Uncertainty, Efficiency and Incentive Compatibility*

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Abstract

The conflict between efficiency and incentive compatibility, that is, the fact that some Pareto optimal (efficient) allocations are not incentive compatible is a fundamental fact in information economics, mechanism design and general equilibrium with asymmetric information. This important result was obtained assuming that the individuals are expected utility (EU) maximizers. Although this assumption is central to Harsanyi's approach to games with incomplete information, it is not the only one reasonable. In fact, a huge literature criticizes EU's shortcomings and considers many alternative preferences. Thus, it is natural to ask: does the mentioned conflict extend to other preferences? Is there any preference where this conflict does not exist? Can we characterize those preferences? We show that in an economy where individuals have complete, transitive, continuous and monotonic preferences, every efficient allocation is incentive compatible if and only if all individuals have maximin preferences.

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1 Introduction

One of the fundamental problems in mechanism design and equilibrium theory with asymmetric information is the conflict between efficiency and incentive compatibility. That is, there are allocations that are efficient but not incentive compatible. This important problem was alluded to in early seminal works by Wilson (1978), Myerson (1979), Holmstrom and Myerson (1983), Prescott and Townsend (1984) and Myerson and Satterthwaite (1983). Since incentive compatibility and efficiency are some of the most important concepts in economics, this conflict generated a huge literature and became a cornerstone of the theory of information economics, mechanism design and general equilibrium with asymmetric information.

It is a simple but perhaps important observation, that this conflict was predicated on the assumption that the individuals were expected utility (EU) maximizers, that is, they would form Bayesian beliefs about the type (private information) of the other individuals and seek the maximization of the expected utility with respect to those beliefs. Since the Bayesian paradigm has been central to most of economics, this assumption seemed not only natural, but the only one worth pursuing.

The Bayesian paradigm is not immune to criticism, however, and many important papers have discussed its problems; e.g. Allais (1953), Ellsberg (1961) and Kahneman and Tversky (1979, 1992) among others. Actually, these early criticisms had stimulated a huge literature considering alternative models, which we mention below. Therefore, it is useful to go back to the original contribution by Harsanyi (1967-8), to understand why Bayesianism is so prevalent.

We are most interested in the *interim* stage, that is, the moment in which each individual knows her type t_i, but not the types of other individuals $(t_j, j \neq i)$, that is, the *interim* stage. The types code the information about the preferences and information of all individuals. Since individuals do not know others' types, Harsanyi assumes that they form Bayesian beliefs about these other types.

Although the first step in Harsanyi's contruction (describing the possible outcomes by types) seems natural, the assumption of Bayesian preferences is less compelling.[1] After Ellsberg (1961)'s critique, Schmeidler (1989) and Gilboa and Schmeidler (1989) initiated a literature that has produced by now a considerable number of different preferences that depart from Bayesianism. This includes

[1]It should be noted that the two parts are indeed independent, because "beliefs about beliefs" can be defined out of the Bayesian framework, as Epstein and Wang (1996) have shown.

the following models: Choquet Expected Utility (CEU)—see Schmeidler (1989); Maximin Expected Utility (MEU)—see Gilboa and Schmeidler (1989); Multiplier preferences—see Hansen and Sargent (2001) and Strzalecki (2008); Second-order expected utility (Smooth model)—see Klibanoff, Marinacci, and Mukerji (2005); Variational preferences—see Maccheroni, Marinacci, and Rustichini (2006); Uncertainty averse preferences—see Cerreia, Maccheroni, Marinacci, and Montrucchio (2008). To this list, we could also add *behavioral* models, like cumulative prospect theory, by Tversky and Kahneman (1992)—see also Wakker (2010); or other models not directly related to ambiguity, as Quiggin (1982) and Yaari (1987). There is also a previous literature on *complete ignorance*.[2,3]

The diversity of choice models summarized above is indicative of the current tension within economics. Namely, we have accumulated strong and multiple evidence that the standard models of choice fail to describe or explain many important economic phenomena. In laboratory studies, the diversity and extension of such failures are both well-known and clear. But the problems are not restricted to laboratory experiments. In macroeconomics and finance, just to mention two of the most practical economic fields, the central problems can be described as a failure of standard models to explain data and facts.[4] Thus, we have both evidence that the standard model does not work and plenty of different models to consider. This should suggest the convenience of revisiting fundamental results in economic theory with alternative preferences and verify to what extent those results are robust.

The purpose of this paper is to understand how a fundamental finding of modern economics, namely the conflict between efficiency and incentive compatibility, is affected by different models of uncertainty. In particular, does this conflict extend to other non-Bayesian preferences? Is there any preference under which

[2]The expected utility was considered an example of *partial ignorance*, see Luce and Raiffa (1989).

[3] Milnor (1954) considers four types of "complete ignorance" preferences: *Laplace*'s Principle of Insufficient Reason, which consider every outcome equally likely; *Wald*'s maximin criterion, which considers the worst-case scenario; *Savage*'s Minimax Regret, which considers the worst-case scenario for the regret (difference between maximum and what you get); and *Hurwicz*'s criterion, which takes a convex combination of the worst and best outcomes.

[4] This includes, among others, the equity premium puzzle—see Mehra and Prescott (1985); the excessive trading puzzle—see Odean (1999); the dividend puzzle—see Long (1978) and Miller and Scholes (1978); and over and underreaction of asset prices—see Bondt and Thaler (1985) and Cutler, Poterba, and Summers (1991). It should be noted that most of these puzzles involve, in a more or less explicit way, efficiency, the treatment of uncertainty and, to a lesser extent, problems of incentive compatibility.

there is no such a conflict?

This is not a usual question in economics, where in general we begin with individuals with fixed preferences and look for the implied economic properties. Thus, it would be more standard if we have fixed an economy with one of these preferences and looked at the conflict between efficiency and incentive compatibility in this economy. Then, we would repeat the same exercise for each preference. Instead of this unending repetition, we ask the same question for all different models at once. More precisely, we consider agents that have complete, transitive, monotonic and continuous preferences, which essentially includes all preferences that have been considered in economics.[5] We ask whether efficient allocations are incentive compatible in such kind of economies. Given the diversity of these preferences and how huge this class of preferences is, it may appear that general and sharp answers would be outside reach.

We show that (a special form of) the maximin expected utility (MEU) introduced by Gilboa and Schmeidler (1989) has the remarkable property that all efficient (Pareto optimal) allocations are also incentive compatible. More than that, and perhaps even more surprising, this is the *only* preference that has this property. This shows that the conflict between efficiency and incentive compatibility is actually much more general than previously known. On the other hand, it also shows that attitude towards ambiguity is not neutral with respect to this conflict, suggesting that ambiguity may reduce the gap between the first-best (efficient allocations) and the second-best (efficient subjected to incentive compatibility). Indeed, our result shows that for at least one class of preferences, this gap is always zero.

This result seems somewhat surprising, since other papers have indicated that ambiguity may actually be bad for efficiency, limiting trading opportunities. In this respect, Mukerji (1998) presents one of the most interesting results.[6] He shows that ambiguity associated to effort (Moral Hazard) may reduce (rather than enhance) efficiency. Therefore, uncertainty may have opposite implications if they occur in an environment with Adverse Selection (the case we consider in this paper) or with Moral Hazard.

Another property of the maximin preferences is that the set of efficient allocations is not small. At least in the case of one-good economies, the set of efficient allocations under maximin preferences strictly includes all allocations that are incentive compatible and efficient for EU individuals.

[5] Lexicographic preferences, not being continuous, and Bewley's preferences, not being complete, are of course ruled out.

[6] We discuss other papers in section 4.

The paper is organized as follows. In section 2, we describe the setting and introduce definitions and notation. Section 3 contains the main result: all interim efficient allocations are incentive compatible if and only if all individuals have maximin preferences. Section 4 reviews the relevant literature and section 5 discusses future directions of research. All proofs are collected in the appendix.

2 Model and Definitions

The set $I = \{1, ..., n\}$ represents the set of individuals in the economy. Each agent $i \in I$ observes a signal in some finite set of possible signals, $t_i \in T_i$. The restriction to finite signals is not crucial and is assumed here just for simplicity. Write $T = T_1 \times \cdots \times T_n$. A vector $t = (t_1, \ldots, t_i, \ldots, t_n)$ represents the vector of all types. T_{-i} denotes $\Pi_{i \neq j}^n T_j$ and, similarly, t_{-i} denotes $(t_1, \ldots, t_{i-1}, t_{t+1}, \ldots, t_n)$. Occasionally, it will be convenient to write t as (t_i, t_j, t_{-i-j}). It should be noted that we describe the uncertainty in terms of types only for simplicity. All results can be easily translated to partition spaces.

For clarity, it is useful to specify the following periods (timing structure) for information and decision making by the individuals:

1. *Ex-ante*: contracts establishing final allocations (depending on types, as described below) are chosen.

2. *Interim*: types are privately known by each individual. Then, individuals announce their types (truthfully or not).

3. *Ex post*: contracts are executed according to the announced types and consumption takes place.

Next, we define endowments, allocations and individuals' preferences.

2.1 Endowments, Allocations and Contracts

Each individual cares about an outcome (e.g. consumption bundle) $b \in \mathcal{B}$. The set of bundles $\mathcal{B}$ is assumed to be a (convex subset of a) topological vector space. To fix ideas, the reader may find it useful to identify $\mathcal{B}$ with $\mathbb{R}_+^\ell$, for some $\ell \in \mathbb{N}$.

Each individual has an *initial endowment* $e_i : T \to \mathcal{B}$. We assume that individual i's endowment depends only on t_i and not on the types of other individuals, that is, we have the following:

Assumption 2.1 (Private information measurability of the endowments) *For every $i \in I$, $t_i \in T_i$ and $t_{-i}, t'_{-i} \in T_{-i}$, the endowments satisfy: $e_i(t_i, t_{-i}) = e_i(t_i, t'_{-i})$, that is, we assume that e_i is $\mathcal{T}_i$-measurable.*[7]

This assumption means that agents know their endowments realization. This is almost always assumed in the literature regarding general equilibrium with asymmetric information, no-trade, auctions and mechanism design. In the latter, endowments are usually assumed to be constant with respect to types (as in Morris (1994)) or not explicitly considered. Note that if endowments are constant, assumption 2.1 is automatically satisfied. In auctions, the players are assumed to be buyers or sellers with explicit fixed endowments, which again implies assumption 2.1. Even when the endowments may vary with types, as in Jackson and Swinkels (2005), where the private information is given by (e_i, v_i), i.e., endowments and values, assumption 2.1 is still satisfied, because the endowment depends only on player i's private information. Note also that since we allow interdependent values, the *ex post value* of the endowment may vary across all states. The assumption is about only the *quantity* endowed, not values. In this sense, it may be considered a mild and natural assumption.

An individual allocation is a function $f : T \to \mathcal{B}$. We will denote by $\mathcal{F}$ the set of individual allocations $f : T \to \mathcal{B}$. An *allocation* is a profile $x = (x_i)_{i \in I}$, where x_i is an individual allocation for individual i. An allocation is *feasible* if $\sum_{i \in I} x_i(t) = \sum_{i \in I} e_i(t)$, for every t. Unless otherwise explicitly defined, all allocations considered in this paper will be feasible.

2.2 Preferences

We consider ex ante, interim and ex post preferences for each individual, which will be denoted by $\succcurlyeq_i$, $\succcurlyeq_i^{t_i}$ and $\succcurlyeq_i^t$, respectively. For simplicity, we assume that all preferences are defined over allocations $\mathcal{F}$, although the values of the allocations at $t' = (t'_i, t'_{-i})$ do not matter for $\succcurlyeq_i^{t_i}$ if $t'_i \neq t_i$ and for $\succcurlyeq_i^t$ if $t' \neq t$.

Since our focus is more on the interim and ex ante preferences, our definition of ex post preferences will not require explicit axioms. That is, we assume that there is a continuous function $u_i : T \times \mathcal{B} \to \mathbb{R}$ such that $u_i(t, b)$ represents individual i's utility for consuming b when types $t \in T$ are realized. The *ex post* preference on $\mathcal{B}$ depending on $t \in T$ is denoted by $\succcurlyeq^t$ and defined by:

$$a \succcurlyeq_i^t b \iff u_i(t, a) \geqslant u_i(t, b), \forall a, b \in \mathcal{B}. \tag{1}$$

[7] $\mathcal{T}_i$ denotes the (σ)-algebra generated by the partition $\cup_{t_i \in T_i} \{t_i\} \times T_{-i}$.

Throughout the paper, we assume the following:

Assumption 2.2 *The function u_i is continuous and the image of $u_i(t_i, \cdot)$ is an interval $[c, d] \subseteq \mathbb{R}$ or $[c, +\infty)$ for any i and $t_i \in T_i$. Moreover, $\succcurlyeq_i$ and $\succcurlyeq_i^{t_i}$ agree with $\succcurlyeq_i^t$ in the following sense: if $f(t) = a$ and $g(t) = a'$ for all $t \in T$,*

$$f \succcurlyeq_i g \iff f \succcurlyeq_i^{t_i} g \iff u_i(t, a) \geqslant u_i(t, a'). \tag{2}$$

The assumption about the image is not essential for our results and could be relaxed, but it simplifies some arguments. In any case, it does not seem overly restrictive. Assumptions 2.1 and 2.2 will be assumed throughout the paper for all preferences.

The following definitions will be useful below.

Definition 2.3 *Let W be a finite set and let $\gtrsim$ be a preference over the set functions $f : W \to \mathcal{B}$. We say that $\gtrsim$ is:*

1. *complete if for every f, g, $f \gtrsim g$ or $g \gtrsim f$.*
2. *transitive if for every f, g, h, $f \gtrsim g$ and $g \gtrsim h$ imply $f \gtrsim h$.*
3. *monotonic if $f(\omega) \geqslant (\gg) g(\omega), \forall \omega \in \Omega$ implies $f \gtrsim (\gtrdot) g$.*[8]
4. *continuous if for all $f, g, h \in \mathcal{F}$, the sets $\{\alpha \in [0, 1] : \alpha f + (1 - \alpha) g \gtrsim h\}$ and $\{\alpha \in [0, 1] : h \gtrsim \alpha f + (1 - \alpha) g\}$ are closed.*

For our discussion below, it will be useful to define more formally Wald's maximin preference, which is a particular case of Gilboa-Schmeidler's MEU. Let us begin by defining it for the private values case, where the individual knows her own preferences:

$$f \succcurlyeq_i^{t_i} g \iff \min_{t_{-i} \in T_{-i}} u_i(t_i, f(t_i, t_{-i})) \geqslant \min_{t_{-i} \in T_{-i}} u_i(t_i, g(t_i, t_{-i})). \tag{3}$$

That is, in face of the uncertainty with respect to the reported type t_{-i}, individual i takes a pessimistic view. Note that in this private values case, the only source of uncertainty is the reported types. In the general case, the actual type of other

[8] By $f(\omega) \gg g(\omega)$ we mean that all coordinates of $f(\omega)$ are strictly above all coordinates of $g(\omega)$. For a general preference $\gtrsim$, we write $f \gtrdot g$ if $f \gtrsim g$ but it is not the case that $g \gtrsim f$.

individuals (chosen by Nature) may affect the individual's utility function. In this case, (3) should be changed to:[9]

$$f \succcurlyeq_i^{t_i} g \iff \min_{t_{-i},t'_{-i}\in T_{-i}} u_i(t_i, t_{-i}, f(t_i, t'_{-i})) \geqslant \min_{t_{-i},t'_{-i}\in T_{-i}} u_i(t_i, t_{-i}, g(t_i, t'_{-i})). \tag{4}$$

It is not difficult to see that this is a special case of Gilboa-Schmeidler's MEU preference; see section 3.2, where we also define an ex ante preference that is consistent with the above interim preference.

2.3 Incentive compatibility

Our definition of incentive compatibility is standard.[10] To introduce it, note that when individual i reports t''_i instead of his true type t'_i, he will receive the allocation $e_i(t'_i, t_{-i}) + x_i(t''_i, t_{-i}) - e_i(t''_i, t_{-i})$ instead of $x_i(t'_i, t_{-i})$, because $x_i(t''_i, t_{-i}) - e_i(t''_i, t_{-i})$ is the trade that i is entitled to receive at the state (t''_i, t_{-i}). Therefore, we have the following:

Definition 2.4 *An allocation x is* incentive compatible (IC) *if there is no i, t'_i, t''_i such that*

$$\left[e_i(t'_i, \cdot) + x_i(t''_i, \cdot) - e_i(t''_i, \cdot)\right] \succ_i^{t'_i} x_i(t'_i, \cdot). \tag{5}$$

Note that we have used the interim preferences, because the individual is at the interim stage when deciding to make a false report, and, therefore, makes all comparisons with respect to his interim preference. For some future results, it will be useful to define also coalitional incentive compatibility.[11]

[9]The reader may think that the most natural definition of the preference would involve the min with respect to only one t_{-i}, that is, compare $\min_{t_{-i}\in T_{-i}} u_i(t_i, t_{-i}, f(t_i, t_{-i}))$. However, one has to remember that in (4) t_{-i} is chosen by Nature, while t'_{-i} is chosen by the individuals. Therefore, there are two different sources of uncertainty. In any case, under the *private values assumption*, which is natural in many settings, these two definitions are equivalent. Our results require (4) only for the general (interdependent values) case.

[10] We focus only on direct mechanisms. There are two justifications for this. First, direct mechanisms are interesting by themselves; second, one could conceive of a "revelation principle" for ambiguity models. Indeed, Bose and Renou (2011) present a version of such a result.

[11]This is the interim version of the transfer coalitional incentive compatibility of Krasa and Yannelis (1994).

Definition 2.5 *An allocation $x = (x_i)_{i \in I}$ is* coalitional incentive compatible (CIC) *if there is no nonempty set $C \subset I$, profiles $t'_C = (t'_i)_{i \in C}$ and $t''_C = (t''_i)_{i \in C}$ and transfers $\tau_C = (\tau_i)_{i \in C} \in \mathcal{B}^{|C|}$ such that*[12]

$$[e_i(t'_i, \cdot) + x_i(t''_i, \cdot) - e_i(t''_i, \cdot) + \tau_i] \succ_i^{t'_i} x_i(t'_i, \cdot), \forall i \in C. \tag{6}$$

2.4 Interim Efficiency

The following definition is also standard:

Definition 2.6 *A feasible allocation $x = (x_i)_{i \in I}$ is* interim efficient *if there is no feasible allocation $y = (y_i)_{i \in I}$ such that $y_i \succsim_i^{t_i} x_i$ for every i and $t_i \in T_i$, with strict preference for some i and t_i.*

3 Main Results

Our main result is the following:

Theorem 3.1 *Let $I = \{1, ..., N\}$ be a set of individuals with interim preferences $\succsim_i^{t_i}$, for $i = 1, ..., N$, which are complete, transitive, monotonic and continuous. Then the following statements are equivalent:*

1. *Every interim efficient allocation is (coalitional) incentive compatible.*[13]
2. *All individuals have maximin preferences, that is, $\succsim_i^{t_i}$ satisfies (4) for all i and t_i.*

A rigorous proof of this Theorem will be given in the appendix. However, subsection 3.1 contains a heuristic proof. Before going to that, let's illustrate the result with a familiar setting.

Myerson-Satterthwaite example

A seller values the object as $v \in [0, 1]$ and a buyer values it as $t \in [0, 1]$. Both values are private information. An allocation will be efficient in this case if trade

[12] A set C for which (6) holds is called a blocking coalition.

[13] We need *coalitional* incentive compatibility only for the case in which $N > 2$, in the implication $(1) \Rightarrow (2)$. That $(2) \Rightarrow (1)$ is true with or without the word coalitional. If $N = 2$, the word *coalitional* can be dropped from the statement of the Theorem.

happens if and only if $t \geqslant v$. Under the Bayesian paradigm, that is, the assumption that both seller and buyer are expected utility maximizers (EUM), Myerson and Satterthwaite (1983) have proved that there is no incentive compatible, individual rational mechanism (without subsidies) that would achieve ex post efficiency in this situation.

Consider now the following simple mechanism: the seller places an ask a and the buyer, a bid b. If the bid is above the ask, they trade at $p = \frac{a+b}{2}$; if it below, there is no trade. Therefore, if they negotiated at price p, the (ex post) profit for the seller will be $p - v$, and for the buyer, $t - p$; if they do not negotiate, both get zero. By Myerson and Satterthwaite (1983)'s result mentioned above, if the individuals are EUM, this mechanism does not always lead to efficient allocations. The problem is that this mechanism would be efficient if and only if both seller and buyer report truthfully, that is, $a = v$ and $b = t$, but these choices are not incentive compatible if the individuals are EUM. Now, we will show that $a = v$ and $b = t$ are incentive compatible choices if both seller and buyer have maximin preferences.

Recall that $a = v$ and $b = t$ are incentive compatible if buyer and seller do not have any incentive to choose a different action. If the buyer chooses $b = t$, the worst-case scenario is to end up with zero (either by buying by $p = t$ or by not trading). Can she do better than this? If she chooses $b > t$, the worst-case scenario is to buy by $p > t$, which leads to a (strict) loss. If she considers $b < t$, the worst-case scenario is to get zero (it always possible that there is no trade). Therefore, neither $b < t$ nor $b > t$ is better (by the maximin criterion) than $b = t$ and she has no incentive to deviate. The argument for the seller is analogous.

Note that our notions of efficiency and incentive compatibility are completely standard. The only difference from the classic framework is the preference considered. Also, although the individuals are pessimistic, they achieve the best possible outcome, even from an EU point of view, that is, the outcome is (ex ante, interim and ex post) efficient.

3.1 Idea of the proof

To grasp the main ideas in the proof of this Theorem, it is useful to separate it in two parts. First, we establish the following:

Proposition 3.2 *Assume that all individuals have interim maximin preferences, that is, $\succcurlyeq_i^{t_i}$ satisfies (4) for all i and t_i. If $x = (x_i)_{i \in I}$ is an interim efficient allocation, then x is (coalitional) incentive compatible.*

Therefore, Proposition 3.2 establishes the implication $2 \Rightarrow 1$ in Theorem 3.1. The idea of its proof is as follows. Note that an individual with maximin preference does not care if he gets something above the worst case scenario in that allocation, that is, he is indifferent between receiving only the worst outcome and receiving something better in some state. Now, if an allocation $x = (x_i)_{i \in I}$ is such that individual j with type t_j can gain something by lying about her type (saying that her type is $t'_j \neq t_j$), this means that x_i is specifying for individual i at the state t_j more than he would get at state t'_j. Indeed, the extra benefit that j gets by lying should come from someone; that someone is our i here. But since i has maximin preferences, i is perfectly happy to get only what is specified under t'_j. This implies that we can find another allocation y, similar to x, in which nobody is worse and j is strictly better. Therefore, we prove that if all individuals have maximin preferences and an allocation is not incentive compatible then it cannot be efficient.

The implication $1 \Rightarrow 2$ in Theorem 3.1 is established by the following:

Proposition 3.3 *Let $I = \{1, ..., N\}$ be a set of individuals with ex ante and interim preferences $\succcurlyeq_i, \succcurlyeq_i^{t_i}$, for $i = 1, ..., N$, which are complete, transitive, monotonic and continuous. If one of the interim preferences is not maximin, i.e., does not satisfy (4), then there is an allocation that is interim efficient but not incentive compatible.*

To establish this result, we first observe that if there is an individual that does not have maximin preferences, then there is a an allocation f such that

$$f \succ_i^{t_i} m_f^{t_i} \equiv \min_{t_{-i}} f(t_i, t_{-i});$$

otherwise the preference would be maximin. The key idea is to use f to define an allocation that is efficient but not incentive compatible. Since $f \succ_i^{t_i} m_f^{t_i}$, this allocation is such that there is a state (depending on the type of another individual j) under which i receives more than his worst-case scenario outcome. The individual i could not receive less, however, otherwise he would be worse-off. This is the key feature to establish that the defined allocation is efficient. Next, since under some types of individual j, i is receiving more, this means that j could lie and get for herself the extra benefits that i is getting. Of course, at this level of generality it is not completely clear that j could benefit in this way; the formalization in the actual proof is exactly to show that j indeed can be strictly better off by lying. Therefore, we have created an allocation that is efficient but not incentive compatible, as we wanted.

3.2 Results for Ex-Ante Preferences

Now, we extend Theorem 3.1 for ex ante preferences. For this, consider the following definition of dynamic consistency between the ex ante and interim preferences.

Definition 3.4 (Ex ante dynamic consistency) *For every $i \in I$, the ex ante and interim preferences $\succcurlyeq_i$, $\succcurlyeq_i^{t_i}$ and $\succcurlyeq_i^t$ satisfy* ex ante dynamic consistency *if:*

(i) $f \succcurlyeq_i^{t_i} g$ *for all* $t_i \in T_i$ *implies that* $f \succcurlyeq_i g$*;*

(ii) if, additionally, there is $t_i' \in T_i$ *such that* $f \succ_i^{t_i'} g$*, then* $f \succ_i g$*.*[14]

Our definition of dynamic consistency is similar but slightly stronger than Epstein and Schneider (2003)'s definition. Indeed, they require (i) above, but instead of (ii), they require:

$(ii)'$ $f \succ_i g$ if $f \succ_i^{t_i'} g$ for all $t_i' \in T_i$.

We will give an example below of a preference that satisfies Epstein and Schneider (2003)'s definition but not ours (see footnote 15).

The stronger requirement (ii), which is valid for Bayesian preferences with full support, allow us to prove that an ex ante efficient allocation is also interim efficient, thus extending a result by Holmstrom and Myerson (1983)—see Proposition 3.10. As Epstein and Schneider (2003) show, dynamic consistency is not a trivial condition on models with general preferences, although it is a very desirable property. For this reason, we did not consider it in the main theorem, but consider it here.

To define ex ante maximin preferences, consider the following notation: for each function $f : T \to \mathcal{B}$, define:

$$\underline{f}(t_i) \equiv \min_{t_{-i}, t'_{-i} \in T_{-i}} u_i(t_i, t_{-i}, f(t_i, t'_{-i})). \tag{7}$$

Following the above notation, the *ex ante* preference would be:

$$f \succcurlyeq_i g \iff \int_{T_i} \underline{f}(t_i)\mu_i(dt_i) \geqslant \int_{T_i} \underline{g}(t_i)\mu_i(dt_i). \tag{8}$$

We will assume also that μ_i puts positive probability on all types on T_i, that is, $\mu_i(\{t_i\}) > 0, \forall t_i \in T_i$. In this case, the ex ante preference $\succcurlyeq_i$ and the interim

[14] We write $f \succ g$ if $f \succcurlyeq g$ and it is not the case that $g \succcurlyeq f$.

preference $\succcurlyeq_i^{t_i}$ will agree for all types t_i. More explicitly, $\succcurlyeq_i$ will be ex ante dynamically consistent.[15]

It is useful to observe that the preference just defined is an instance of the Maximin Expected Utility (MEU) preferences defined by Gilboa and Schmeidler (1989). To see this, let Δ_i denote the set of measures π on $T_i \times T_{-i} \times T_{-i}$. For $\pi \in \Delta$, let $\pi|_{T_i}$ denote the marginal of π in T_i. Define, for each i, the following set:

$$\mathcal{P}_i \equiv \{\pi \in \Delta : \pi|_{T_i} = \mu_i\}. \tag{9}$$

Then, the preference defined by (8) is equivalently defined by:

$$\begin{aligned} f \succcurlyeq_i g \iff \quad & \min_{\pi \in \mathcal{P}_i} \int_{T_i \times T_{-i} \times T_{-i}} u_i(t_i, t_{-i}, f(t_i, t'_{-i}))\, d\pi(t_i, t_{-i}, t'_{-i}) \\ & \geqslant \min_{\pi \in \mathcal{P}_i} \int_{T_i \times T_{-i} \times T_{-i}} u_i(t_i, t_{-i}, g(t_i, t'_{-i}))\, d\pi(t_i, t_{-i}, t'_{-i}), \end{aligned}$$

which is easily seen to be a particular case of Gilboa and Schmeidler's MEU.[16]

The following result extends Proposition 3.3 for the ex ante preference.

Proposition 3.5 *Let $I = \{1, ..., N\}$ be a set of individuals with ex ante and interim preferences $\succcurlyeq_i, \succcurlyeq_i^{t_i}$, for $i = 1, ..., N$, which are complete, transitive, monotonic and continuous. If we have ex ante dynamic consistency and one of the interim preferences is not maximin, then there is an allocation that is ex ante efficient but not incentive compatible.*

3.3 One-good, private values economy

In this subsection, we consider two restrictions on the basic framework considered so far. First, we restrict to one-good economies. Second, we particularize to private-values. The following definitions formalize these notions.

[15] If we had defined $\succcurlyeq_i$ using a maximin criterion as in (4), then condition (ii) of the definition of dynamic consistency would not be satisfied—although $(ii)'$ would be. In other words, a preference defined by the maximin criterion both in the ex ante and interim stages—i.e. taking minima also over own types—would satisfy Epstein and Schneider (2003)'s definition of dynamic consistency, but not ours.

[16] This is a particular case of the maximin expected utility axiomatized by Gilboa and Schmeidler (1989) because we require $\mathcal{P}_i$ to have the format given by (9), while the set $\mathcal{P}_i$ in Gilboa and Schmeidler (1989) has to be only compact and convex.

Definition 3.6 (One-good economy) *We say that the economy is one-good if $\mathcal{B} \subseteq \mathbb{R}$, and for every $i \in I$ and $t \in T$, $a \mapsto u_i(t, a)$ is strictly increasing.*

Definition 3.7 (Private values) *We say that we have private values if the utility function of agent i depends on t_i but not on t_j for $j \neq i$, that is, $u_i(t_i, t_{-i}, a) = u_i(t_i, t'_{-i}, a)$ for all i, t_i, t_{-i}, t'_{-i} and a.*

In Theorem 3.1, the absence of conflict between efficiency and incentive compatibility was a one-directional implication (efficiency implies incentive compatibility). In an important particular case, namely that of private value, one-good economy, we actually have an equivalence under maximin preferences. This is the content of the following:

Proposition 3.8 *Consider a one-good economy with private values and assume that all individuals have maximin preferences. Then, x is an interim efficient allocation if and only if it is incentive compatible.*

3.4 Ex Ante, Interim and Ex Post Efficiency

It is useful to define the ex ante, interim and ex post efficiency as follows.

Definition 3.9 *Consider a feasible allocation $x = (x_i)_{i \in I}$ and let $\succcurlyeq_i, \succcurlyeq_i^{t_i}$ and $\succcurlyeq_i^t$ represent respectively the ex ante, interim and ex post preferences of agent $i \in I$, as defined above (see section 2.2). We say that x is:*

1. ex post efficient *if there is no feasible allocation $y = (y_i)_{i \in I}$ such that $y_i(t) \succcurlyeq_i^t x_i(t)$ for every i and $t \in T$, with strict preference for some i and t.*

2. interim efficient *if there is no feasible allocation $y = (y_i)_{i \in I}$ such that $y_i \succcurlyeq_i^{t_i} x_i$ for every i and $t_i \in T_i$, with strict preference for some i and t_i.*

3. ex ante efficient *if there is no feasible allocation $y = (y_i)_{i \in I}$ such that $y_i \succcurlyeq_i x_i$ for every i, with strict preference for some i.*

4. strongly efficient *if it is ex ante, interim and ex post efficient.*

Let E_A, E_I and E_P denote, respectively, the sets of ex ante, interim and ex post efficient allocations. Therefore, x is strongly efficient if $x \in E_A \cap E_I \cap E_P$. The set of strongly efficient allocations is denoted $E \equiv E_A \cap E_I \cap E_P$.

Let $\mathcal{A}$ denote the set of allocations $a : T \to \mathcal{B}$ and $D_A(x)$, $D_I(x)$ and $D_P(x)$ denote, respectively, the set of ex post, interim and ex ante deviations of $x \in \mathcal{A}$. That is, $D_A(x)$ is the set of those $y \in \mathcal{A}$ that satisfy the property defined in the item 1 above. Thus, $E_A = \{x : D_A(x) = \emptyset\}$. Analogous statements hold for $D_I(x)$, $D_P(x)$, E_I and E_A.

Holmstrom and Myerson (1983) note that $E_A \subset E_I \subset E_P$ for Bayesian preferences. However, we have the following:

Proposition 3.10 *If the preferences satisfy ex ante dynamic consistency, $E_A \subsetneq E_I$ but we may have $E_P \not\subset E_I$, $E_I \not\subset E_P$ and $E_A \not\subset E_P$.*

The fact that the inclusion $E_I \subset E_P$ may fail for maximin preferences is, however, not essential. First, it holds in one-good economies. Second, we could require the ex post efficiency together with the interim and the ex ante efficiency. We clarify both issues in the sequel.

Lemma 3.11 *In an one-good economy, $E_I \subset E_P$.*

Proof. Suppose that $x \in E_I \setminus E_P$. Then there exists y, j, t' such that $y_i \succcurlyeq_i^t x_i$ for all $i \in I, t \in T$ and $y_j \succ_j^{t'} x_j$. Since utilities are strictly increasing, we have $\sum_{i\in I} y_i(t') > \sum_i x_{i\in I}(t) = \sum_{i\in I} e_i(t)$, that is, y is not feasible. ∎

We are interested in the following:

Proposition 3.12 *If the preferences satisfy ex ante dynamic consistency, there exist strongly efficient allocations, that is, $E \neq \emptyset$.*

Proof. Trivially, there exists $x \in E_A$. By Proposition 3.10, $x \in E_I$. If $x \notin E_P$, then there exists ex post efficient y such that $y_i \succcurlyeq_i^t x_i$ for all i, t (and it improves upon x at least for one i, t). But then this implies that $y_i \succcurlyeq_i^{t_i} x_i$ and $y_i \succcurlyeq_i x_i$ also hold for all i, t_i. Since $x \in E_A \cap E_I$, $y \in E = E_A \cap E_I \cap E_P$. ∎

4 Discussion

4.1 General Equilibrium with Asymmetric Information

It is well known that in a finite economy with asymmetric information once people exhibit standard expected utility, then it is not possible in general to find allocations which are Pareto optimal and also incentive compatible—see, for example, Wilson (1978), Myerson (1979), Holmstrom and Myerson (1983), and Prescott and Townsend (1984). The key issue is the fact that in a finite economy each agent's private information has an impact and therefore an agent will take advantage of this private informational effect to influence the equilibrium allocation to favor herself. This is what creates the incentive compatibility problem. To get around this problem, Yannelis (1991) imposes the private information measurability condition, and in this case indeed, any ex ante private information Pareto optimal allocation is incentive compatible (see Krasa and Yannelis (1994), (Koutsougeras and Yannelis, 1993) and Hahn and Yannelis (1997) for an extensive discussion of the private information measurability of allocations). In fact, the private information measurability is not only sufficient for proving that ex ante efficient allocations are incentive compatible, but it is also necessary in the one-good case.

It is useful to try to understand why measurability was used to solve the problem of the conflict between efficiency and incentive compatibility. If an agent trades a non-measurable contract, this means that the contract makes promises depending on conditions that she cannot verify. Therefore, other agents may have an incentive to cheat her and do not deliver the correct amount in those states. This possibility is exactly the failure of incentive compatibility. To the contrary, if she insists to trade only measurable contracts (allocations), then she cannot be cheated and incentive compatibility is preserved.

However, the requirement of private information measurability raises two main concerns. First, it is an exogenous, theoretical requirement, which may be difficult to justify in real economies. The second concern, which is more relevant, is that the private information measurability restriction may lead to reduced efficiency and in certain cases even to no-trade. Thus, on the one hand, the private information measurability restriction implies incentive compatibility, but on the other hand, it reduces efficiency. To the contrary, the maximin expected utility allows for trade and results in a Pareto efficient outcome which is also incentive compatible.

Different solutions to the conflict between efficiency and incentive compatibility for the standard (Bayesian) expected utility for replica economies have been proposed by Gul and Postlewaite (1992) and McLean and Postlewaite (2002). Those authors impose an "informational smallness" condition and show the existence of incentive compatible and Pareto optimal allocations in an approximate sense for a replica economy. The informational smallness can be viewed as an approximation of the idea of perfect competition and as a consequence only approximate results can be obtained in this replica economy framework. Sun and Yannelis (2007) and Sun and Yannelis (2008) formulate the idea of perfect competition in an asymmetric information economy with a continuum of agents. In this case each individual's private information has negligible influence and as a consequence of the negligibility of the private information, they are able to show that any ex ante Pareto optimal allocation is incentive compatible. The above results are obtained in the set up of standard (Bayesian) expected utilities and they are only approximately true in large but finite economies.

Subsequently to the completion of this paper, de Castro, Pesce, and Yannelis (2010) revisited the Kreps (1977)'s example of the non-existence of the rational expectation equilibrium. They showed that there is nothing wrong with the rational expectation equilibrium notion other than the assumption that agents are expected utility maximizers. Using the maximin preferences studied here, de Castro, Pesce, and Yannelis (2010) recomputed the Kreps' example and showed that the rational expectation equilibrium not only exists, but it is also unique, efficient and incentive compatible. Furthermore, de Castro, Pesce, and Yannelis (2011) have obtained existence and incentive compatibility results for the maximin core.

Another related paper is Morris (1994). He departures from the Milgrom and Stokey (1982) no-trade theorem, which requires the common prior assumption, and shows that the incentive compatibility requirement allows for obtaining equivalent no-trade theorems under assumptions weaker than the common prior assumption. In this context, no trade theorems may be interpreted as a loss of efficiency created by the constraint of incentive compatibility.

Correia-da Silva and Hervés-Beloso (2009) used a MEU for a general equilibrium model with uncertain deliveries, and proved the existence of a new equilibrium concept, which they called prudent equilibrium. Although they considered MEU preferences, their focus was different and did not consider the incentive compatibility studied here.

4.2 Decision Theory

The maximin criterion has a long history. It was proposed by Wald (1950) and Rawls (1971), and axiomatized by Milnor (1954), Maskin (1979), Barbera and Jackson (1988), Nehring (2000) and Segal and Sobel (2002). Binmore (2008, Chapter 9) presented an interesting discussion of the principle, making the connection of the large worlds of Savage (1972). Gilboa and Schmeidler (1989) generalized at the same time the maximin criterion (see footone 16) and Bayesian preferences by allowing for multiple priors. Bewley (2002) introduced a model of decision under incomplete information.

Gilboa, Maccheroni, Marinacci, and Schmeidler (2010) consider decision makers who have two preferences. One of these preferences is incomplete and corresponds to the part of her preference that she can justify for third persons. They call this preference objective and model it as a Bewley incomplete preference. The other preference corresponds to a subjective preference, where the decision maker cannot be proven wrong and this is modeled as a maximin expected utility preference.

Rigotti, Shannon, and Strzalecki (2008), de Castro and Chateauneuf (2011), characterized conditions for ex ante efficiency for convex preferences (the first) and MEU preferences (the second).

Mukerji (1998) used a model with ambiguity to analyze the problem of investment holdup and incomplete contracts in a model with *moral hazard*. Interestingly, he obtained results that go in the opposite direction than those obtained here: in the moral hazard model that he considered, ambiguity makes it harder to obtain incentive compatibility, not easier as we proved for our general equilibrium with asymmetric information model.[17] The connection between ambiguity and information has been addressed before by Mukerji (1997) and Ghirardato (2001). With respect to efficiency and incentive compatibility, Haller and Mousavi (2007) presented evidence that ambiguity improves the second-best in a simple Rothschild and Stiglitz (1976)'s insurance model.

The analysis of games with ambiguity averse players has also a limited literature. Klibanoff (1996) considered games where players have MEU preferences. Salo and Weber (1995), Lo (1998) and Ozdenoren (2000, Chapter 4) analyzed auctions where players have ambiguity aversion. More recently, Bose, Ozdenoren, and Pape (2006) and Bodoh-Creed (2010) studied optimal auction mechanisms when individuals have MEU preferences, while Lopomo, Rigotti, and Shannon (2009) investigated mechanisms for individuals with Bewley's preferences. However, none of these papers have uncovered the property of no conflict between efficiency and incentive compatibility for the maximin preferences considered here.

5 Concluding Remarks and Open questions

We showed that maximin preferences present no conflict between incentive compatibility and efficiency. Moreover, it is the the only preference that has this property.

[17] We are grateful to Sujoy Mukerji for bringing this paper to our attention.

This result presents a different way of characterizing preferences. In standard decision theory, axioms for individual behavior, grounded in intuitive reasoning or, sometimes, experimental evidence, lead to unique representations. On the other hand, our result considers an economic property that is meaningful in the context of an economy, not isolated individual decisions. Focusing on one phenomenon—efficiency implying incentive compatibility—we were able to completely characterize the behavior that lead to that situation. The preference was therefore defined by a collective property, not by individual features. Of course we are not suggesting that this is a better way of characterizing preferences; just that this is an alternative way, that could prove useful to the study of economic systems.

In this vein, we close by discussing some open questions and other directions for future research.

It is of interest to know the incentive compatibility properties for all uncertainty averse preferences (as defined by Cerreia, Maccheroni, Marinacci, and Montrucchio (2008)). In other words, fixing a profile of uncertainty averse preferences, we would like to know how close the sets of efficient and incentive compatible allocations are. Or yet: how close are the set of second-best outcomes (that is, outcomes that are efficient subject to being incentive compatible) and first-best (just efficient) outcomes?

In an earlier version of this paper, we introduced notions of maximin core and maximin perfect equilibrium. It is natural to investigate these concepts in more detail. Also, we have not pursued the issue of implementation. It is our conjecture that in view of the inherent efficiency and incentive compatibility of the new equilibrium notions, one should be able to show that they are implementable as a maximin perfect equilibrium and thus provide non cooperative foundations for the maximin core and maximin value.

It would be interesting to study an evolutionary model of populations of agents with different preferences. Will a society formed only by maximin agents outperform societies formed by individuals with diverse preferences? What happens if some mutations lead to Bayesian subjects inside this maximin society?

In sum, we hope this paper stimulates new venues of investigation.

A Appendix

For the examples below, it will be convenient to use a concise notation for the allocations. Consider two-individual economies, with set of types $T_1 = \{U, D\}$ and $T_2 = \{L, R\}$. The allocation $x = (x_1, x_2)$ will be represented by:

x_1	L	R
U	$x_1(U,L)$	$x_1(U,R)$
D	$x_1(D,L)$	$x_1(D,R)$

and

x_2	L	R
U	$x_2(U,L)$	$x_2(U,R)$
D	$x_2(D,L)$	$x_2(D,R)$

where $x_i(t_1, t_2) \in \mathcal{B}$. Sometimes, we will write the above in just one table and often omit the types in the columns and rows.

Proof of Proposition 3.10.
Assume that $x \in E_A \setminus E_I$. Then there exists y, j, t_j such that $y_i \succcurlyeq_i^{t_i} x_i$ for all $i \in I, t_i \in T_i$ and $y_j \succ_j^{t_j} x_j$. Because of ex ante dynamically consistent, this implies that $y_i \succcurlyeq_i x_i$, for all i and $y_j \succ_j x_j$ for some j, that is, $y \in D_A(x)$, which contradicts $x \in E_A$.

Now we offer counterexamples for the other inclusions, using maximin preferences.

- $E_I \not\subset E_A$. Let $n = 2$, $\mathcal{B} = \mathbb{R}_+$, $T_i = \{t_i', t_i''\}$, $u_i(t,a) = a$, for $i = 1,2$ and any $t \in T$. Put $\mu_1(\{t_1'\}) = 0.3$ and $\mu_2(\{t_2'\}) = 0.6$. Consider the allocations $x = (x_1, x_2)$ and $y = (y_1, y_2)$ defined as follows:

(x_1, x_2)	t_2'	t_2''
t_1'	$(2,2)$	$(2,2)$
t_1''	$(3,3)$	$(2,2)$

  and

(y_1, y_2)	t_2'	t_2''
t_1'	$(1,3)$	$(2,2)$
t_1''	$(3,3)$	$(3,1)$

  Thus, $\underline{x}_1(t_1') = \underline{x}_1(t_1'') = 2$; $\underline{y}_1(t_1') = 1; \underline{y}_1(t_1'') = 3$, which implies that $y_1 \succ_1 x_1$ because $\mu_1(\{t_1'\}) = 0.3 < \mu_1(\{t_1''\}) = 0.7$. On the other hand, $\underline{x}_2(t_2') = 2; \underline{x}_2(t_2'') = 2$; $\underline{y}_2(t_2') = 3; \underline{y}_2(t_2'') = 1$, which implies $y_2 \succ_2 x_2$ because $\mu_2(\{t_2'\}) = 0.6 > \mu_2(\{t_2''\}) = 0.4$. Therefore, $y \in D_A(x)$, that is, $x \notin E_A$. Now suppose that there is z such that $z \in D_I(x)$, that is, $z_i \succcurlyeq_i^{t_i} x_i$, $\forall i, t_i \in T_i$ and $z_j \succ_j^{t_j} x_j$ for some $j \in I$. This means that $\underline{z}_1(t_1'), \underline{z}_1(t_1''), \underline{z}_2(t_2'), \underline{z}_2(t_2'') \geqslant 2$ and at least one of these inequalities has to be strict. Observe that this requires $z_1(t_1, t_2) \geqslant 2$ and $z_2(t_1, t_2) \geqslant 2$, for any $(t_1, t_2) \in T_1 \times T_2$. But then feasibility implies $z_1(t_1, t_2) = z_2(t_1, t_2) = 2$, for any $(t_1, t_2) \neq (t_1'', t_2')$. In turn, this implies that none of the inequalities $\underline{z}_1(t_1'), \underline{z}_1(t_1''), \underline{z}_2(t_2'), \underline{z}_2(t_2'') \geqslant 2$ can be strict. Therefore, $z \notin D_I(x)$, which is a contradiction that shows $x \in E_I$.

- $E_P \not\subset E_I$. Consider that $n = 2$, $\mathcal{B} = \mathbb{R}_+$ and $u_1(t_1, t_2, a) = u_2(t_1, t_2, a) = a$, where $T_1 = T_2 = \{1, -1\}$. Let $e_1(t) + e_2(t) = 1$ for all t. Consider the allocation $x = (x_1, x_2)$ defined by:

(x_1, x_2)	$t_2 = 1$	$t_2 = -1$
$t_1 = 1$	$(1,0)$	$(0,1)$
$t_1 = -1$	$(0,1)$	$(1,0)$

Then x is feasible and ex post efficient, that is, $x \in E_P$. However, $x \notin E_I$. Indeed, consider the deviation $y = (y_1, y_2)$ defined by $y_1(t) = \frac{1}{2} = y_2(t)$. This satisfies: $y_i \succ_i^{t_i} x_i, i = 1, 2$ because:

$$\frac{1}{2} = \min_{t'_{-i} \in \{1,-1\}} u_i(t_i, y_i(t_i, t'_{-i})) > \min_{t'_{-i} \in \{1,-1\}} u_i(t_i, x_i(t_i, t'_{-i})) = 0.$$

This shows that $E_P \not\subset E_I$.

- $E_I \not\subset E_P$ and $E_A \not\subset E_P$. Let $n = 2$, $\mathcal{B} = \mathbb{R}^2_+$, $T_1 = T_2 = \{1, 2\}$, $u_i(t, (a_1, a_2)) = a_1 a_2$ and $e_i(t) = (t_i, t_i)$, for $i = 1, 2$. Consider the following allocation:

(x_1, x_2)	$t_2 = 1$	$t_2 = 2$
$t_1 = 1$	$((1,1),(1,1))$	$((1.5,1.5),(1.5,1.5))$
$t_1 = 2$	$((1.5,1.5),(1.5,1.5))$	$((3,1),(1,3))$

In this case, we have $\underline{x}_i(1) = 1$; $\underline{x}_i(2) = 2.25$, $i = 1, 2$, which are the best possible levels for both players (it is not possible to improve these minima for both players). Therefore, $x = (x_1, x_2)$ is interim efficient and ex ante efficient. However, it is clearly not ex post efficient, because we can define $y_i(t) = x_i(t)$ for all $t \neq (2, 2)$ and $y_i(2, 2) = (2, 2)$, $i = 1, 2$ and this is clearly better than $(x_1(2, 2), x_2(2, 2)) = ((3, 1), (1, 3))$. This shows that $E_I \not\subset E_P$ and $E_A \not\subset E_P$. ∎

A.1 Proof of Theorem 3.1

Proof of Proposition 3.2: Suppose that x is not incentive compatible. This means that there exists an individual i and types t'_i, t''_i such that:

$$\begin{aligned} &\min_{t_{-i}, t'_{-i} \in T_{-i}} u_i\left(t'_i, t_{-i}, e_i\left(t'_i, t'_{-i}\right) + x_i\left(t''_i, t'_{-i}\right) - e_i\left(t''_i, t'_{-i}\right)\right) \\ &> \min_{t_{-i}, t'_{-i} \in T_{-i}} u_i\left(t'_i, t_{-i}, x_i\left(t'_i, t'_{-i}\right)\right). \end{aligned} \tag{10}$$

We will prove that x cannot be interim efficient by constructing another feasible allocation $y = (y_i)_{i \in I}$ that Pareto improves upon x. For this, define

$$y_j\left(t_i, t_{-i}\right) = \begin{cases} x_j\left(t_i, t_{-i}\right), & \text{if } t_i \neq t'_i \\ e_j\left(t'_i, t_{-i}\right) + x_j\left(t''_i, t_{-i}\right) - e_j\left(t''_i, t_{-i}\right), & \text{if } t_i = t'_i \end{cases} \tag{11}$$

To see that $(y_j)_{j \in I}$ is feasible, it is sufficient to consider what happens when $t_i = t'_i$:

$$\begin{aligned} \sum_{j \in I} y_j\left(t'_i, t_{-i}\right) &= \sum_{j \in I} e_j\left(t'_i, t_{-i}\right) + \sum_{j \in I} x_j\left(t''_i, t_{-i}\right) - \sum_{j \in I} e_j\left(t''_i, t_{-i}\right) \\ &= \sum_{j \in I} e_j\left(t'_i, t_{-i}\right), \end{aligned}$$

because $\sum_{j \in I} x_j(t_i'', t_{-i}) = \sum_{j \in I} e_j(t_i'', t_{-i})$, from the feasibility of x_j at (t_i'', t_{-i}).

From (10) and (11), we have $y_i \succ_i^{t_i'} x_i$ and $y_i \sim_i^{t_i} x_i$ for any $t_i \neq t_i'$. It remains to prove that $y_j \succcurlyeq_j^{t_j} x_j$ for any $j \neq i$ and t_j. The fact that e_j depends only on t_j implies that $e_j(t_i', t_{-i}) = e_j(t_i'', t_{-i})$ for all $t_{-i} \in T_{-i}$. Then, for every $t_{-i} \in T_{-i}$,

$$y_j(t_i', t_{-i}) = e_j(t_i', t_{-i}) + x_j(t_i'', t_{-i}) - e_j(t_i'', t_{-i}) = x_j(t_i'', t_{-i}). \tag{12}$$

For each $t_j \in T_j$, define $X_j^{t_j}$ as the set $\{x_j(t_j, t_{-j}) : t_{-j} \in T_{-j}\}$ and $Y_j^{t_j} \equiv \{y_j(t_j, t_{-j}) : t_{-j} \in T_{-j}\}$. Fix a $t = (t_i, t_j, t_{-i-j})$. If $t_i \neq t_i'$, the definition (11) of y_j implies that $y_j(t) = x_j(t) \in X_j(t_j)$. If $t_i = t_i'$, (12) gives $y_j(t_i', t_{-i}) = x_j(t_i'', t_{-i}) \in X_j(t_j)$. Thus, $Y_j(t_j) \subset X_j(t_j)$, for all $t_j \in T_j$. Therefore,

$$\underline{y}_j(t_j) = \min_{t_{-j}, y \in Y_j^{t_j}} u_j(t_j, t_{-j}, y) \geqslant \min_{t_{-j}, x \in X_j^{t_j}} u_j(t_j, t_{-j}, x) = \underline{x}_j(t_j). \tag{13}$$

This shows that $y_j \succcurlyeq_j^{t_j} x_j$ for all $j \neq i$ and $t_j \in T_j$. Thus, y is a Pareto improvement upon x, that is, x is not interim efficient. ∎

The reader can observe that the only place where we used the specific definition of the interim preference as the minimum was to conclude (13). Indeed if we were to use other preferences (in particular the expected utility preferences), this step would not go through.

Corollary A.1 *If x is maximin efficient, x is coalitional incentive compatible.*

Proof. It is enough to adapt Proposition 3.2's proof, substituting i by the blocking coalition C. ∎

For proving the converse implication in Theorem 3.1, it will be useful to introduce some notation. Let the finite set W represent the alternatives w about which a decision maker may be ignorant. The decision maker has a preference $\gtrsim$ over the set $\mathcal{F}_W$ of all real valued functions $f : W \to [a, b]$, with $>$ denoting its strict part.[18]

We can identify $\mathcal{F}_W$ with (a subset of) the Euclidean space $\mathbb{R}^{|W|}$ and use its Euclidian norm, topology, etc. Recall that $\gtrsim$ is maximin if for all $f, g \in \mathcal{F}$,

$$f \gtrsim g \iff \min_{w \in W} f(w) \geqslant \min_{w \in W} g(w).$$

Let us also define, for each $f \in \mathcal{F}_W$, $m_f \equiv \min_{w \in W} f(w)$, and abuse notation by denoting also by m_f the constant act that always pays m_f. The following result might be of interest in its own.

Proposition A.2 *Suppose that $\gtrsim$ is complete, transitive, monotonic and continuous. If $\gtrsim$ is not maximin, there exists*

$$h \in \mathcal{E} \equiv \{f \in \mathcal{F}_W : \exists w' \in W \textit{ such that } f(w') > \min_{w \in W} f(w)\},$$

such that for every $g \neq h$ satisfying $h \geqslant g$, we have $h > g$.

[18] There is no problem in allowing $b = \infty$, in which case the reader should understand $[a, b]$ as $[a, +\infty)$.

Proof. Since $\gtrsim$ is not maximin, there exists $f \in \mathcal{E}$ such that $f \succ m_f$. Let $W = \{w_1, w_2, ..., w_K\}$. We will define functions $f_k, g_k^\alpha : W \to \mathbb{R}_+$, for $k = 1, 2, ..., K$ and $\alpha \in [0,1]$. The definition of f_k will be recursive. Let $f_1 \equiv f$ and suppose that f_k is defined satisfying $f_k \sim f$. Define g_k^α as follows:

$$g_k^\alpha(w) = \begin{cases} f_k(w), & \text{if } w \neq w_k \\ \alpha f_k(w) + (1-\alpha)a, & \text{if } w = w_k \end{cases}$$

The set $A_k = \{\alpha \in [0,1] : g_k^\alpha \sim f_k\}$ contains 1 and is closed. Moreover, by monotonicity and continuity, there is the smallest $\alpha_k \in A_k$. Define f_{k+1} as $g_k^{\alpha_k}$. Then by definition, for $k = 1, ..., K$,

$$f_{k+1} \sim f_k \sim f \text{ and } f_{k+1} \leqslant f_k. \tag{14}$$

We claim that $h \equiv f_{K+1}$ satisfies the properties in the statement above.

Indeed, suppose that there is a $g : W \to I$ such that $g \neq h$, $h \geqslant g$ and $h \sim g$. Since $g \neq h$, the set $\{k : g(w_k) < h(w_k)\}$ is non-empty. Let k be the largest element of this set. Observe that $f_{k+1} \sim h$, $f_{k+1} \geqslant h \geqslant g$ and $h(w_j) = f_k(w_j) = g_k^\alpha(w_j)$ for every $j < k$ and $\alpha \in [0,1]$. Observe that $g_k^{\alpha_k}(w_k) = f_{k+1}(w_k) \geqslant h(w_k) > g(w_k) \geqslant a$. This implies that $\alpha_k > 0$ and there exists $\alpha < \alpha_k$ such that $g(w_k) < g_k^\alpha(w_k) < g_k^{\alpha_k}(w_k) = f_{k+1}(w_k)$. However, by definition of α_k, for any $\alpha < \alpha_k$, $f_{k+1} \succ g_k^\alpha$. It is easy to see that $g_k^\alpha \geqslant g$ and, therefore, $g_k^\alpha \gtrsim g$. But then $h \sim f_{k+1} \succ g_k^\alpha \gtrsim g$, which contradicts $h \sim g$. ∎

We need another result for the proof of Theorem 3.1.

Lemma A.3 (Alternative for corner allocations) *Let the preferences $\{\gtrsim_i\}_{i\in I}$ be adequate. Suppose that $x = (x_j)_{j\in I}$ is a i-corner allocation, that is, $x_j(\omega) = 0 \in \mathcal{B} = \mathbb{R}_+^\ell$ for all $\omega \in \Omega$ and all $j \neq i$. Then one (and only one) of the following alternatives is true:*

1. *x is an ex ant efficient allocation;*
2. *there exists $z : T \to \mathcal{B}$ and $j \neq i$ such that:*
   - *(a)* $z \succ_j 0$;
   - *(b)* $z \geqslant 0$;
   - *(c)* $x_i \geqslant z$;
   - *(d)* $x_i - z \sim_i x_i$.

Proof. It is easy to see that if there exists z satisfying the conditions above, it is possible to transfer z to individual j, strictly improving j and without making any individual worse off; therefore x is not ex ante efficient. Conversely, if x is not ex ante efficient, then there exists a Pareto improving $y = (y_j)_{j\in I}$ satisfying $y_k \gtrsim_k x_k$, for all $k \in I$ and $y_j \succ_j x_j$ for some j. Fix such j. Of course, this j cannot be i, since i already has all the endowment of the economy and cannot be strictly better by a feasible transfer. Therefore, define $z \equiv y_j \succ_j x_j = 0$, which gives (a) above. Since $y_k \geqslant 0$ for all k, then we also have (b). This also allows to conclude that $\sum_{k\in I} x_k = x_i = \sum_{k\in I} y_k \geqslant y_j = z$, which establishes (c). For the same reason, $x_i \geqslant y_i + y_j = y_i + z$, that is, $x_i - z \geqslant y_i$. Since $z \geqslant 0$, we have $x_i \gtrsim_i x_i - z$. By monotonicity, $x_i - z \gtrsim_i y_i$. On the other hand, the fact that y is Pareto improving gives $y_i \gtrsim_i x_i$. Transitivity then establishes (d). ∎

Proof of Propositions 3.3 and 3.5.
Assume for now that $N = 2$. It is enough to prove Proposition 3.5, since it implies Proposition 3.3. To see this, note that given interim preferences, we can always find ex ante preferences that are ex ante dynamically consistent using the procedure described in section 3.2—see particularly equation (8) and the correspondent discussion. Now, for ex ante dynamically consistent preferences, ex ante interim efficiency implies interim efficiency by Proposition 3.10.

Suppose that individual 1's preference is not maximin, that is, there exists some type t_1' such that $\succsim_1^{t_1'}$ is not maximin. We will show that there is an allocation that is ex ante efficient which is not incentive compatible.

Let U denote the image of $(t_2, b) \mapsto u_1(t_1', t_2, b)$, and let $\bar{e} = (1, 1, ..., 1) \in \mathcal{B} = \mathbb{R}_+^\ell$ be the unitary bundle. Then, for each $\alpha \in U$, there exists $\lambda(t_2) \in \mathbb{R}_+$ such that $u_1(t_1, t_2, \lambda(t_2)\bar{e}) = \alpha$. Let $E = \{\lambda\bar{e} : \lambda \in \mathbb{R}_+\}$. Thus, given a function $f : \{t_1'\} \times T_2 \to U \subseteq \mathbb{R}$, we can find for each $t_2 \in T_2$ a bundle $f^{u_1}(t_1', t_2) \in E$ such that :

$$u_1(t_1', t_2, f^{u_1}(t_1', t_2)) = f(t_1', t_2).$$

Let $W = \{t_1'\} \times T_2$ and define $\succeq^*$ over functions $f : W \to U \subset \mathbb{R}$ by:

$$f \succeq^* g \iff f^{u_1} \succsim_1^{t_1'} g^{u_1}.$$

By Proposition A.2, there exists

$$f(t_1', \cdot) \in \mathcal{E} = \{f : W \to I : \exists w' \in W \text{ such that } f(w') > \min_{w \in W} f(w)\}$$

such that for every $g(t_1', \cdot) \neq f(t_1', \cdot)$ satisfying $f(t_1', \cdot) \geqslant g(t_1', \cdot)$, we have $f(t_1', \cdot) \succ^* g(t_1', \cdot)$. By the definition of $\succeq^*$, f^{u_1} and $\succsim_1$'s properties, for any $g : T \to \mathcal{B}$ distinct from f^{u_1}, we have:

$$f^{u_1} \geqslant g \Rightarrow f^{u_1} \succ_1^{t_1'} g. \tag{15}$$

Let $M_f \equiv \{t_2 : u_1(t_1', t_2, f^{u_1}(t_1', t_2)) = \min_{t_2 \in T_2} u_1(t_1', t_2, f^{u_1}(t_1', t_2))\}$. Fix $t_2' \in M_f$ and define: $e_1(t_1', \cdot) = f^{u_1}(t_1', t_2')$. For any $t_2 \in T_2$, define $e_2(\cdot, t_2) \equiv f^{u_1}(t_1', t_2) - e_1(t_1', t_2)$. By the definition of $f^{u_1}(t_1', \cdot)$, $e_2(\cdot, t_2) \geqslant 0$. Note also that $e_2(\cdot, t_2') = 0$. Now, for $t_1 \neq t_1'$, define $e_1(t_1, t_2) = 0$ and $f^{u_1}(t_1, t_2) = e_2(t_1, t_2)$. It is easy to see that $(f^{u_1}, 0)$ is then a feasible 1-corner allocation.[19]

Let $z : T \to \mathcal{B}$ be such that $z \geqslant 0$. Monotonicity implies then that $f^{u_1} \succsim_1^{t_1} g \equiv f^{u_1} - z$ for all $t_1 \in T_1$ and (15) implies that $f^{u_1} \succ_1^{t_1'} g$. Since $\succsim_1$ is adequate, $f^{u_1} \succ_1 g$. Therefore, there is no z satisfying all the assumptions in item 2 of Lemma A.3, which implies that the i-corner allocation $(f^{u_1}, 0)$ is (ex ante and interim) efficient.

On the other hand, since $f(t_1, \cdot) \in \mathcal{E}$, there is a type $t_2'' \notin M_f$ such that

$$f(t_1', t_2'') > f(t_1', t_2') \Rightarrow f^{u_1}(t_1', t_2'') - f^{u_1}(t_1', t_2') \gg 0. \tag{16}$$

[19] We have defined the initial endowments here only to make the example completely specified.

Then, if individual 2 is of type t''_2, he has an incentive to report t'_2. Indeed, if $t_2 = t''_2$ and individual 2 reports t'_2 instead of t''_2, he will consume, for any $t_1 \in T_1$,

$$\begin{aligned} e_2(t_1, t''_2) - e_2(t_1, t'_2) &= [f^{u_1}(t'_1, t''_2) - e_1(t'_1, t''_2)] - [f^{u_1}(t'_1, t'_2) - e_1(t'_1, t'_2)] \\ &= f^{u_1}(t'_1, t''_2) - f^{u_1}(t'_1, t'_2) \gg 0, \end{aligned}$$

where the first equality comes from the definition of $e_2(\cdot, t_2)$, the second comes from the definition of $e_1(t'_1, \cdot)$ and the inequality comes from (16). Since individual 2's allocation under $(f, 0)$ is always zero and the preference is monotonic, he would be strictly better off. Thus, the allocation is not incentive compatible.

Now, when $N > 2$, the proof above works by substituting individual 1 by i and individual 2 by a coalition of all individuals other than i. ∎

A.2 Other Proofs

Proof of Proposition 3.8

We need just to prove that coalitional incentive compatible allocations are efficient. Assume that x is coalitional incentive compatible. We claim that x_j is $\mathcal{F}_j$-measurable for each $j \in I$.

We establish this claim by contradiction. Suppose that x is incentive compatible but x_j is not $\mathcal{F}_j$-measurable for some $j \in I$, that is, suppose that there exist $t_{-j}, t'_{-j} \in T_{-j}$ such that $x_j(t_j, t_{-j}) \neq x_j(t_j, t'_{-j})$. Without loss of generality, we may assume that $x_j(t_j, t_{-j}) > x_j(t_j, t'_{-j})$. Since e_j is $\mathcal{F}_j$-measurable, $e_j(t_j, t'_{-j}) = e_j(t_j, t_{-j})$. Therefore

$$x_j(t_j, t_{-j}) - e_j(t_j, t_{-j}) > x_j(t_j, t'_{-j}) - e_j(t_j, t'_{-j}). \tag{17}$$

Let $C \equiv I \setminus \{j\}$. From feasibility of x and (17), we have:

$$\begin{aligned} \sum_{i \in C} [x_i(t_j, t_{-j}) - e_i(t_j, t_{-j})] &= -[x_j(t_j, t_{-j}) - e_j(t_j, t_{-j})] \\ &< -\left[x_j(t_j, t'_{-j}) - e_j(t_j, t'_{-j})\right] \\ &= \sum_{i \in C} \left[x_i(t_j, t'_{-j}) - e_i(t_j, t'_{-j})\right]. \end{aligned}$$

Thus,

$$\delta \equiv \sum_{i \in C} \left[x_i(t_j, t'_{-j}) - e_i(t_j, t'_{-j}) - x_j(t_j, t_{-j}) + e_j(t_j, t_{-j})\right] > 0.$$

For each $i \in C$, let

$$\tau_i \equiv -x_i(t_j, t'_{-j}) + e_i(t_j, t'_{-j}) + x_i(t_j, t_{-j}) - e_i(t_j, t_{-j}) + \frac{\delta}{n-1},$$

so that $\sum_{i \in C} \tau_i = 0$ and

$$e_i(t_j, t_{-j}) + x_i(t_j, t'_{-j}) - e_i(t_j, t'_{-j}) + \tau_i > x_i(t_j, t_{-j}).$$

By the monotonicity of u_i, we can conclude that for all $i \in C$,

$$u_i\left(t_i, e_i(t_j, t_{-j}) + x_i(t_j, t'_{-j}) - e_i(t_j, t'_{-j}) + \tau_i\right) > u_i\left(t_i, x_i(t_j, t_{-j})\right),$$

which contradicts the assumption that x is coalitionally incentive compatible. This establishes the claim that x_j is $\mathcal{F}_j$-measurable.

Now, assume that x is not efficient. This means that there exists a feasible allocation y such that $y_j \succcurlyeq_j^{t_j} x_j$ for all $j \in I, t_j \in T_j$ and there is $i \in I, t'_i \in T_i$ such that $y_i \succ_i^{t_i} x_i$, that is, $\underline{y}_i(t'_i) > \underline{x}_i(t'_i)$. Since x_i is $\mathcal{F}_i$-measurable, this implies that $u_i(t'_i, y_i(t'_i, t_{-i})) > u_i(t'_i, x_i(t'_i, t_{-i}))$ for every t_{-i}. The monotonicity of u_i now gives $y_i(t'_i, t_{-i}) > x_i(t'_i, t_{-i})$. Similarly, $y_j \succcurlyeq_j^{t_j} x_j$ and the fact that x_j is $\mathcal{F}_j$-measurable imply that $y_j(t'_i, t_{-i}) \geqslant x_j(t'_i, t_{-i})$ for all $j \neq i$. But then, $\sum_{i \in I} y_i(t'_i, t'_{-i}) > \sum_{i \in I} x_i(t'_i, t'_{-i}) = \sum_{i \in I} e_i(t'_i, t'_{-i})$ and y is not feasible, which is a contradiction. ∎

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