)}$. The vector of probabilities of landing Heads for the first time on toss $n = (1, 2, ..., k)$ then becomes $\boldsymbol{p}_{(k)} = (p_{1(k)}, p_{2(k)}, ..., p_{k(k)})$. Thus the probabilities of the $k+1$ possible payoffs $(2^1, 2^2, ..., 2^k, 2^{k+1})$ in the k-truncated StPG may be written as $\boldsymbol{p}_{(k,1)} = (p_{1(k)}, p_{2(k)}, ..., p_{k(k)}, p_{k+1(k)})$. Therefore the $k = 6$ truncated StPG is characterized by the probability vector $\boldsymbol{p}_{(6,1)} = \left(\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}, \frac{1}{64}, \frac{1}{64}\right)$. Here $p_{6(6)} = p_{7(6)} = \frac{1}{2^6} = \frac{1}{64}$, so the $k = 6$ reduced form StPG may be implemented with an urn containing $2^6 = 64$ balls. In general the k-truncated StPG may be implemented with an urn containing no fewer than 2^k balls.

Multiple-price list The multiple-price list developed here subsumes a 'costless choice' weak-form violation of Reduction test and five incrementally more costly strong-form violation of Reduction tests, which do double duty in recording across-subjects heterogeneity of alternation bias strength corresponding to e.g. the magnitude of $P_S(H|T)$.

The range of the multiple-price list is a function of the number of items (questions) and the increment size. The number of items may be further decomposed into $j \in \mathbb{Z}_{++}$ items on either side of the central 'costless choice' item, giving a total of $2j+1$ items. Defining the inter-item increment to be $\triangle \in \mathbb{R}_{++}$, then the total range of the multiple price list becomes $2j\triangle$.

Across all experiments and treatments reported here, we fix $j = 5$, giving 11 items in total. In Experiments 2 and 3(a) we employ an increment size of €1, giving a total range of $2 \cdot 5 \cdot 1 =$ €10. In part (b) of Experiment 3 we employ $\triangle = 0.20$ Euro cents, giving a total range of $2 \cdot 5 \cdot 0.2 =$ €2.

Table 1: Example multiple-price list with $\triangle = 1$

Q.	Alternative A	Alternative B
1.	Urn variant + 5 Euro	Coin variant
2.	Urn variant + 4 Euro	Coin variant
3.	Urn variant + 3 Euro	Coin variant
4.	Urn variant + 2 Euro	Coin variant
5.	Urn variant + 1 Euro	Coin variant
6.	Urn variant	Coin variant
7.	Urn variant	Coin variant + 1 Euro
8.	Urn variant	Coin variant + 2 Euro
9.	Urn variant	Coin variant + 3 Euro
10.	Urn variant	Coin variant + 4 Euro
11.	Urn variant	Coin variant + 5 Euro

6 Experiment I

This experiment is designed to test whether Reduction holds in the 'judged valuation' task that is Certainty Equivalent elicitation.

6.1 Subjects and procedures

The experiment was conducted in z-Tree at the experimental laboratory of the Karlsruhe Institute of Technology (KIT). Sixty-three students from different fields of study were recruited into sessions of no more than 10 Subjects using ORSEE (Greiner, 2004). At the time of participating in the study, all subjects were enrolled in an engineering or computer science degree program at KIT, and had already completed at least one course in mathematics or statistics as part of their degree program. Subjects ranged in age from 22 to 28, with an average of 24.1 years. 70% of the subjects were male.

This Certainty Equivalent (CE) test section comprised two banks of 15 questions, with each question presented separately on the screen. All subjects answered both banks of 15 questions. At the beginning of each session, Subjects were given written instructions describing the lottery and their choice alternatives. After subjects had time to read the instructions, the experimenter

Table 2: Fifteen choices (presented individually) between the lottery (either the reduced-form Urn-implemented StPG or the compound-form coin-implemented StPG) and a fixed sure amount.

Q.	**Alternative A**	**Alternative B: certain sum**
1.	Lottery	1 Euro
2.	Lottery	2 Euro
3.	Lottery	3 Euro
4.	Lottery	4 Euro
5.	Lottery	5 Euro
6.	Lottery	6 Euro
7.	Lottery	7 Euro
8.	Lottery	8 Euro
9.	Lottery	9 Euro
10.	Lottery	10 Euro
11.	Lottery	11 Euro
12.	Lottery	12 Euro
13.	Lottery	13 Euro
14.	Lottery	14 Euro
15.	Lottery	15 Euro

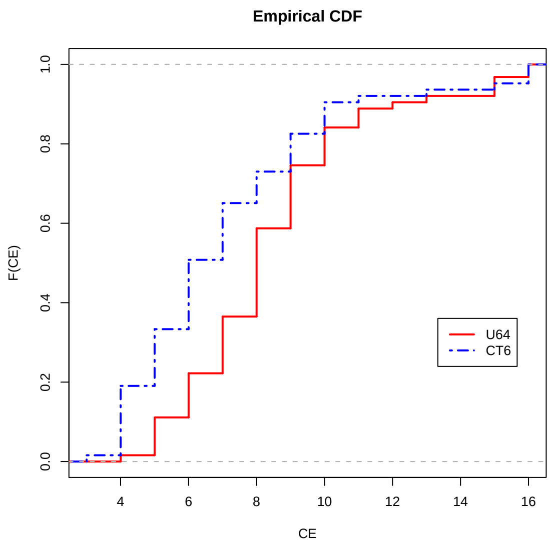
demonstrated the Urn and the Coin Toss randomization devices to be used to 'play for real' one choice according to the Random Lottery Incentive scheme. Subjects were given the opportunity to seek clarification on any aspect of the experiment before commencing the z-Tree program. Subjects wishing to ask a question were individually led outside the laboratory room, where neither the question nor the answer could be heard by the other subjects. After all session Subjects had complete their questions, the experimenter proceeded to implement one of each Subject's choices. Standard laboratory protocols to minimize the risk of experimenter demand effects were followed.

6.2 Results

Comparison of the Empirical Cumulative Distribution Functions for the compound- and reduced-form StPGs reveals a First-Order Stochastic Dominance relationship: the reduced-form (Urn) First-Order Stochastically Dominates the compound-form (Coin). After dropping 4 subjects due to aberrant response patterns (those coded as 16 in the Empirical CDF plot), 59 paired observations are available for nonparametric testing. The null hypothesis under Reduction is that there is no median difference within-subjects between the reduced-form lottery CE and the compound-form lottery CE. The Wilcoxon signed-rank test rejects the null of no median difference ($p < 0.0001$). For robustness to skewness we also implement the sign test, and find that the null of no median difference is rejected with $p = 0.0000$.

Bearing in mind the discreteness of the monetary sums, Subjects' choices reveal a willingness to pay on average €1.66 more, and €2 in the median, for the reduced form over the compound form.

Figure 1: Empirical CDFs of the Certainty Equivalents of the reduced-form StPG (U64, red) and the compound-form StPG (CT6, blue).



7 Experiment II

This experiment is designed to implement weak-form and strong-form tests for violation of Reduction.

7.1 Subjects and procedures

The subject pool, method of recruitment, laboratory, laboratory procedures and incentive scheme (RLI) are the same as in the Certainty Equivalent elicitation Experiment I above. All subjects (N=20) answer 11 questions, one at a time on the z-Tree screen, for both the $k = 2$ truncated StPG as well as the $k = 6$ truncated StPG. The multiple price list used is as set out in Table 1: $j = 5$, $\triangle = 1$.

7.2 Results

Table 3: Proportion of Subjects (of $N = 20$) choosing the Urn with the associated 95% confidence intervals (Jeffreys prior) for the 'max 2 tosses' StPG(2) and 'max 6 tosses' StPG(6) price lists.

	Urn +5	**Urn +4**	**Urn +3**	**Urn +2**	**Urn +1**	**Urn +0**	**Coin +1**	**Coin +2**	**Coin +3**	**Coin +4**	**Coin +5**
StPG(2)	1	1	1	1	1	.95	.20	.10	0	0	0
(95% CI)	(.88,1)	(.88,1)	(.88,1)	(.88,1)	(.88,1)	(.79,.99)	(.07, .41)	(.02, .28)	(0,.12)	(0,.12)	(0,.12)
StPG(6)	1	1	1	1	1	.90	.45	.15	0	0	.05
(95% CI)	(.88,1)	(.88,1)	(.88,1)	(.88,1)	(.88,1)	(.72,.98)	(.25,.66)	(.04,.35)	(0,.12)	(0,.12)	(.005,.21)

For the Urn + 0 entries we can test for weak-form violation of Reduction ($H_0 : \hat{p} = p_0$ and $H_1 : \hat{p} > p_0$ where $p_0 = 0.5$) using a Binomial test with $N = 20$ and $p_0 = 0.5$. On the StPG(2) task, under the null hypothesis, the probability of observing $\hat{p} \geq .95$ is 0.000020. On the StPG(6) task, under the null hypothesis, the probability of observing $\hat{p} \geq .90$ is 0.000201. 90% of Subjects display weak-form violation of Reduction on the StPG(6) task.

There is strong asymmetry to the left and right of the costless choice item (Urn+0). All bonus sums under the $\triangle = 1$ increment are systematically recognized as 'give-away money' when added to the Urn. We infer that $\delta < 1$ and that dominance is satisfied. Hence entries on the right-hand-side of the table reflect strong-form violation of Reduction, consistent with alternation bias. 45% of Subjects forgo a sure bonus of €1 in order to obtain the reduced form instead of the compound form.

8 Experiment III

This experiment is designed to test the robustness of Experiment II's results by (a) replicating the experiment at a different laboratory with a different subject pool and (b) testing for range and increment effects.

8.1 Subjects and procedures

The experiment was conducted using z-Tree at the Magdeburg Experimental Laboratory of Economic Research (MaXLab). 40 students from different fields of study were recruited into sessions of no more than 10 Subjects using ORSEE (Greiner, 2004). At the time of participating in the study, all subjects were enrolled in an engineering or computer science degree program at the Otto-von-Guericke University Magdeburg in Germany, and had already completed at least one course in mathematics or statistics as part of their degree program. Subjects ranged in age from 22 to 29, with an average of 25 years. 67% of the subjects were male.

Although the subject pool differs from that of Experiments I and II, the method of recruitment, the laboratory procedures and the incentive scheme (RLI) are the same as in Experiments I and II above. All subjects (N=40) answer (a) 11 questions, one at a time on the z-Tree screen, for both the $k=2$ truncated StPG as well as the $k=6$ truncated StPG in the $\triangle=1$ multiple-price list format, and then (b) another battery of 2×11 questions in the $\triangle=0.20$ multiple-price list format.

8.2 (a) Results

For the $\text{Urn}+0$ entries we can test for weak-form violation of Reduction ($H_0 : \hat{p} = p_0$ and $H_1 : \hat{p} > p_0$ where $p_0 = 0.5$) using a Binomial test with $N = 40$ and $p_0 = 0.5$. On the StPG(2) task, under the null hypothesis, the probability of observing $\hat{p} \geq .95$ is 0.000000. On the StPG(6) task, under the null hypothesis, the probability of observing $\hat{p} \geq .90$ is 0.000000. 90% of Subjects display weak-form violation of Reduction on the StPG(6) task.

Just as in Experiment II, here in Experiment III there is strong asymmetry to the left and right of the costless choice item (Urn+0). Once again we can infer that $\delta < 1$ and that dominance is satisfied. Entries on the right-hand-side of the table reflect strong-form violation of Reduction,

Table 4: Proportion of Subjects (of $N = 40$) choosing the urn with the associated 95% confidence intervals (Jeffreys prior) for the 'max 2 tosses' StPG(2) and 'max 6 tosses' StPG(6) price lists.

	Urn +5	**Urn +4**	**Urn +3**	**Urn +2**	**Urn +1**	**Urn +0**	**Coin +1**	**Coin +2**	**Coin +3**	**Coin +4**	**Coin +5**
StPG(2)	1	1	1	1	1	.95	.25	.10	0	0	0
(95% CI)	(.94,1)	(.94,1)	(.94,1)	(.94,1)	(.94,1)	(.85,.99)	(.14, .40)	(.03, .22)	(0,.06)	(0,.06)	(0,.06)
StPG(6)	1	1	1	1	1	.90	.475	.175	0	0	.025
(95% CI)	(.94,1)	(.94,1)	(.94,1)	(.94,1)	(.94,1)	(.78,.97)	(.33,.63)	(.08,.31)	(0,.06)	(0,.06)	(.003,.11)

consistent with alternation bias. 48% of Subjects forgo a sure bonus of €1 in order to obtain the reduced form instead of the compound form.

The results in Table 4 constitute an overwhelmingly successful replication of Experiment II's findings.

8.3 (b) Results

Table 5: Proportion of Subjects (of $N = 40$) choosing the urn with the associated 95% confidence intervals (Jeffreys prior) for the 'max 2 tosses' StPG(2) and 'max 6 tosses' StPG(6) price lists.

	Urn +1	**Urn +.80**	**Urn +.60**	**Urn +.40**	**Urn +.20**	**Urn +0**	**Coin +.20**	**Coin +.40**	**Coin +.60**	**Coin +.80**	**Coin +1**
StPG(2)	1	1	1	1	1	1	.70	.525	.35	.275	.20
(95% CI)	(.94,1)	(.94,1)	(.94,1)	(.94,1)	(.94,1)	(.94,1)	(.55, .82)	(.37,.67)	(.22,.50)	(.16,.43)	(.10,.34)
StPG(6)	1	1	1	1	1	1	.875	.75	.65	.575	.50
(95% CI)	(.94,1)	(.94,1)	(.94,1)	(.94,1)	(.94,1)	(.94,1)	(.75,.95)	(.60,.86)	(.50,.78)	(.42,.72)	(.35,.65)

Here, with restricted increment and range, the Urn + 0 'costless choice' entries no longer require the Binomial test, as the fraction of subjects choosing the Urn is 1. Again there is strong asymmetry to the left and right of the costless choice item (Urn+0). Here we can now infer that $\delta < 0.20$ and that dominance is satisfied. Entries on the right-hand-side of the table reflect strong-form violation of Reduction, consistent with alternation bias. 88% of Subjects forgo a sure bonus of 20 Euro cents in order to obtain the reduced form instead of the compound form.

9 Conclusion

Together the three experiments reported here cast doubt on the descriptive validity of Reduction for StPGs. Since Reduction is an axiom – and furthermore one that is necessary for the application of EU to resolve the St. Petersburg Paradox – the weak- and strong-form violations uncovered here also impinge upon the role conceived for EU by Daniel Bernoulli.

The present empirical findings are consistent with the operation of alternation bias, which is a subjective distortion of conditional probability, distinct from the major avenues pursued thus far for resolving the St. Petersburg Paradox: outcome distortion (concave utility for money) and probability distortion (unconditional probability weighting).

Finally, alternation bias offers the prospect of relaxing the need to completely reparameterize CPT as brought about by re-emergence of the St. Petersburg Paradox under conventional parameterizations of CPT (Blavatskyy, 2005; Rieger and Wang, 2006).

Acknowledgements

The authors wish to thank the Chair of Professor Siegfried Berninghaus of Karlsruhe Institute of Technology for financial support. Personal thanks and gratitude are due to Professor Berninghaus for his advice, consistent encouragement and moral support. This manuscript benefitted greatly from comments by session audience members at the 2011 European ESA meeting in Luxembourg. The usual disclaimer applies.

References

Asparouhava, E., Hertzel, M., Lemmon, M., 2009. Inference from streaks in random outcomes: Experimental evidence on beliefs in regime shifting and the law of small numbers, Management Science 55(1), 1766–1782.

Aumann, R.J., 1962. Utility theory without the completeness axiom, Econometrica 30(3), 445-462.

Bar-Hillel, M., Wagenaar, W.A., 1991. The perception of randomness, Advances in Applied Mathematics 12(4), 428–454.

Bernasconi, M., Loomes, G., 1992. Failures of the reduction principle in an Ellsberg-type problem, Theory and Decision 32(1), 77–100.

Blavatskyy, P.R., 2005. Back to the St. Petersburg Paradox?, Management Science 51(4), 677-678.

Budescu, D.V., 1987. A markov model for generation of random binary sequences, Journal of Experimental Psychology: Human Perception and Performance 13(1), 25-39.

Chernoff, H., 1954. Rational selection of decision functions, Econometrica 22(2), 422–443.

Clotfelter, C., Cook, P., 1993. The 'gambler's fallacy' in lottery play, Management Science, 39(12), 1521-1525.

Cox, J.C., Sadiraj, V., Vogt, B., 2009. On the empirical relevance of St. Petersburg lotteries, Economics Bulletin 29(1), 214–220.

Croson, R., Sundali, J., 2005. The gambler's fallacy and the hot hand: Empirical data from casinos, Journal of Risk and Uncertainty 30(3), 195–209.

Fishburn, P.C., 1978. Mixture axioms in linear and multilinear utility theories, Theory and Decision 9(2), 161–171.

Fishburn, P.C., 1986. Comment on G. Shafer, "Savage revisited", Statistical Science 1(4), 492–495.

Greiner, B., 2004. The online recruitment system ORSEE 2.0 – A guide for the organization of experiments in economics. Cologne: University of Cologne.

Halevy, Y., 2007. Ellsberg revisited: An experimental study, Econometrica 75(2), 503–536.

Harrison, G.W., 1994. Expected utility theory and the experimentalists, Experimental Economics 19(2), 223–253.

Harrison, G.W., Martinez-Correa, J., Swarthout, J.T., 2012. Reduction of compound lotteries with objective probabilities: Theory and evidence, CEAR working paper, Georgia State University, GA.

Hauser, J.H., 1978. Consumer preference axioms: Behavioral postulates for describing and predicting stochastic choice, Management Science 24(13), 1331–1341.

Kahneman, D., Tversky, A., 1979. Prospect theory: An analysis of decision under risk, Econometrica 47(2), 263–291.

Kaivanto, K., 2008. Alternation bias and the parameterization of cumulative prospect theory. In Abdellaoui, M., Hey, J.D. (Eds.) Advances in decision making under risk and uncertainty. Springer-Verlag, Heidelberg, 91–107.

Kaivanto, K., Kroll, E.B., 2012. Negative recency, randomization device choice, and reduction of compound lotteries, Economics Letters 115(2), 263–267.

Kareev, Y., 1995. Positive bias in the perception of covariation, Psychological Review, 102(3), 490–502.

Keller, L.R., 1985. Testing of the ‘reduction of compound alternatives’ principle, Omega 13(4), 349–358.

Luce, R.D., 2001. Reduction invariance and Prelec’s weighting functions, Journal of Mathematical Pscychology 45(1), 167–179.

Luce, R.D., Raiffa, H., 1957. Games and decisions, (John Wiley, New York NY).

Mandler, M., 2005. Incomplete preferences and rational transitivity of choice, Games and Economic Behavior 50(x), 255–277.

von Neumann, J., Morgenstern, O., 1947. Theory of games and economic behavior, 2nd edition, (Princeton University Press, Princeton NJ).

Prelec, D., 1998. The probability weighting function, Econometrica 66(3), 497–452.

Prelec, D., 2000. Compound invariant weighting functions in prospect theory. In Kahneman, D., Tversky, A. (Eds.) Choices, values and frames. Cambridge University Press, Cambridge, 67–92.

Rabin, M., 2002. Inference by believers in the law of small numbers, Quarterly Journal of Economics 117(3), 775–816.

Rapoport, A., Budescu, D.V., 1997. Randomization in individual choice behavior, Psychological Review 104(3), 603-617.

Reichenbach, H., 1934. Wahrscheinlichkeitslehre: Eine Untersuchung uber die logischen und mathematischen Grundlagen der Wahrscheinlichkeitsrechnung. Translated as Reichenbach, H., 1949. The theory of probability; An inquiry into the logical and mathematical foundations of the calculus of probability (Berkeley, CA: University of California Press).

Rieger, M.O., Wang, M., 2006. Cumulative prospect theory and the St. Petersburg Paradox, Economic Theory 28(3), 24–55.

Samuelson, P.A., 1952. Probability, utility, and the independence axiom, Econometrica 20(4), 670–678.

Smith, V.L., 1982. Microeconomic systems as an experimental science, American Economic Review 72(5), 923–955.

Terrell, D., 1994. A test of the gambler's fallacy: Evidence from pari-mutuel games, Journal of Risk and Uncertainty, 8(3), 309317.

Terrell, D., 1998. Biases in assessments of probabilities: New evidence from greyhound races, Journal of Risk and Uncertainty 17(2), 151–166.

Tversky, A., Kahneman, D., 1981. The framing of decision sand the psychology of choice, Science 211(4481), 453–458.

Wakker, P., 2010. Prospect theory for risk and ambiguity (Cambridge University Press, Cambridge).