# Ambiguity and compound risk attitudes: an experiment*

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**Abstract**

The identification of compound risk attitudes and ambiguity attitudes has recently received experimental support (Halevy, 2007) and been incorporated in decision models (Seo, 2009; Halevy and Ozdenoren, 2008; Segal, 1987). Non reduction of compound lotteries is this literature's explanation of Ellsberg type behavior.

We conduct an experiment measuring individual behavior under simple risk, under various types of compound risk and under ambiguity. We examine how each of these behaviors changes as the probability (or size) of the winning event varies. We find that attitudes towards all three types of uncertainties move from seeking to aversion as the probability level increases. Controlling for probability level, we find that the link between ambiguity and compound risk attitudes is partial and sensitive to the type of compound risk considered. We do not support the equivalence between reduction of these compound risks and ambiguity neutrality.

**JEL Classification:** C91 D81

**Keywords:** Ambiguity, Ellsberg paradox, Reduction of compound lotteries

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# 1 Introduction

Contrary to theories such as expected utility (Savage, 1954), developments in the theory of decision making under ambiguity (i.e., subjective uncertainty about probabilities) recognize that ambiguity is not always treated the same as a known risk. Segal (1987), for example, suggests that ambiguous bets (including Ellsberg (1961) bets[1]), are comparable to a two-stage risk, where the first stage lottery describes the probabilities of getting various lotteries in the second stage; Segal's model relies on the hypothesis of nonreduction of two-stage lotteries to generate ambiguity sensitive behavior. Other theories explicitly using violation of reduction of compound lotteries to model ambiguity attitudes include Seo (2009) and Halevy and Ozdenoren (2008).[2] In these papers, reduction of compound risk implies neutrality to ambiguity. Non reduction of compound lotteries is this strand of literature's explanation of Ellsberg type behavior. Since Segal (1987)'s intuition, very few papers have empirically explored the link between attitude towards ambiguity and attitude towards compound risk. There appear to be two main (and contradictory) empirical contributions dealing with this relationship. Bernasconi and Loomes (1992) test Segal's hypothesis using a compound risk version of Ellsberg's three color urn and, based on their finding of less Ellsberg type behavior than typically found under ambiguity, conclude:

> "(...) Segal's resolution of the Ellsberg's paradox is, at best, only a partial explanation." (p. 89)
>
> "(...) Our findings that 'ambiguous lotteries' in the sense of Ellsberg cannot be fully characterized by 'distributed lotteries' as suggested by Segal also undermine the possibility of viewing ambiguity aversion and risk aversion as 'the two sides of the same coin'." (p. 91)

More recently, Halevy (2007)[3] finds that attitude towards ambiguity is related to attitude towards compound risk. In fact, Halevy goes even further and claims that the lack of neutrality towards compound risk, i.e., non reduction of compound lotteries, is necessary for non neutral attitude towards ambiguity:

> "(...) subjects who reduced compound lotteries were almost always ambiguity neutral, and most subjects who were ambiguity neutral reduced compound lotteries appropriately." (p. 531)

[1]In Ellsberg's two-urn paradox, a subject faces two 100-ball urns containing red and black balls; the risky urn has a known $50 : 50$ composition and the ambiguous urn has an unspecified $x : 100 - x$ composition. Ellsberg shows that ambiguity aversion, defined as the preference for betting on red (black) being drawn from the risky urn over the same bets using the ambiguous urn, is incompatible with additive probability and therefore violates expected utility.

[2]We highlight these papers as they explicitly include objective compound lotteries and have clear implications for the relationship between behavior toward compound lotteries and behavior toward ambiguous acts. A number of related models, including Klibanoff et al. (2005), Nau (2006), Ergin and Gul (2009) and Neilson (2010) do not include objective compound lotteries among the objects of choice and need not hypothesize non reduction of such lotteries to generate ambiguity sensitivity.

[3]This study uses stimuli similar to those used in Yates and Zukowski (1976) and Chow and Sarin (2002) and inspired by Ellsberg's (1961) two-urn paradox. Specifically, subjects evaluate bets on the color of balls drawn from four different 10 ball urns: the first urn (simple risk) has 5 black and 5 red balls, the second urn (ambiguity) has black and red in unknown proportion, the last two urns (compound risks) have their composition determined by first-stage random draws. For the third urn, a draw generates an equal chance of each color composition, while for the fourth urn, a draw generates a half chance of all black and a half chance of all red.

> "(...) The results suggest that failure to reduce compound (objective) lotteries is the underlying factor of the Ellsberg paradox, and call upon decision theory to uncover the theoretical relationship between ambiguity aversion and different forms in which reduction may fail." (p. 532)

The design of the Halevy (2007) study has a number of advantages compared to that of Bernasconi and Loomes (1992). Most importantly, Halevy observes choices under both compound risk and ambiguity for each subject while Bernasconi and Loomes observe behavior only under compound risk and compare with behavior under ambiguity from previous studies. One of the main motivations of the present study is to further explore the relationship between attitudes towards ambiguity and compound risks using a design sharing this and other positive features with Halevy. Is it the case that any careful design of this form will confirm Halevy's findings or is there still substantial empirical uncertainty about the relationship?

We design and conduct an experiment using the valuation of bets on Ellsberg-like urns to measure individual behavior under simple risk, under compound risk and under ambiguity. For each type of uncertainty, we examine bets with different probabilities of winning (in the case of ambiguity, different sizes of the winning event) in order to examine the way (if any) in which these valuations and their relationship change with these probabilities. Like Halevy (2007), our design allows within subject comparisons across the types of uncertainty.

We find that attitudes towards all three types of uncertainty are strongly influenced by the event on which the good outcome is realized: larger events in the sense of set containment tend to make risk, ambiguity and compound risk less attractive. Just as simple risk attitudes are usually measured by comparing the valuations of simple risks to their expected values, we measure attitudes towards compound risks by comparing the valuations of compound risks to the valuations of the reduced simple risks. Ambiguity attitudes are measured by a similar comparison of valuations of ambiguous bets to those of simple risks. The most novel result with respect to existing literature shows that attitudes towards most of the compound risks we examine move from compound risk seeking for low winning probabilities to compound risk aversion for high winning probabilities. In our view, behavior toward compound risk is relatively understudied in the literature. This seems especially true in light of the low (if any) cost of manipulating the form of compound risk as compared to the cost of changing the actual probability distribution – to the extent that behavior differs substantially toward compound risks as compared to simple risks, the marketing, policy and economic implications of understanding this behavior may be large.

Our data confirm that some compound risk attitudes help predict (although quite partially) ambiguity attitudes. However, except in the sense that, defined stringently enough, essentially all subjects both fail to reduce compound risk and are non-neutral toward ambiguity, our data do not generally support the equivalence between reduction of specific compound risks and ambiguity neutrality. In particular, for the case most comparable to Halevy (2007), we find that roughly half of subjects who reduce uniform compound lotteries are non-neutral to ambiguity, with the majority of those non-neutral exhibiting Ellsberg-type aversion toward ambiguity. Similarly, only about half of subjects who are ambiguity neutral reduce uniform compound risks. We find similar results even when using compound lotteries (hypergeometric in our case) that closely match a plausible mental model of the process used to generate the ambiguous urn. Thus, our data support neither the conclusion in the quote from Halevy nor the models of Segal (1987), Seo (2009) and Halevy and Ozdenoren (2008).[4] Our results suggest that the relationship between attitudes toward compound risk and toward ambiguity is more complex than simple identification. We provide estimates of this relationship.

Section 2 details the experimental design. Section 3 describes the attitudes of subjects toward simple risk, compound risk and ambiguity. Section 4 discusses the findings on the relationship between ambiguity and compound risk attitudes. Section 5 concludes.

# 2 Experiment

## 2.1 Procedure

In the overall experiment, subjects faced thirty-two gambles. The experiment was divided into, successively, simple risk (R), ambiguity (A), and two-stage compound risk (CR) parts. These three types of uncertainty were each represented by Ellsberg-like urns and subjects had to consider bets whose outcome depended on which color ball(s) were drawn from the urn(s). See Figure 1 for an example of bets involving each type of uncertainty. Stimuli were displayed on a computer monitor in order to describe each choice situation. At any point during each of the parts, subjects could return to and revise the responses regarding earlier bets in that part. For the simple (resp. compound) risk, the subjects saw the color of the balls in one (resp. two successive stages of) urn(s), and thus could infer the probability of winning the bet. For ambiguity, the colors of the balls in the urn were hidden by making the urn opaque. A training question was asked at the beginning of each part to check whether subjects had a correct understanding of the design and of the type of uncertainty faced.

All bets had a winning payoff of 50 euros and a losing payoff of 0 euros. To elicit certainty equivalents, we use an iterative multiple price list procedure adapted from Abdellaoui et al. (2011a). For each bet, subjects make choices between the bet and (an ascending range of) sure payments. This is done in three steps. The first step consists of six choices between the bet and a sure payment; the sure payments are equally spaced between 0 and 50 euros (see Figure 15 in Appendix C). The second step consists of a new set of eleven choices, where the sure payments span the narrower range between the lowest sure payment that the respondent had rejected and the highest sure payment he had accepted in the previous step (Figure 16). At the third step, all the choices made in the first two steps are displayed and the subject is given an opportunity to revise any of the choices if desired (Figure 17).[5]

We conducted three probability treatments – for most types of uncertainty, certainty equivalents were elicited for 3 different probabilities $(1/12, 1/2, 11/12)$ of winning 50 euros. For ambiguity, instead of probability levels, we varied the fraction of winning colors: $(1/12, 1/2, 11/12)$.

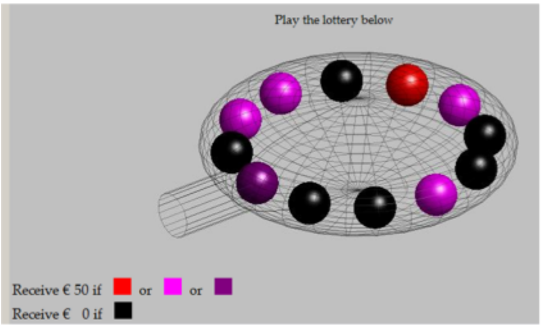
[4] All these except Halevy and Ozdenoren also make the reverse claim that neutrality towards ambiguity implies neutrality towards compound lotteries. Our data reject this claim as well.

[5] While the software forced monotonicity in the first two steps, violations of monotonicity were allowed in step three. Hence, subjects were not allowed in the first two steps to choose, for example, 10 euros for sure rather than the bet and then choose the same bet rather than 20 euros; but this was allowed in the final step if the subject wished. None of the subjects violated monotonicity.

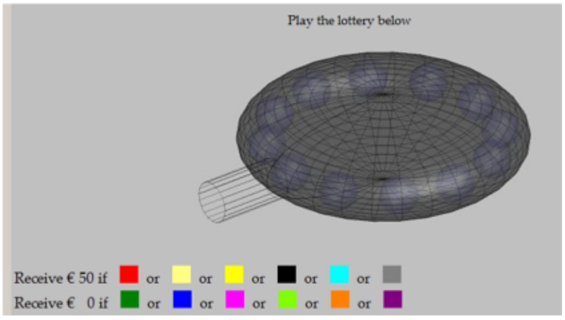
The specific simple risk, ambiguous and compound risk bets used for each probability treatment are listed in Tables 1 and 2. The notation $(p,\ 50; 0)$ represents a simple lottery with probability $p$ of winning 50 and $(1-p)$ of winning 0. Similarly, $(q_1, (r_1, 50; 0); ...; q_m, (r_m, 50; 0))$ represents a two-stage compound lottery with first stage probability $q_i$ of a second stage lottery giving 50 with probability $r_i$ and 0 with probability $1 - r_i$. Additionally, $(q, (r, 50; 0); c)$ represents a two-stage compound lottery with first stage probability $q$ of a second stage lottery giving 50 with probability $r$ and 0 with probability $1-r$ and first stage probability $1-q$ of giving amount $c \in \{0, 50\}$.[6] Finally, $(k\ colors,\ 50; n-k\ colors,\ 0)$ represents a bet on an ambiguous urn that yields 50 if one of $k$ colors is drawn and 0 if one of the other $(n-k)$ colors is drawn. Appendix B gives the visual depiction of the full set of stimuli for the probability level one-half.

Figure 1: Ellsberg-like bets

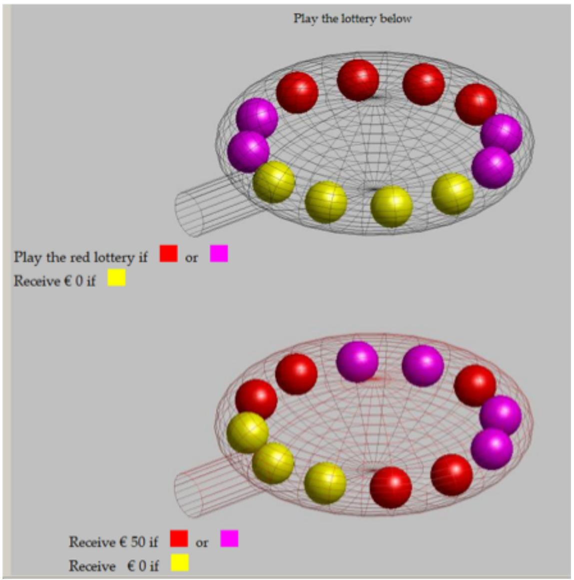
(a) Simple risk




(b) Ambiguity




(c) Compound risk




[6]Even though, probabilistically, this second compound risk notation is encompassed by the first notation, we use it to better reflect the different depictions of some of the compound risks in the experiment. For some of the compound risks, first stage outcomes that resolve all uncertainty are depicted as leading to second stage urns that contain all balls of the same color, while in other compound risks those outcomes are depicted as simply giving a monetary amount directly. This is the distinction between the "CR high with explicit degenerate urn" and "CR high" stimuli (as well as between the "CR low with explicit degenerate urn" and "CR low" stimuli) listed in Table 2.

| **Probability ↓** | **Simple risk**[a] | | **Ambiguity** | |
|---|---|---|---|---|
| **Urn(s)⟶** | 12 ball | 2 ball | 12 ball | 2 ball |
| **1/12** | $(1/12, 50; 0)$ | - | $(1\ color, 50; 11\ colors, 0)^b$ | - |
| **1/2** | $(1/2, 50; 0)$ | $(1/2, 50; 0)$ | $(6\ colors, 50; 6\ colors, 0)$ | $(1\ color, 50; 1\ color, 0)^c$ |
| **11/12** | $(11/12, 50; 0)$ | - | $(11\ colors, 50; 1\ color, 0)$ | - |

Table 1: Simple Risk and Ambiguity Stimuli

[a]The stimuli also included a 6 ball version of $(1/2, 50; 0)$ but we make no use of it in this paper.
[b]Subjects faced this stimuli three times with the winning color varied.
[c]Subjects faced this stimuli two times with the winning color varied.

| **Probability ↓** | **Diverse uniform CR** | **Degenerate uniform CR** | **Hypergeometric CR** |
|---|---|---|---|
| **Urn(s)⟶** | 12 ball in the first stage and 2 ball in the second stage | | |
| **1/12** | - | - | $(1/6, (1/2, 50; 0); 5/6, (0, 50; 0))$ |
| **1/2** | $(1/4, (1, 50; 0); 1/4, (1/2, 50; 0);$ $1/4, (1/2, 50; 0); 1/4, (0, 50; 0))$ | $(1/2, (1, 50; 0); 1/2, (0, 50; 0))$ | $(5/22, (1, 50; 0); 12/22, (1/2, 50; 0);$ $5/22, (0, 50; 0))$ |
| **11/12** | - | - | $(5/6, (1, 50; 0); 1/6, (1/2, 50; 0))$ |

| **Probability ↓** | **CR high** | **CR low** | **CR high with explicit degenerate urn** | **CR low with explicit degenerate urn** |
|---|---|---|---|---|
| **Urn(s)⟶** | 12 ball in both the first and second stage | | | |
| **1/12** | $(1/2, (1/6, 50; 0); 0)$ | $(1/6, (1/2, 50; 0); 0)$ | $(1/2, (1/6, 50); 1/2(1, 0))$ | $(1/6, (1/2, 50; 0); 5/6(1, 0))$ |
| **1/2** | $(3/4, (2/3, 50; 0); 0)$ | $(2/3, (3/4, 50; 0); 0)$ | $(3/4, (2/3, 50; 0); 1/4(1, 0))$ | $(2/3, (3/4, 50; 0); 1/3(1, 0))$ |
| **11/12** | $(5/6, (1/2, 50; 0); 50)$ | $(1/2, (5/6, 50; 0); 50)$ | $(5/6, (1/2, 50; 0); 1/6(1, 50))$ | $(1/2, (5/6, 50; 0); 1/2(1, 50)$ |

Table 2: Compound Risk Stimuli

## 2.2 Choice of compound risks

Our motivation for the various types of compound risk we used in the experiment includes the following. Motivated by Halevy (2007), we included the diverse uniform CR and degenerate uniform CR to mimic the two compound risks that study used. The hypergeometric CR were designed to be similar to the process used to generate the ambiguous urns in our study – in both cases balls are drawn (without replacement) from a larger urn and placed in a new urn, a draw from which determines the outcome of the bet. Notice that the diverse uniform CR corresponds to a similar process of urn formation, the only difference being that the balls from the larger urn would be drawn with replacement. This makes these compound risks natural candidates for a subject to potentially identify with their "mental model" of the ambiguous urn.[7] The CR high and CR low compound risks were included both to allow a test of the reach of any connection between ambiguity and compound risk and to allow for the possibility (as examined in e.g., Friedman, 2005) that exchanging the first-stage and second-stage probabilities might have a systematic effect. We do not find a consistent effect of this kind and therefore do not emphasize the distinction between CR high and CR low in our analysis. Finally the CR high and low with degenerate urn compound risks were included to see if making the stage structure of the risks more explicit has any effect. We do find such a framing effect, but leave this for future work to investigate.

## 2.3 Ambiguity implementation

The ambiguous urns were generated through a two-stage process involving physical bags of balls. Before the experiment, each subject was asked to generate his own two ambiguous urns by drawing twelve balls (resp. two balls) from a bag containing 144 balls (resp. 4 balls) evenly divided among the 12 (resp. 2) colors. For these bags, the subject was told that the balls could be of 12 (resp. 2) possible colors, but was informed of neither the number of balls nor the distribution of colors in the bags. Each ball was marked with one of the colors beforehand by the experimenter, but this mark was hidden from both the experimenter and the subject during the experiment.

This two-stage process has two main advantages. First, the final color composition of the ambiguous urns is unknown to both the subject and the experimenter reducing possible suspicion[8] (see e.g., Hey et al. 2010) and comparative ignorance effects (see e.g., Fox and Tversky, 1995); these effects have been shown to elevate ambiguity aversion. Second, this process may induce in subjects a two-stage representation of ambiguity (however, it does not guarantee that subjects used such a representation); hence, one cannot argue that possible similarities in attitudes towards two-stage compound risk and ambiguity are undermined by the way ambiguity has been implemented. If anything, our set-up might be biased in the direction of making such a connection stronger.

[7]We thank Yoram Halevy for emphasizing this to us in conversations.

[8]Note that in our experiment, suspicion is also reduced by the variety of bets considered. Bets entailed alternative subsets of winning colors among the twelve available; consequently, the experimenter could hardly bias the determination of the composition of the ambiguous urn to the subject's disadvantage.

### 2.4 Incentives

A Random Lottery Incentive mechanism (RLI) was used to provide incentives.[9] Subjects performed the choice tasks knowing that one of their choices would be randomly drawn at the end of the session and played for real. If the randomly selected choice was for a bet involving (compound) risk, the corresponding urn(s) were created and the subject physically selected the ball(s) that determined the payment, and then were given the money. In the case of a bet on an ambiguous urn, they drew the ball that determined the payment from the urn (either 12 or 2 ball depending on the question) they had generated at the beginning of the experiment, and then were given the money. In the case of choosing a sure amount of money, subjects were given that amount of money. Thus, subjects' total payoffs in the RLI treatment ranged between 0 and 50 euros.

### 2.5 Sample

64 subjects were recruited, 51 from Arts et Métiers ParisTech (Engineering School) and 13 from a Master in quantitative economics (Paris 1). Subjects were all well acquainted with probability but had no knowledge of decision theory. The experiment consisted of individual and computer-based interviews using specific software built for the experiment. The subject and the experimenter sat in front of a computer together; in an introductory phase, the experimenter explained the study through examples of stimuli involving different (combination of) urn(s), and the ambiguous urns were generated as described earlier; the software was started and the subject began with the training involving a risky stimulus. During this training phase, the principle of the multiple choice list was explained and the random lottery incentive mechanism described. Then, for each screen where choices were needed, the subject verbally indicated his choices to the experimenter who then entered them onto the screen. We chose this method of entry rather than having the subjects click to enter their choices themselves to reduce the possibility of subjects carelessly and rapidly clicking through many answers without thinking. Subjects who asked to were allowed to calculate with pencil and paper (only one subject did so). The software then moved to the next screen and this process continued until all the choices for all of the stimuli were made. The experiment ended with the random selection of a choice and the implementation of what the subject had indicated for that choice.

## 3 Attitudes toward uncertainty and their probability dependence

Table 3 reports statistics on the observed certainty equivalents for the four different uncertainty treatments. We begin by discussing behavior toward simple risk and toward ambiguity. Subsequently we discuss behavior toward compound risk.

[9]The RLI has been theoretically criticized (Holt, 1986) but this criticism is not supported by empirical evidence (Starmer and Sugden, 1991); see Wakker (2007) for a detailed discussion.

## 3.1 Attitudes toward simple risk and toward ambiguity

As Table 3 illustrates, from the observed certainty equivalents (R) for simple risks, on average subjects were risk seeking for the treatments with low probability of winning and risk averse for the treatments with high probability of winning. This pattern is consistent with a large body of experimental literature on risk attitudes (see e.g., Wakker 2010, p. 204 for references). Also consistent with previous evidence is the substantial individual heterogeneity in risk attitudes as illustrated by the relatively large standard deviations.

Similarly, on average, subjects were ambiguity seeking for the treatment with one winning color and ambiguity averse for the treatments with more winning colors. In contrast to simple risk, there is only a small body of literature exploring how ambiguity attitude varies with the size of the winning event. Furthermore, some of that literature operationalizes ambiguity as compound risk (e.g., Kahn and Sarin, 1988) and so, in our context, should be viewed as evidence on compound risk attitude rather than on ambiguity attitude. As with simple risk, the pattern of moving from ambiguity seeking for small winning events to ambiguity aversion for large winning events is consistent with previous evidence (Abdellaoui et al., 2011a; Tversky and Fox, 1995; Einhorn and Hogarth, 1985; Curley and Yates, 1985).

To classify subjects as ambiguity averse/neutral/seeking we compared their certainty equivalents in the ambiguous treatment with $i$ winning colors to the certainty equivalents in the simple risk treatment with probability $i/12$ of winning. A subject is ambiguity averse/neutral/seeking when the former certainty equivalent is below/equal/above the latter. Our use of the number of winning colors as a sufficient measure of the "probability" of winning relies on the assumption that subjects view the colors symmetrically. Specifically, in terms of certainty equivalents, we assume that if two events $E$ and $E'$ consist of the same number of colors, then a bet on $E$ has the same certainty equivalent as a bet with the same stakes on $E'$. Such symmetry was emphasized and experimentally verified in a previous study (Abdellaoui et al., 2011a) that operationalized ambiguity in exactly the same way as in this paper. We conducted a further verification with our subject pool for the case of one winning color and find that the hypothesis that the colors yield the same certainty equivalent cannot be rejected within subject (MANOVA for repeated measure, p=0.49).

| **Winning probability/Number of winning color(s)** $\longrightarrow$ | **1/12** | | | **6/12** | | | **11/12** | | |
|---|---|---|---|---|---|---|---|---|---|
| **Certainty equivalents (in euros)** $\downarrow$ | Mean | Median | SD | Mean | Median | SD | Mean | Median | SD |
| Expected value (EV) | 4.17 | 4.17 | 0 | 25 | 25 | 0 | 45.83 | 45.83 | 0 |
| Risk (R) - 2 ball | - | - | - | 24.13 | 24.50 | 7.91 | - | - | - |
| Risk (R) - 12 ball | 7.81 | 7.50 | 5.22 | 24.53 | 24.50 | 7.65 | 43.27 | 44.50 | 5.84 |
| Ambiguity (A) - 2 ball[a] | - | - | - | 22.73 | 24.50 | 8.10 | - | - | - |
| Ambiguity (A) - 12 ball | 9.36[b] | 8.83 | 6.87 | 20.20 | 20.50 | 8.25 | 37.53 | 41.50 | 12.50 |
| Diverse uniform compound risk (CRU) | - | - | - | 22.47 | 24.50 | 7.64 | - | - | - |
| Degenerate uniform compound risk (CRUd) | - | - | - | 24.38 | 24.50 | 6.63 | - | - | - |
| Hypergeometric compound risk (CRG) | 6.78 | 4.50 | 5.12 | 21.58 | 23.00 | 7.17 | 41.73 | 44.50 | 7.19 |
| Compound risk high (CRH) | 9.72 | 9.50 | 6.19 | 24.23 | 24.50 | 7.11 | 36.72 | 39.00 | 8.57 |
| Compound risk high with explicit degenerate urn (CRHe) | 9.63 | 9.50 | 6.15 | 23.08 | 23.00 | 8.94 | 39.38 | 40.50 | 7.69 |
| Compound risk low (CRL) | 8.78 | 9.50 | 5.19 | 24.05 | 24.50 | 7.24 | 37.50 | 40.50 | 8.98 |
| Compound risk low with explicit degenerate urn (CRLe) | 8.75 | 7.50 | 5.44 | 22.70 | 21.00 | 9.19 | 39.58 | 43.50 | 8.83 |

Table 3: Certainty equivalents and attitudes towards simple risk, compound risks and ambiguity

[a]The reported numbers are the average of two certainty equivalents for each subject, as they evaluated an ambiguous two-ball urn twice.

[b]In this case, for 1/12 *only*, the figures are based on the average of three certainty equivalents for each subject, as they evaluated an ambiguous 12-ball urn (with 1 winning color) three times.

| Winning probability/Number of winning color(s) ⟶<br>Attitudes displayed (% of subjects) ↓ | 1/12 | 6/12 | 11/12 |
|---|---|---|---|
| Risk averse (R < EV) | 12.50 | 35.94 | 54.69 |
| Risk Neutral (R = EV) | 25.00 | 37.50 | 17.19 |
| Risk seeking (R > EV) | 62.50 | 26.56 | 28.12 |
| Ambiguity averse (A < R) | 31.25 | 45.31 | 48.44 |
| Ambiguity Neutral (A = R) | 29.69 | 39.06 | 23.44 |
| Ambiguity seeking (A > R) | 39.06 | 15.63 | 28.12 |
| CRU averse (CRU < R) | - | 37.50 | - |
| CRU neutral (CRU = R) | - | 34.37 | - |
| CRU seeking (CRU > R) | - | 28.13 | - |
| CRUd averse (CRUd < R) | - | 28.12 | - |
| CRUd neutral (CRUd = R) | - | 42.19 | - |
| CRUd risk seeking (CRUd > R) | - | 29.69 | - |
| CRG averse (CRG < R) | 48.44 | 56.25 | 42.19 |
| CRG neutral (CRG = R) | 26.56 | 23.44 | 23.44 |
| CRG seeking (CRG > R) | 25.00 | 20.31 | 34.37 |
| CRH averse (CRH < R) | 29.69 | 42.19 | 68.75 |
| CRH neutral (CRH = R) | 32.81 | 20.31 | 18.75 |
| CRH seeking (CRH > R) | 37.50 | 37.50 | 12.5 |
| CRHe averse (CRHe < R) | 28.13 | 45.31 | 59.38 |
| CRHe neutral (CRHe = R) | 26.56 | 17.19 | 18.75 |
| CRHe seeking (CRHe > R) | 45.31 | 37.50 | 21.87 |
| CRL averse (CRL < R) | 31.25 | 40.63 | 70.31 |
| CRL neutral (CRL = R) | 32.81 | 21.88 | 15.63 |
| CRL seeking (CRL > R) | 35.94 | 37.50 | 14.06 |
| CRLe averse (CRLe < R) | 31.25 | 48.44 | 50.00 |
| CRLe neutral (CRLe = R) | 31.25 | 17.19 | 23.44 |
| CRLe seeking (CRLe > R) | 37.50 | 34.37 | 26.56 |

Table 4: Attitudes towards simple risk, compound risks and ambiguity by probability treatment

## 3.2 Attitude toward compound risk and reduction of compound lotteries

Just as classic Bayesian expected utility theories ignore the distinction between ambiguity and simple risk, they also ignore the distinction between simple risk and compound risk. One of the main assumptions of most economic models of decision making under risk is that the nature and the complexity of a lottery should not affect its evaluation. Most models implicitly or explicitly incorporate a reduction of compound lotteries axiom that requires a decision maker to be indifferent between the extensive and the reduced form of a compound lottery. We investigate attitude toward compound risk by comparing certainty equivalents between the compound risk treatment and the simple risk treatment giving the same reduced probability. We say a subject is compound risk averse/neutral/seeking when the certainty equivalent for the compound risk is below/equal/above the certainty equivalent for the reduced simple risk. Recall that for each reduced probability level we examined either five or seven (if probability 1/2) two-stage risks differing only in the way probability was divided up across the stages.

Although there is substantial heterogeneity across subjects, we find, on average, compound risk seeking for low probabilities moving to compound risk aversion for high probabilities for all but the hypergeometric compound risk. For CRH, CRHe, CRL and CRLe, a Cuzick (1985) non parametric test for trend strongly rejects no trend (p=0.000 for each of the first three and p=0.012 for the last) in compound risk premium with respect to expected value, confirming this pattern of compound risk premium increasing with probability. Recent literature has not investigated the relationship between probability and compound risk attitudes. However, an older experimental literature examining ambiguity operationalized as compound risk could be interpreted as providing some evidence for the pattern of compound risk seeking for low probabilities and compound risk aversion for high probabilities (Kahn and Sarin, 1988) and some against (Larson, 1980). That compound risk seeking is common for low probability levels is consistent with Friedman (2005) who examines only low probability compound lotteries (with winning probabilities ranging from 0.0625 to 0.5625) and finds that individuals tend to value the compound lotteries more than their reduced simple risks. Friedman (2005) also finds a lack of effect of exchanging probabilities between the two stages of a compound lottery.

A natural theory of failure to value compound risks identically to their corresponding reduced simple risks is that calculating the correct reduced probabilities is mentally burdensome. Since it is hard to see how finding the reduced probabilities for the 1/12 and 11/12 cases is not computationally equally difficult, the fact that for most of our compound risks there are such opposite departures from reduction for those two treatments (leading to overvaluation in the case of 1/12 and undervaluation in the case of 11/12 relative to reduced simple risk) strongly suggests that this theory of non-reduction cannot be whole explanation. The fact that there are differences in the evaluations of CRH versus CRHe, and CRL versus CRLe indicate that there may be interesting pure framing issues in the presentation of compound lotteries for future work to explore. See Budescu and Fischer (2001) for additional studies and discussion of behavior toward compound lotteries.

### 3.3 The impact of probability on attitudes towards ambiguity, simple risk and compound risk

Given Tables 3 and 4 and the previous discussion, a key finding is that increases in probability of the good outcome (under ambiguity, making the winning event larger) changes typical behavior toward simple risk, compound risk (excepting CRG) and ambiguity from seeking to aversion. This highlights the importance of controlling for probability when measuring uncertainty attitudes. It also suggests that any descriptive model intended to apply to the full range of uncertain situations must allow attitudes to change with probabilities. We conclude this section with Table 5, summarizing the dependence of the average premia (i.e., difference in the corresponding average certainty equivalents) on probabilities.

An earlier, preliminary study (Abdellaoui et al., 2011b) we conducted using simple risk, ambiguity and only CRL and CRH types of compound risks but using more probability levels (1/12, 1/6, 1/3, 1/2, 2/3, 5/6, 11/12) confirms a similar pattern of moving from seeking to aversion as the probability increases.

| | Average premia[10] | | | | | | |
|---|---|---|---|---|---|---|---|
| **Probability** | **Simple risk** | **Ambiguity** | **Compound risk** | | | | |
| | **EV-R** | **R-A** | **R-CRG** | **R-CRH** | **R-CRHe** | **R-CRL** | **R-CRLe** |
| **1/12** | -3.65*** | -1.55* | 1.03* | -1.91** | -1.81** | -0.97 | -0.94 |
| **1/2** | 0.47 | 4.33*** | 2.95*** | 0.30 | 1.45 | 0.48 | 1.83 |
| **11/12** | 2.57*** | 5.73*** | 1.53 | 6.55*** | 3.89*** | 5.77*** | 3.69*** |

Table 5: Uncertainty premia

# 4 The relationship between ambiguity attitudes and compound risk attitudes

## 4.1 Relating reduction of compound risk and ambiguity neutrality

### 4.1.1 A comparison with Halevy (2007)

The strongest and most striking evidence in Halevy (2007) for the identification of ambiguity attitude with compound risk attitude is a simple contingency table relating neutrality/non-neutrality towards ambiguity and reduction/non-reduction of compound risk. As all bets in Halevy (2007) either have objective probability 1/2 of winning or, in the case of ambiguity, win if one of the two possible colors is drawn, the most direct comparison is to our results for probability 1/2. We focus initially on the 2 ball risky urn, ambiguous urn, diverse uniform compound urn and degenerate uniform compound urn as they are the closest to the four urns used by Halevy (2007).[11] In this case, for our simple risk treatment, the median subject is risk neutral while the average subject is slightly risk averse. This is similar to the behavior of subjects toward simple risk in Halevy (2007) – there, the median subject is risk neutral and the average subject in the low dollar value treatment (expected value of 1 dollar) is very slightly risk loving while for the high dollar value treatment (expected value of 10 dollars), the average subject is modestly risk averse. With regard to ambiguity, our average subject is somewhat ambiguity averse, while the median subject is ambiguity neutral. In Halevy (2007), both the median and average behavior is ambiguity averse. A possible source of this increased incidence of ambiguity aversion, following Fox and Tversky (1995), is that Halevy's design involves thinking about the ambiguous and simple risk alternatives simultaneously while, in our study, these are separated in that there are a number of simple risk questions, followed by a number of ambiguity questions. Fox and Tversky show that increasing the salience of the comparison increases ambiguity aversion. With regard to the two uniform compound risks, both our data and Halevy find that many subjects violate reduction (83.8 % for Halevy and 73.4% in our data).

[11]The only differences are that our urns (final stage urns in the case of compound risk) contain two balls rather than 10 balls and our diverse uniform compound urn is uniform over permutations of ball colors, so it is a mean-preserving squeeze compared to Halevy's third urn that is uniform on color compositions. Since the motivation for having the third urn in Halevy(2007) is to have it generate a mean-preserving squeeze compared to his degenerate uniform compound urn, this change should, if anything, generate a more pronounced effect.

In Table 6, we construct a contingency table for our data alongside the table reported by Halevy. With the four urns we are focusing on, we observe a certainty equivalent for the simple lottery (1/2, 50; 1/2, 0), the compound lottery (1/4, (1, 50); 1/4, (1/2, 50); 1/4, (1/2, 50); 1/4, (0, 50)), the compound lottery (1/2, (1,50); 1/2, (0,50)), and the ambiguous bet (50 if 1 color; 0 otherwise). Reduction of compound risk is satisfied if the certainty equivalents for the simple risk (denoted by R) and the two compound risks (denoted by CRU and CRUd, respectively) are equal. Ambiguity neutrality is satisfied if the certainty equivalent for the simple risk and for the ambiguous bet are equal.

| | | | **Compound risk attitudes** | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | | **Halevy (2007)** | | | **The present study Reduce means R=CRU=CRUd** | | |
| | | | **Reduce** | **Do not reduce** | **Total** | **Reduce** | **Do not reduce** | **Total** |
| **Ambiguity attitudes** | **Neutral** | Count | 22 | 6 | 28 | 8 | 9 | 17 |
| | | Expected | 4.5 | 23.5 | | 4.5 | 12.5 | |
| | **Non neutral** | Count | 1 | 113 | 114 | 9 | 38 | 47 |
| | | Expected | 18.5 | 95.5 | | 12.5 | 34.5 | |
| | **Total** | | 23 | 119 | 142 | 17 | 47 | 64 |
| **Fisher's exact test p-value (2-tailed)** | | | 0.0000 | | | 0.051 | | |

Table 6: Contingency table relating ambiguity and two uniform compound risks for probability one-half

Both sets of data show a relationship between reduction/non reduction and ambiguity neutrality/non neutrality. While Halevy's data suggests something close to identification – conditional on reducing compound risk, one out of 22 subjects is non neutral toward ambiguity, while conditional on ambiguity neutrality, 6 out of 28 subjects fail to reduce the compound risks – our data suggests a weaker connection – conditional on reducing the compound risks, 9 out of 17 subjects are non neutral toward ambiguity (6 are ambiguity averse and 3 are ambiguity seeking), while conditional on ambiguity neutrality, 9 out of 17 subjects fail to reduce the compound risks.

More generally, the contingency table in Halevy can be read narrowly or broadly – is it reduction of these *specific* compound risks and neutrality toward this *specific* ambiguity that are tied together, or is it reduction of compound risks in general and neutrality toward general ambiguities that are intimately related? As our data includes a variety of compound risks as well as ambiguous bets involving events of varying sizes, we can further investigate this issue.

### 4.1.2 Using other compound risk premia

Some evidence that it is not sufficient to consider reduction/non-reduction of just any compound risks giving reduced probability one-half is provided by examining the compound risks denoted earlier by CRL and CRH. This gives rise to the following contingency table.

| | | | **Compound risk attitudes Reduce means R=CRL=CRH** | | |
|---|---|---|---|---|---|
| | | | **Reduce** | **Do not reduce** | **Total** |
| **Ambiguity attitudes** | **Neutral** | Count | 3 | 22 | 25 |
| | | Expected | 2.7 | 22.3 | |
| | **Non neutral** | Count | 4 | 35 | 39 |
| | | Expected | 4.3 | 34.7 | |
| | **Total** | | 7 | 57 | 64 |
| **Fisher's exact test p-value (2-tailed)** | | | 1.0000 | | |

Table 7: Contingency table relating ambiguity and two compound risks for probability one-half

Table 7 provides no support for a link between ambiguity neutrality and reduction of these compound risks. Of those who reduce, the majority are ambiguity non-neutral. Of those who are ambiguity neutral, the vast majority do not reduce.

Another interpretation of what a general link between reduction of compound risk and neutrality toward ambiguity might mean is that individuals who reduce all compound risks are neutral to all ambiguity and vice-versa. It is impossible to literally observe behavior toward all risks and ambiguities, but we can examine how the reduction of all 17 compound risks in our study relates to neutrality toward all 7 ambiguous bets in our study. The result is Table 8.

| | | | **Compound risk attitudes Reduce means same probability implies same CE** | | |
|---|---|---|---|---|---|
| | | | **Reduce** | **Do not reduce** | **Total** |
| **Ambiguity attitudes** | **Neutral** | Count | 1 | 0 | 1 |
| | | Expected | 0.0 | 1.0 | |
| | **Non neutral** | Count | 0 | 63 | 63 |
| | | Expected | 1.0 | 62 | |
| | **Total** | | 1 | 63 | 64 |
| **Fisher's exact test p-value (2-tailed)** | | | 0.016 | | |

Table 8: Contingency table relating ambiguity and compound risks for all questions

We take from Table 8 the conclusion that if you ask subjects to evaluate enough bets, essentially all of them will sometimes fail to reduce compound risk and will sometimes behave in a non-neutral way toward ambiguity. The one subject who did neither evaluated all options according to expected value – this subject was neutral to risk, compound risk and to ambiguity. Thus, descriptively, a link between ambiguity non-neutrality and non-reduction of compound risks becomes tautological with enough data on each subject – both describe almost all subjects.

Returning to the specific risk interpretation, arguably the hypergeometric compound risks we described earlier come closest to the process actually used to generate the ambiguous urn.

Under the theory that subjects use such a compound risk as their mental model of the ambiguity, hypergeometric compound risks might be ideal candidates for generating a relationship. Table 9 gives the results when substituting the hypergeometric risk for the two uniform compound risks in Table 6.

| | | | **Compound risk attitudes Reduce means R=CRG** | | |
|---|---|---|---|---|---|
| | | | **Reduce** | **Do not reduce** | **Total** |
| **Ambiguity attitudes** | **Neutral** | Count | 8 | 9 | 17 |
| | | Expected | 4.8 | 12.2 | |
| | **Non neutral** | Count | 10 | 37 | 47 |
| | | Expected | 13.2 | 33.8 | |
| | **Total** | | 18 | 46 | 64 |
| **Fisher's exact test p-value (2-tailed)** | | | 0.060 | | |

Table 9: Contingency table relating ambiguity and the hypergeometric compound risk for probability one-half

Just as with the uniform compound risks, the data indicate a relationship between reduction/non reduction and ambiguity neutrality/non neutrality, but it appears quite partial – conditional on reducing the compound risk, 10 out of 18 subjects are non neutral toward ambiguity (4 are ambiguity averse and 6 are ambiguity seeking), while conditional on ambiguity neutrality, 9 out of 17 subjects fail to reduce the compound risk.

## 4.2 Estimating the relationship between ambiguity attitudes and compound risk attitudes

Our results so far suggest that the relationship between attitude towards compound risk and ambiguity is more complex than simple identification. To further explore this relationship, we use regression analysis to relate various ambiguity premia to various compound risk premia. Define the ambiguity premium ($AP$) for an ambiguous bet for which the proportion of winning colors is $p$ as the certainty equivalent for a simple lottery with probability of winning $p$ minus the certainty equivalent for the ambiguous bet. Similarly the compound risk premium ($CRP$) for a compound risk with probability of winning $p$ (under reduction) is the certainty equivalent for a simple lottery with probability of winning $p$ minus the certainty equivalent of the compound risk. We estimate the link between $AP$ and $CRP$ for several levels of $p$ and several varieties of compound risks.

To begin, we return to the case most comparable to Halevy (2007) using our 2 ball urns to estimate the model

$$\text{Model A: } AP_i = a + b * CRPU_i + c * CRPUd_i + e_i$$

where $i = 1, \ldots, 64$ indexes subjects, $CRPU$ is the diverse uniform compound risk premium and $CRPUd$ is the degenerate uniform compound risk premium. The OLS estimates for model A are reported in Table 10. We report coefficient estimates (in euro), OLS standard errors for each coefficient as well as heteroskedasticity robust standard errors.[12] The estimates indicate a significant positive effect of the compound risk premia on the ambiguity premium – greater compound risk aversion is associated with greater ambiguity aversion. This relationship, however, is far from identification. Knowledge of the compound risk premia explains only 21.54% of the variation in the ambiguity premium.

| **Dep. variable:** | **Model A** | | |
|---|---|---|---|
| **AP (2 ball)** | **Coef** | **Standard errors**[13] | |
| **n=64** | | **OLS** | **Robust** |
| constant | 1.1151 | 0.8900 | 0.8641 |
| $CRU$ | 0.2155 | 0.1414* | 0.1335** |
| $CRUd$ | 0.3259 | 0.1228** | 0.1051** |
| Prob>F | | 0.0006 | 0.0019 |
| $R^2$ | 0.2154 | | |

Table 10: Regression relating ambiguity premia to uniform compound risk premia

We next estimate a baseline model allowing the ambiguity premium to vary with the probability treatment $p$. This model reflects what can be explained without using the compound risk premium.

$$\text{Model B1: } AP_{it} = a_t + e_{it}$$

where $i = 1, \ldots, 64$ indexes subjects and $t = 1, 2, 3$ indexes the three probability treatment levels. We estimate model B1 using OLS regression with dummy variables for the probability treatment levels and the constant term omitted.[14] The coefficients on the dummy variables should be interpreted as the average ambiguity premium for the corresponding treatment.

We then expand the model to include the one or more compound risk premia as explanatory variables.

$$\text{Model B2: } AP_{it} = a_t + b_1 * CRP1_{it} + ... + b_k * CRPk_{it} + e_{it}$$

[12]These robust standard errors are as implemented through the vce(robust) option in Stata 11.1. For more details see footnote 15.

[14]To maximize comparability across the different probability levels, we use the 12 ball urns to calculate the premia at each level.

Finally, we allow the slopes on the compound risk premia to vary with the probability treatment.

$$\text{Model B3: } AP_{it} = a_t + b_{1t} * CRP1_{it} + ... + b_{kt} * CRPk_{it} + e_{it}$$

Slope dummy variables are used to allow the slopes on the compound risk premium to vary. The coefficient of a slope dummy variable should be interpreted as the estimated slope on CRP for the corresponding treatment.

The OLS estimates for model B1 are reported in Table 11, while those for models B2 and B3 with hypergeometric compound risk premia are reported in Table 12. For all models, we report coefficient estimates (in euro), OLS standard errors for each coefficient as well as heteroskedasticity robust standard errors and cluster robust standard errors (robust to within-subject correlation in the errors in addition to heteroskedasticity).[15]

| **Dep. variable: AP (12 ball) n=192** | **Coef** | **Model B1 Standard errors[16] OLS** | **Robust** | **Cluster** |
|---|---|---|---|---|
| Prob 1/12 | -1.5521 | 1.2744 | 0.8027* | 0.8069* |
| Prob 1/2 | 4.3281 | 1.2744*** | 1.0581*** | 1.0637*** |
| Prob 11/12 | 5.7344 | 1.2744*** | 1.7630*** | 1.7723*** |
| **Prob>F** | | 0.0002 | 0.0000 | 0.0000 |
| $R^2$ | | 0.0887 | | |

Table 11: Ambiguity premia by probability treatment

Table 12 shows a highly significant positive effect of the hypergeometric compound risk premium on the ambiguity premium – holding the probability treatment fixed, greater aversion to hypergeometric compound risk is associated with greater ambiguity aversion. This relationship, however, is far from identification. Knowledge of the hypergeometric compound risk premium boosts the variation in ambiguity premium explained from 8.87% (from Table 11) to 32.2%. Furthermore, the intercept is significantly negative for the probability 1/12 treatment and significantly positive for the probability one-half and 11/12 treatments. This indicates that reduction of hypergeometric compound lotteries (i.e., a compound risk premium of zero) is associated with ambiguity seeking for the lowest probability treatment and ambiguity aversion for the probability one-half and 11/12 treatments. The estimated magnitudes are non trivial – for the probability one-half and 11/12 treatments in the most complete model (B3), the point estimate of the ambiguity premium for an individual who reduces compound lotteries ranges from 9% to 12% of the expected value. This finding, together with the modest explanatory power of the regression argues against the identification of ambiguity attitude with hypergeometric compound risk attitude.

[15] These robust standard errors are as implemented through the vce(robust) and vce(cluster subject) options in Stata 11.1. They are based on "sandwich" estimators as in e.g., White (1980) and its generalization to account for clustering. See e.g., Cameron and Miller (2011) or chapter 8 of Angrist and Pischke (2009) for a survey and discussion.

| **Dep. variable:** | **Model B2** | | | | **Model B3** | | | |
|---|---|---|---|---|---|---|---|---|
| **AP (12 ball)** | **Coef** | **Standard errors[a]** | | | **Coef** | **Standard errors** | | |
| **n=192** | | **OLS** | **Robust** | **Cluster** | | **OLS** | **Robust** | **Cluster** |
| Prob 1/12 | -2.2923 | 1.1230** | 0.6222*** | 0.6292*** | -2.5714 | 1.1370** | 0.6651*** | 0.6686*** |
| Prob 1/2 | 2.2083 | 1.1533* | 1.0762** | 1.1192* | 3.0818 | 1.2026** | 1.0242*** | 1.0296*** |
| Prob 11/12 | 4.6352 | 1.1281*** | 1.5067*** | 1.5179*** | 4.4372 | 1.1264*** | 1.4899*** | 1.4977*** |
| CRPG | 0.7178 | 0.0949*** | 0.1123*** | 0.1014*** | | | | |
| Prob 1/12*CRPG | | | | | 0.9885 | 0.2472*** | 0.1537*** | 0.1545*** |
| Prob 1/2*CRPG | | | | | 0.4220 | 0.1583*** | 0.1204*** | 0.1210*** |
| Prob 11/12*CRPG | | | | | 0.8471 | 0.1324*** | 0.1935*** | 0.1945*** |
| **Prob>F[b]** | | 0.0000 | 0.0000 | 0.0000 | | 0.0000 | 0.0000 | 0.0000 |
| $R^2$ | 0.3015 | | | | 0.3220 | | | |

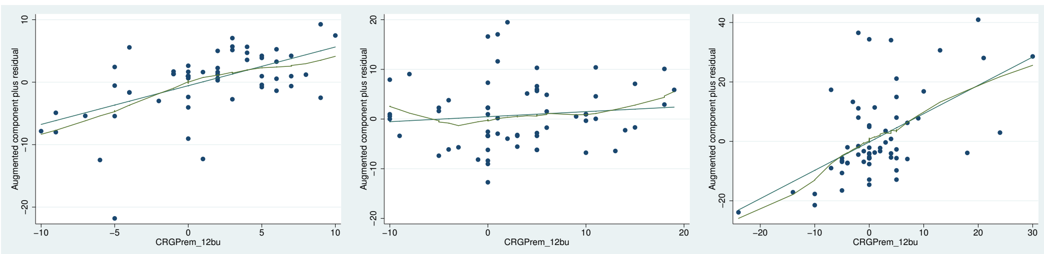
Table 12: Regressions relating ambiguity premia to probability treatments and hypergeometric compound risk premia

[a]***, ** and * on the standard error estimates mean that the corresponding coefficient is significant at a 1, 5 and 10% level respectively when using that standard error.

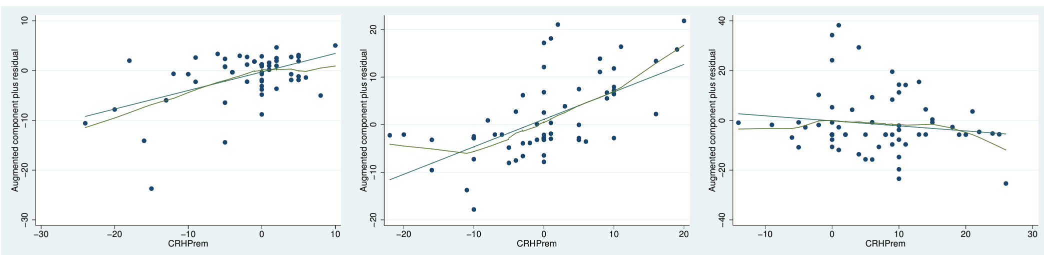
[b]The reported Prob>F and $R^2$ are those of the equivalent regressions including a constant term and omitting the term(s) involving the Prob 1/2 dummy variable(s) as $R^2$ is only calculated meaningfully in regressions including a constant term. For Model 3, a test that all the slopes on CRPG are equal yields Prob>F of 0.0621, 0.0104 and 0.0168 respectively.

In Table 13, we report the results of including the additional compound risk premia we observed across probability treatments. When allowing the slopes to vary with probability treatment, the only premium adding significant explanatory power beyond CRPG is CRPH. Inspection of the coefficients and comparison with Table 12 suggests that the contribution of CRPH is primarily at probability one-half. The overall proportion of the ambiguity premium explained rises to 42.11%. As before, conditional on compound risk premia of zero, the regression predicts non-trivial and statistically significant ambiguity aversion at probabilities one-half and 11/12 and modest ambiguity seeking (significant using the robust and cluster standard errors) at probability 1/12.

In tables 14 and 15 in Appendix A, we reestimate each model allowing additionally for subject fixed effects. An inspection of those table shows that the key findings are robust to the inclusion of such effects, although, for the probability 11/12 treatment, CRPL becomes significant as well. Finally, one might be concerned that there is a stronger but non-linear relationship between compound risk premia and ambiguity premia that we are missing with our linear specification. However, an inspection of the augmented component-plus-residual plots[17] for regressions of ambiguity premium on CRPG and CRPH for each probability treatment separately (in figure 2) suggests that linearity looks like a reasonable assumption for these data.




(a) CRPG for probability 1/12, 1/2, 11/12 treatments




(b) CRPH for probability 1/12, 1/2, 11/12 treatments

Figure 2: Augmented component-plus-residual plots for checking linearity of ambiguity premium in compound risk premium holding fixed the probability treatment

[17]These plots were proposed by Mallows (1986) as a tool for detecting non-linearities in regressions. They were implemented using the *acprplot* command in Stata 11.1 with the *lowess* option used to include the smoothed estimate.

| Dep. variable: | Model B2 | | | | Model B3 | | | |
|---|---|---|---|---|---|---|---|---|
| AP (12 ball) | Coef | Standard errors[a] | | | Coef | Standard errors | | |
| n=192 | | OLS | Robust | Cluster | | OLS | Robust | Cluster |
| Prob 1/12 | -1.0870 | 1.1292 | 0.6514* | 0.6134*** | -1.6031 | 1.4004 | 0.5402*** | 0.5430*** |
| Prob 1/2 | 2.6472 | 1.1301** | 0.9501*** | 0.9159*** | 3.7550 | 1.1678*** | 0.8892*** | 0.8939*** |
| Prob 11/12 | 2.9434 | 1.2477** | 1.7635* | 1.8133 | 5.8891 | 1.5637*** | 2.4157** | 2.4285** |
| CRPG | 0.3682 | 0.1290*** | 0.1625** | 0.1485** | | | | |
| Prob 1/12*CRPG | | | | | 0.6296 | 0.3925 | 0.1762*** | 0.1771*** |
| Prob 1/2*CRPG | | | | | -0.0529 | 0.1835 | 0.1031 | 0.1037 |
| Prob 11/12*CRPG | | | | | 0.8614 | 0.2295*** | 0.3209*** | 0.3226*** |
| CRPL | -0.0426 | 0.1158 | 0.1043 | 0.0977 | | | | |
| Prob 1/12*CRPL | | | | | -0.1427 | 0.3428 | 0.1509 | 0.1517 |
| Prob 1/2*CRPL | | | | | 0.0096 | 0.2337 | 0.1644 | 0.1652 |
| Prob 11/12*CRPL | | | | | -0.1998 | 0.1504 | 0.1420 | 0.1428 |
| CRPLe | 0.0991 | 0.1121 | 0.1291 | 0.1367 | | | | |
| Prob 1/12*CRPLe | | | | | 0.1750 | 0.4355 | 0.2780 | 0.2794 |
| Prob 1/2*CRPLe | | | | | 0.1381 | 0.1537 | 0.1241 | 0.1248 |
| Prob 11/12*CRPLe | | | | | 0.0415 | 0.1769 | 0.2384 | 0.2397 |
| CRPH | 0.1648 | 0.1140 | 0.1162 | 0.1205 | | | | |
| Prob 1/12*CRPH | | | | | 0.3747 | 0.3407 | 0.2348 | 0.2360 |
| Prob 1/2*CRPH | | | | | 0.3674 | 0.2080* | 0.1657** | 0.1666** |
| Prob 11/12*CRPH | | | | | -0.2272 | 0.1708 | 0.1730 | 0.1739 |
| CRPHe | 0.2643 | 0.1201** | 0.1337** | 0.1196** | | | | |
| Prob 1/12*CRPHe | | | | | -0.0782 | 0.2738 | 0.0908 | 0.0913 |
| Prob 1/2*CRPHe | | | | | 0.2499 | 0.1683 | 0.1600 | 0.1608 |
| Prob 11/12*CRPHe | | | | | 0.2603 | 0.2414 | 0.3314 | 0.3331 |
| **Prob>F**[b] | | 0.0000 | 0.0000 | 0.0000 | | 0.0000 | 0.0000 | 0.0000 |
| $R^2$ | 0.3587 | | | | 0.4211 | | | |

Table 13: Regressions relating ambiguity premia to probability treatments and various compound risk premia

[a] ***, ** and * on the standard error estimates mean that the corresponding coefficient is significant at a 1, 5 and 10% level respectively when using that standard error.

[b] The reported Prob>F and $R^2$ are those of the equivalent regressions including a constant term and omitting the term(s) involving the Prob 1/2 dummy variable(s) as $R^2$ is only calculated meaningfully in regressions including a constant term. For Model 3, the slopes on all CRP except CRPG and CRPH appear jointly insignificant (Prob>F of 0.4976, 0.3643 and 0.1044 respectively.)

# 5 Summary and conclusion

We find that risk, compound risk and ambiguity attitudes are all strongly influenced by the probability (under ambiguity, size) of the events under consideration. To our knowledge, this is the first study to explore this influence while distinguishing between ambiguity and compound risks. The pattern for all three types of uncertainty is seeking for low probabilities of winning, and aversion for high probabilities of winning.

Our data show that attitudes towards ambiguity and compound risks are related but distinct, and we estimate the relationship controlling for the size of the winning event. Moreover, this relationship is quite sensitive to the type of compound risks considered. Of our compound risk stimuli observed for multiple probability levels, the hypergeometric CR and CR high appear to have the strongest relationship with ambiguity attitude.

The most striking finding of Halevy (2007) is the identification of ambiguity neutrality and reduction of uniform compound risk. In our cases most comparable to Halevy (2007), we do not support this identification. Taking our findings together with Halevy (2007)'s raises a puzzle about why the results are different in this regard. Even using compound risks more closely connected to the process generating the ambiguity in our experiment does not restore Halevy (2007)'s findings. Possible explanations including mathematical sophistication of the subjects (our subjects appear to have had more training in this regard) or coarseness in the elicitation of certainty equivalents (we observed the certainty equivalent to the nearest euro while Halevy's subjects could report to the nearest penny) would seem to suggest differences opposite to those we observed. Better understanding the source of these differences is an interesting topic for future research. We do however provide some support for Halevy's hypothesis that compound risks such as uniform or hypergeometric that are plausibly connected to the process generating the ambiguous urn generate more of a connection between ambiguity neutrality and reduction of compound risk than other types of compound risks with the same reduced probability. For example, that relationship disappears when using our CRH and CRL stimuli.

Our data suggest that a descriptively valid theory of decision making under uncertainty should account for simple risk attitude, compound risk attitude and ambiguity attitude as distinct aspects of preference and allow each to vary with the probabilities of events.

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## Appendix A - Fixed effects regressions

| Dep. variable: | Model B1 | | | Model B2 | | | Model B3 | | |
|---|---|---|---|---|---|---|---|---|---|
| AP (12 ball) | Coef | Standard errors[a] | | Coef | Standard errors | | Coef | Standard errors | |
| n=192 | | OLS | Robust & Cluster[b] | | OLS | Robust & Cluster | | OLS | Robust & Cluster |
| Prob 1/12 | -1.5521 | 1.1985 | 0.9636 | -2.3003 | 1.0426** | 0.8187*** | -2.8497 | 1.0611*** | 0.8506*** |
| Prob 1/2 | 4.3281 | 1.1985*** | 4.3281*** | 2.1856 | 1.0861** | 0.8559** | 2.9418 | 1.1504** | 0.8081*** |
| Prob 11/12 | 5.7344 | 1.1985*** | 1.1991*** | 4.6234 | 1.0500*** | 0.9923*** | 4.6505 | 1.0467*** | 0.9620*** |
| CRPG | | | | 0.7255 | 0.1100*** | 0.1286*** | | | |
| Prob 1/12*CRPG | | | | | | | 1.2583 | 0.2803*** | 0.2711*** |
| Prob 1/2*CRPG | | | | | | | 0.4694 | 0.1795*** | 0.1596*** |
| Prob 11/12*CRPG | | | | | | | 0.7079 | 0.1508*** | 0.2212*** |
| **Prob>F** | | 0.0000 | 0.0000 | | 0.0000 | 0.0000 | | 0.0000 | 0.0000 |
| $R^2$ (within subject) | 0.1417 | | | 0.3632 | | | 0.3920 | | |

Table 14: Regressions relating ambiguity premia to probability treatments and hypergeometric compound risk premia with subject fixed effects

[a]***, ** and * on the standard error estimates mean that the corresponding coefficient is significant at a 1, 5 and 10% level respectively when using that standard error.
[b]Note that with subject fixed effects, the robust and subject cluster robust standard error options in *Stata 11.1* are the same.

| Dep. variable: AP (12 ball) n=192 | Model B2 Coef | Model B2 Standard errors[a] OLS | Model B2 Standard errors[a] Robust & Cluster [b] | Model B3 Coef | Model B3 Standard errors OLS | Model B3 Standard errors Robust & Cluster |
|---|---|---|---|---|---|---|
| Prob 1/12 | -1.0702 | 1.0728 | 0.9198 | -2.0629 | 1.4158 | 1.3043 |
| Prob 1/2 | 2.7311 | 1.0692** | 0.8227*** | 3.9351 | 1.1116*** | 0.6589*** |
| Prob 11/12 | 2.6340 | 1.2158** | 1.2384** | 6.4575 | 1.6266*** | 1.8029*** |
| CRPG | 0.3884 | 0.1499* | 0.1431*** | | | |
| Prob 1/12*CRPG | | | | 0.9499 | 0.4469** | 0.4685** |
| Prob 1/2*CRPG | | | | -0.0215 | 0.2072 | 0.1279 |
| Prob 11/12*CRPG | | | | 0.6797 | 0.2585*** | 0.2888** |
| CRPL | -0.0096 | 0.1393 | 0.1152 | | | |
| Prob 1/12*CRPL | | | | -0.1587 | 0.3932 | 0.3531 |
| Prob 1/2*CRPL | | | | 0.2105 | 0.2624 | 0.1935 |
| Prob 11/12*CRPL | | | | -0.3176 | 0.1733* | 0.1530** |
| CRPLe | 0.0334 | 0.1325 | 0.1156 | | | |
| Prob 1/12*CRPLe | | | | 0.4601 | 0.4966 | 0.4771 |
| Prob 1/2*CRPLe | | | | 0.0676 | 0.1737 | 0.1085 |
| Prob 11/12*CRPLe | | | | 0.0202 | 0.2001 | 0.2416 |
| CRPH | 0.2406 | 0.1221* | 0.1144** | | | |
| Prob 1/12*CRPH | | | | 0.0127 | 0.3864 | 0.4056 |
| Prob 1/2*CRPH | | | | 0.5145 | 0.2363** | 0.1851*** |
| Prob 11/12*CRPH | | | | -0.2303 | 0.1922 | 0.1683 |
| CRPHe | 0.2216 | 0.1422 | 0.1391 | | | |
| Prob 1/12*CRPHe | | | | 0.0921 | 0.3085 | 0.2826 |
| Prob 1/2*CRPHe | | | | 0.0539 | 0.1901 | 0.1238 |
| Prob 11/12*CRPHe | | | | 0.3856 | 0.2732 | 0.2997 |
| **Prob>F** | | 0.0000 | 0.0000 | | 0.0000 | 0.0000 |
| $R^2$ | 0.4213 | | | 0.5134 | | |

Table 15: Regressions relating ambiguity premia to probability treatments and various compound risk premia with subject fixed effects

[a]***, ** and * on the standard error estimates mean that the corresponding coefficient is significant at a 1, 5 and 10% level respectively when using that standard error.

[b]Note that with subject fixed effects, the robust and subject cluster robust standard error options in *Stata 11.1* are the same.

## Appendix B - Stimuli for probability level 1/2

As indicated by Tables 1 and 2, for each probability/winning colors treatment, there were a number of different stimuli. Below, the stimuli corresponding to probability 1/2 are displayed.

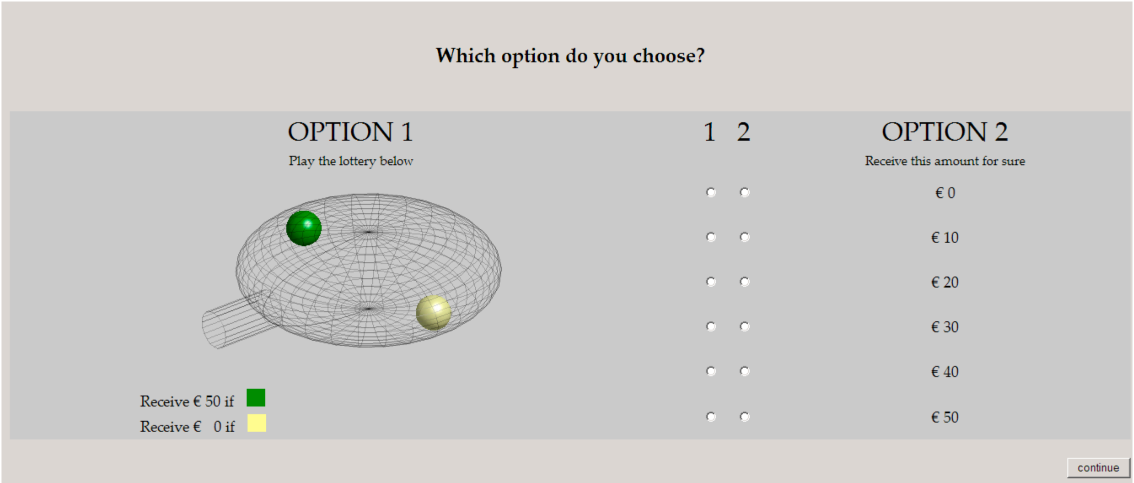


Figure 3: Display - Simple risk for probability 1/2 treatment (2 ball urn)

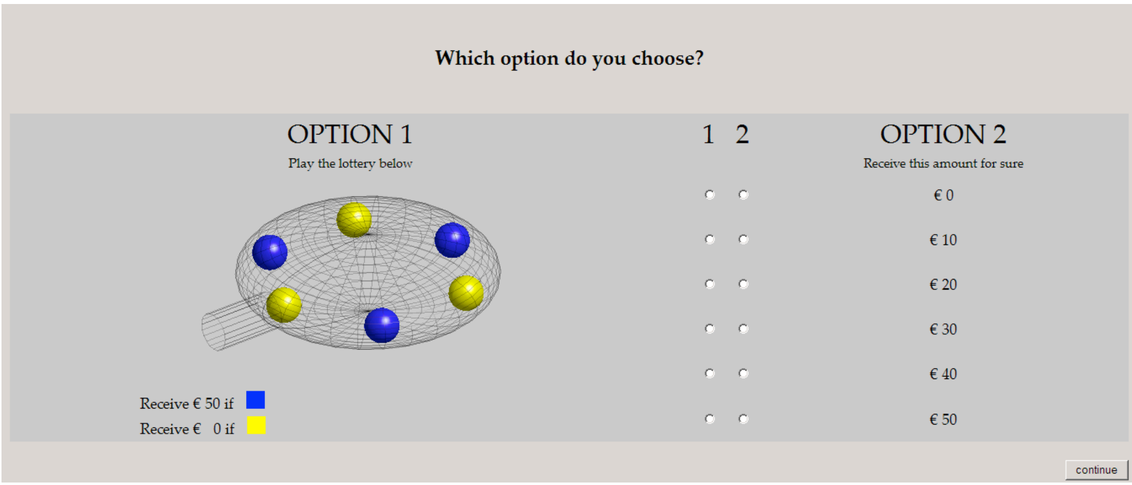


Figure 4: Display - Simple risk for probability 1/2 treatment (6 ball urn)

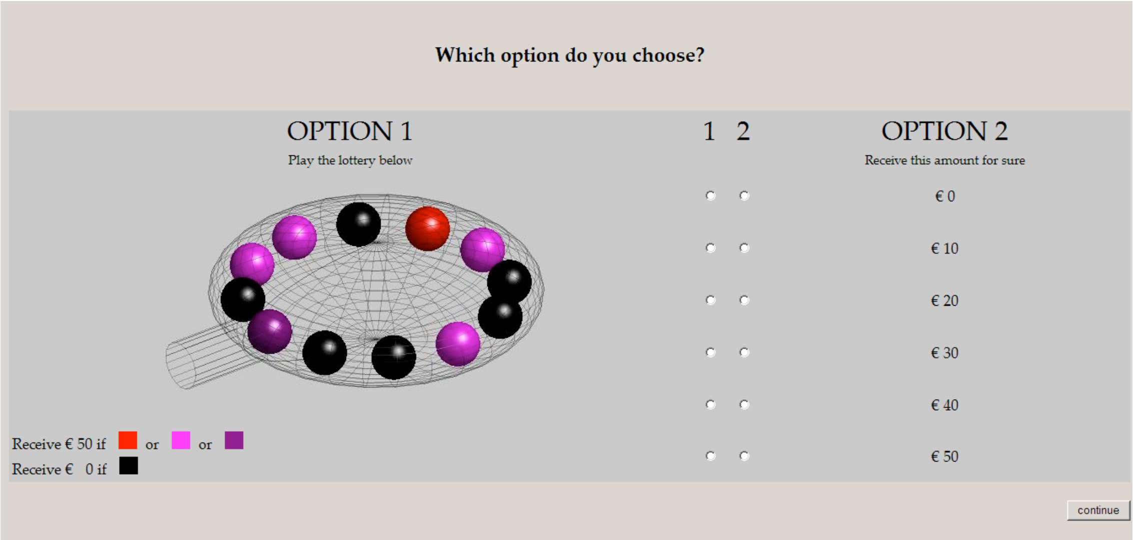


Figure 5: Display - Simple risk for probability 1/2 treatment (12 ball urn)

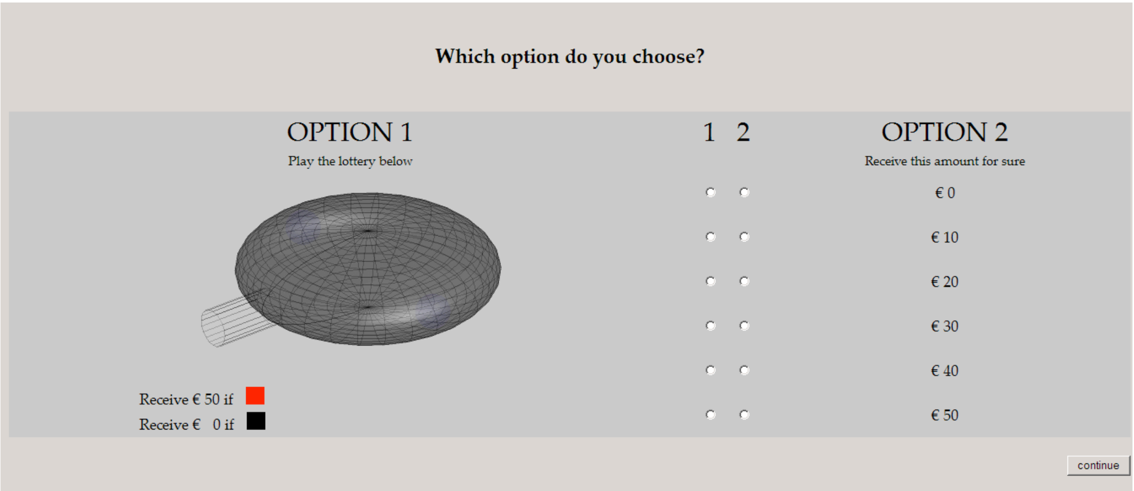


Figure 6: Display - Ambiguity for probability 1/2 treatment (2 ball urn)

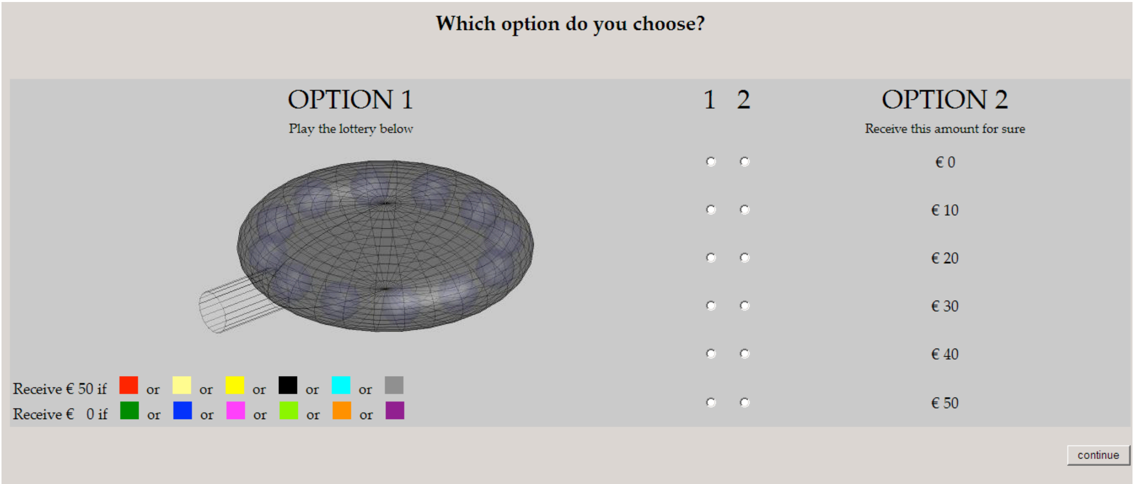


Figure 7: Display - Ambiguity for probability 1/2 treatment (12 ball urn)

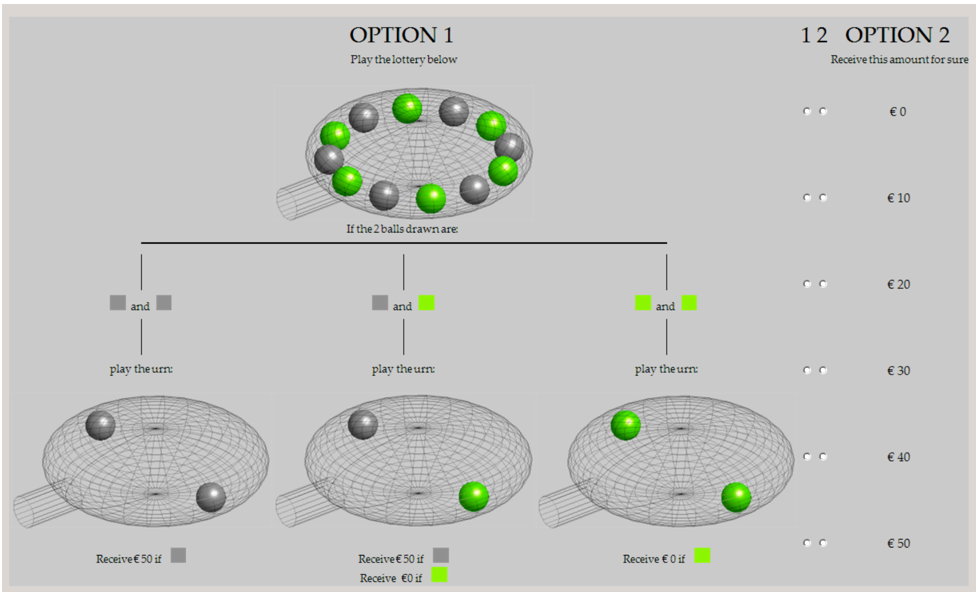


Figure 8: Display - Hypergeometric compound risk for probability 1/2 treatment

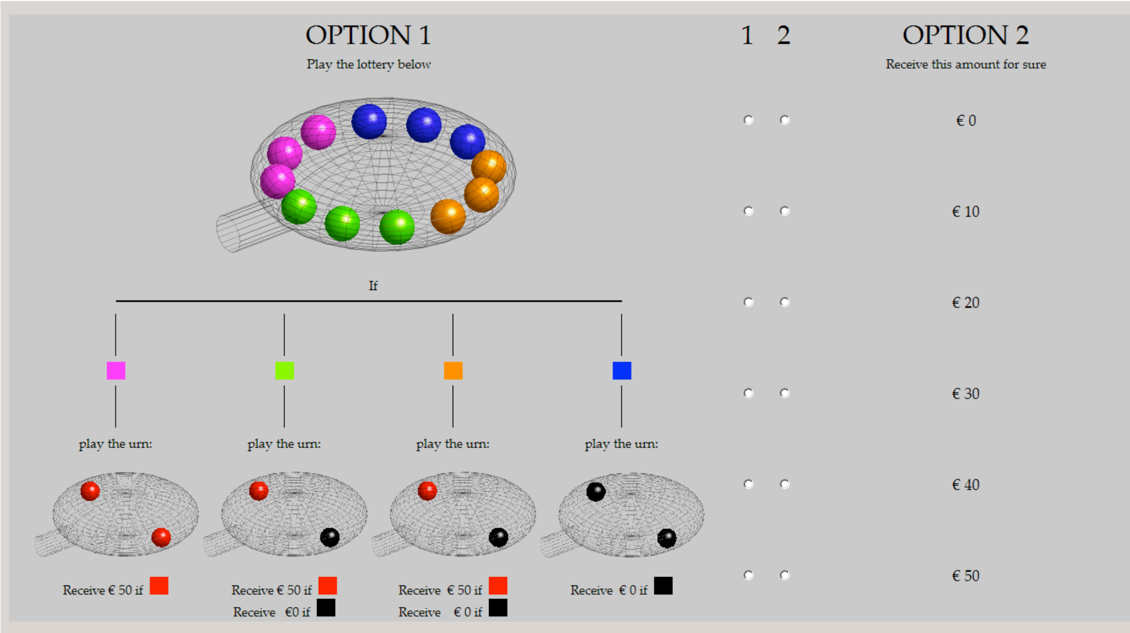


Figure 9: Display - Diverse uniform compound risk for probability 1/2 treatment

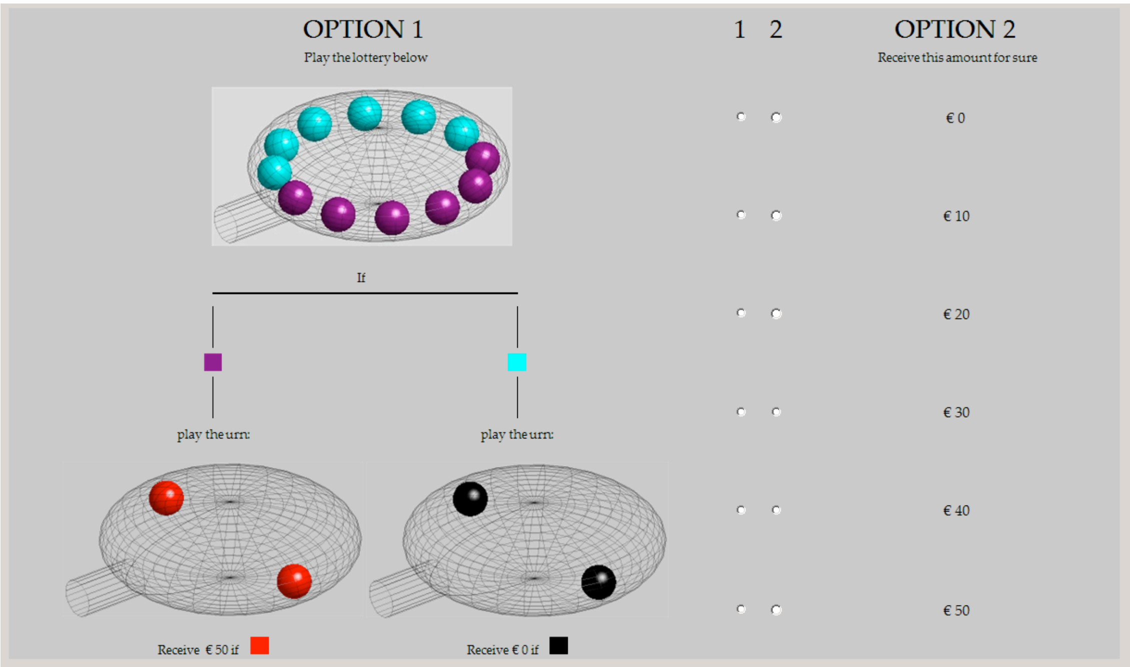


Figure 10: Display - Degenerate uniform compound risk for probability 1/2 treatment

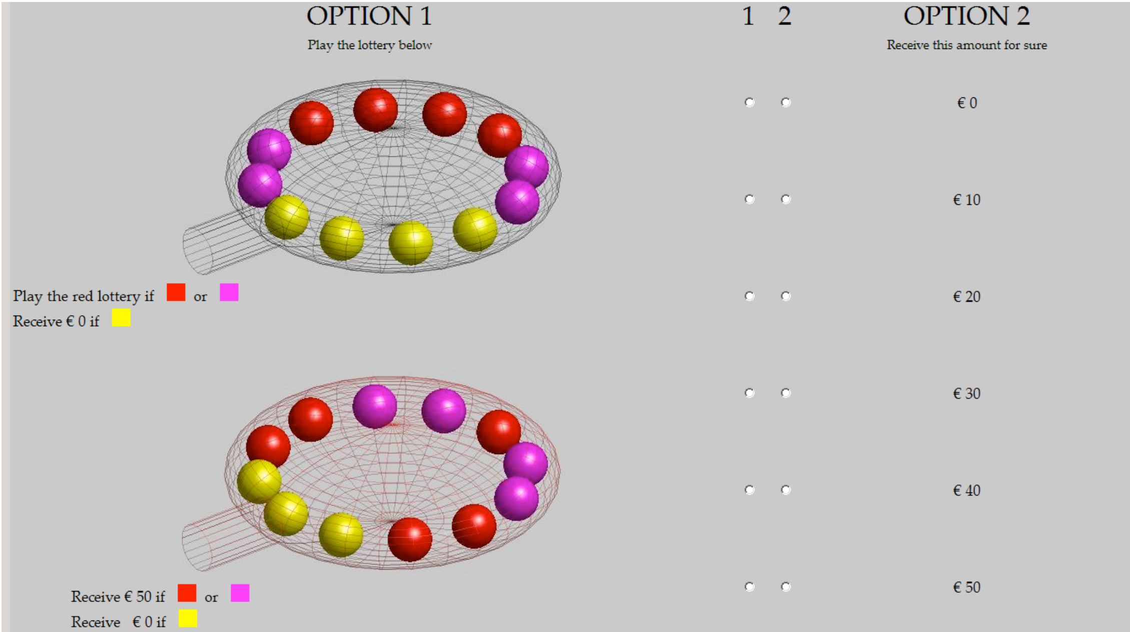


Figure 11: Display - Compound risk low for probability 1/2 treatment

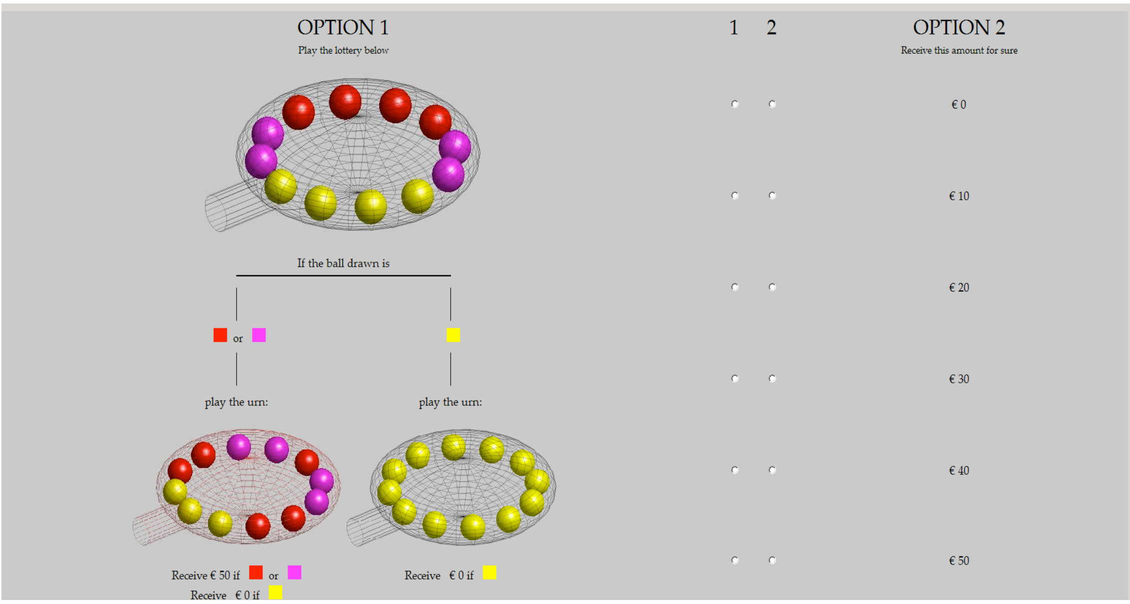


Figure 12: Display - Compound risk low with explicit degenerate for probability 1/2 treatment

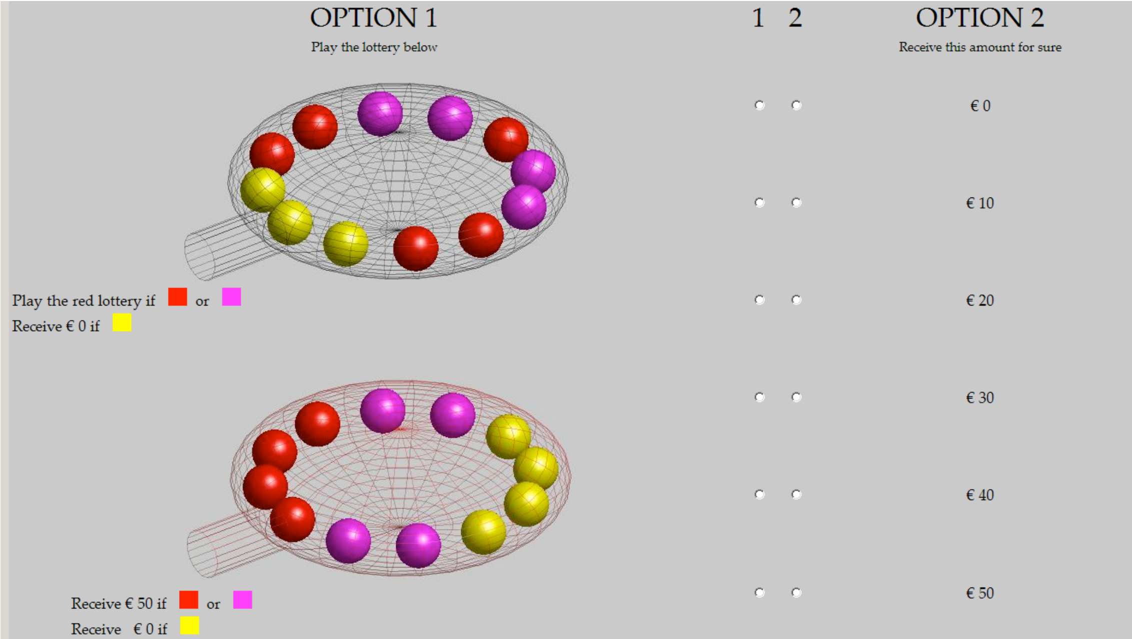


Figure 13: Display - Compound risk high for probability 1/2 treatment

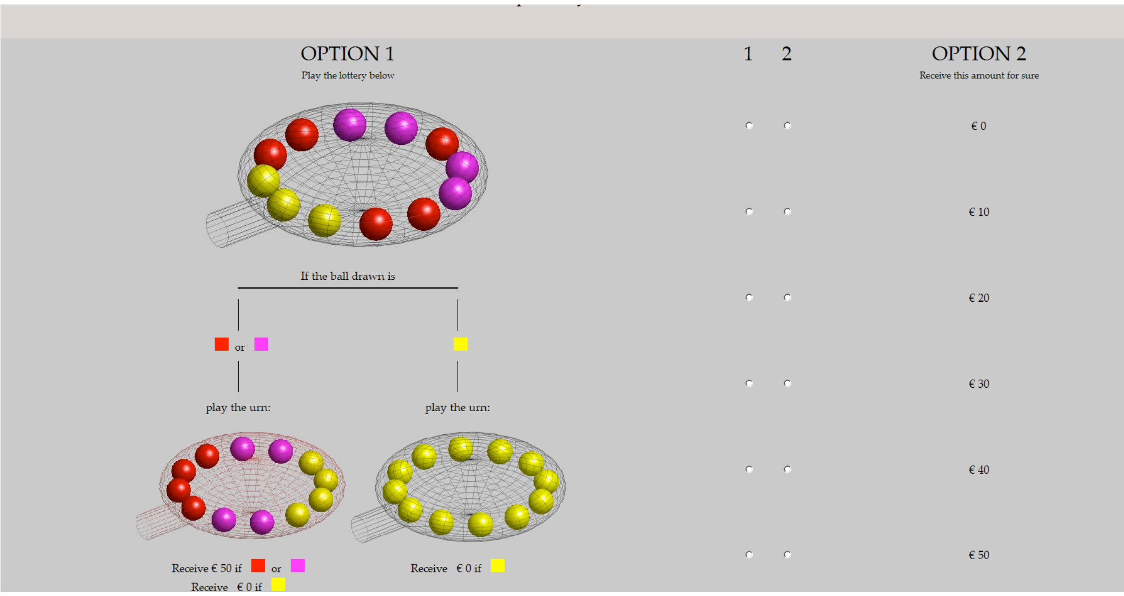


Figure 14: Display - Compound risk high with explicit degenerate for probability 1/2 treatment

## Appendix C - Illustration of the multiple price list method

For each stimulus three screens were presented sequentially to the subject. On the first screen they chose for each of six amounts evenly spaced between 0 and 50 euros between the stimulus and the sure amount. On the second screen, for the same stimulus, they chose for each of eleven amounts evenly spaced between the highest amount for which they chose the stimulus on the first screen and the lowest amount for which they chose the sure amount on the first screen. On the third screen, the choices in one euro increments implied by the choices from the first two screens and monotonicity are displayed for the subject. At that point the subject is given the opportunity to change any of those choices if desired and the final response for that stimulus is recorded. In the example below, this final recorded certainty equivalent is 8.5 euros.

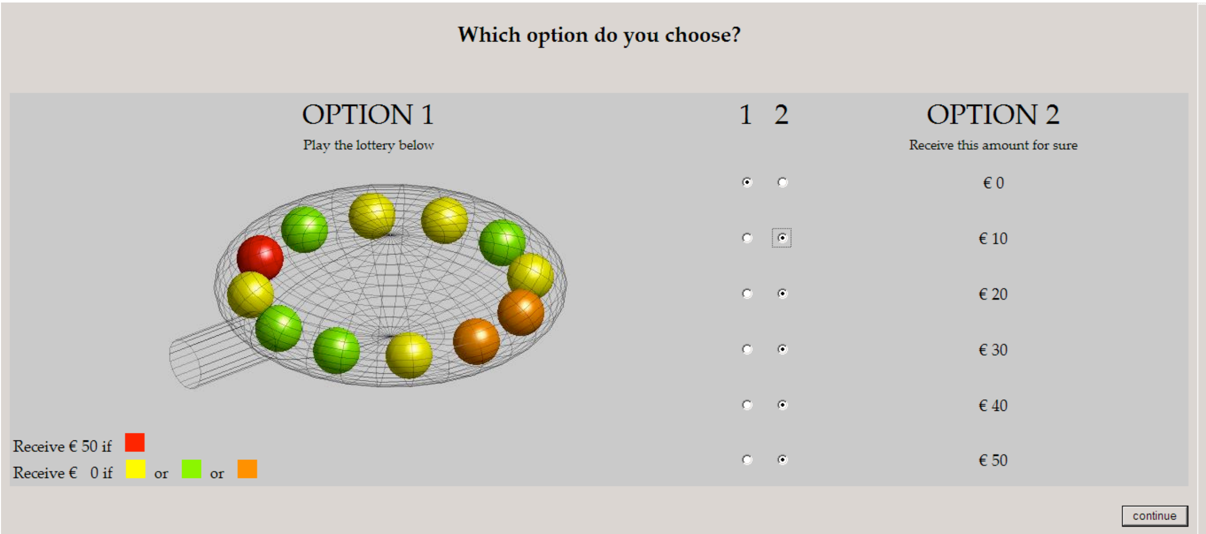


Figure 15: Simple risk for probability 1/12 treatment (first list)

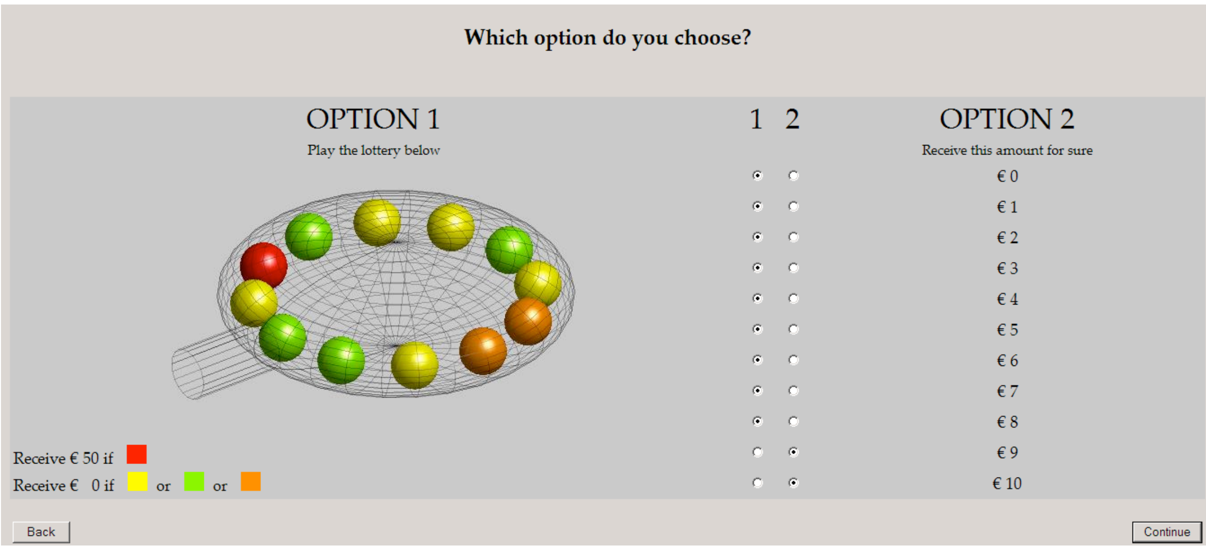


Figure 16: Simple risk for probability 1/12 treatment (second list: refinement of the first)

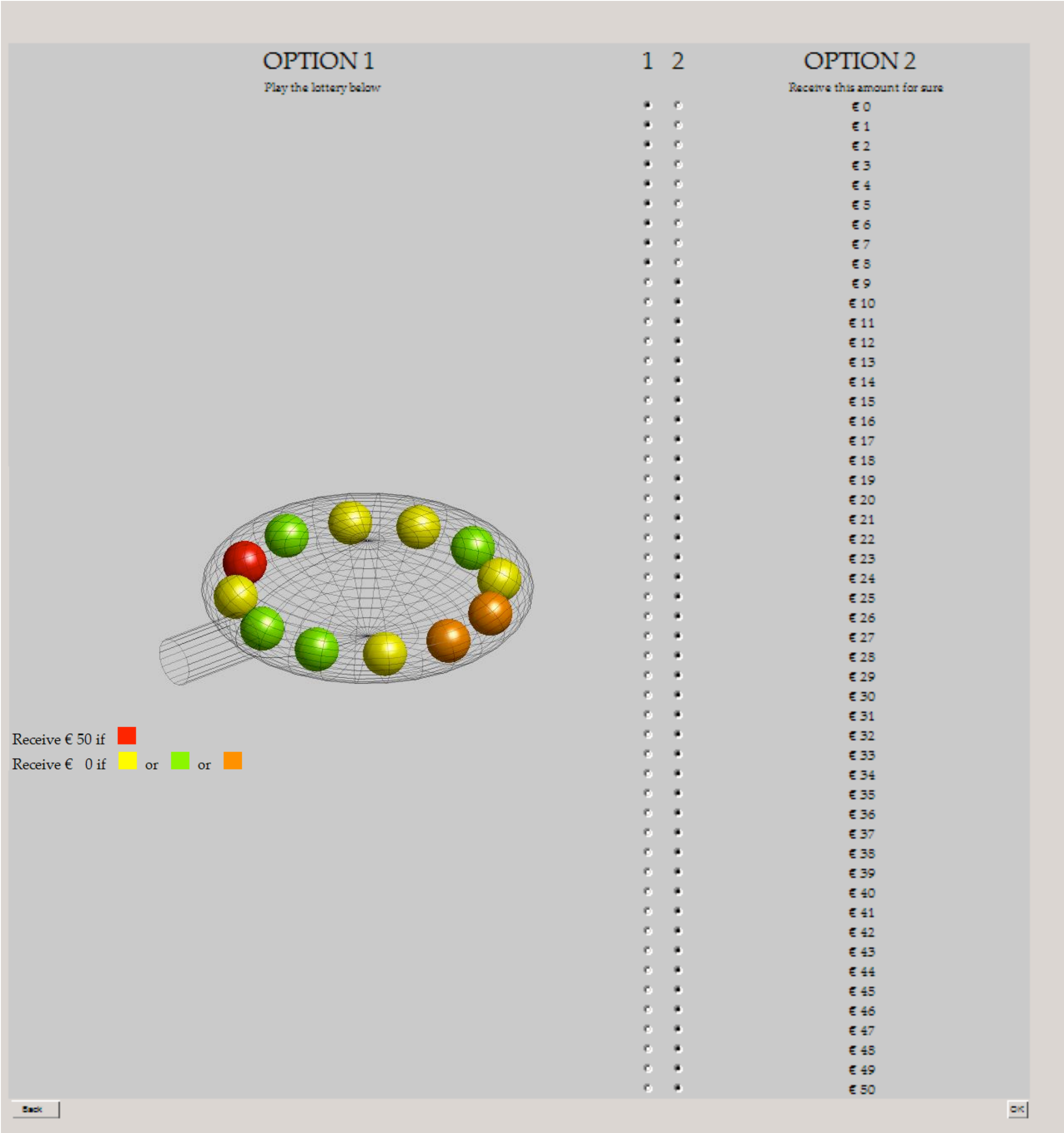


Figure 17: Simple risk for probability 1/12 treatment (third step: confirmation)