Reference Prices, Double Comparisons, and Anomalies in Consumption-Payment Decisions

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Abstract

We propose a positive theory of consumer choice, one that evaluates the hedonic benefits of various consumption and payment streams. In any given period, mental accounting finds consumers engage in a double comparison: one between the benefit of consumption and a reference price, and another between the reference price and the actual price. The reference price is influenced by the payment stream and adapts over time. Under standard assumptions, mainly loss aversion and adaptation, our model predicts various anomalies observed in consumer choice: sunk cost effects, the flat-rate bias, preference for advance payment, and payment depreciation. None of the existing model can explain all these biases.

1 Introduction

Commuters taking the subway could purchase a single journey ticket, a charge card, or a monthly pass. Users of a gym could sign up for a pay-as-you-go contract, a monthly contract, or a yearly contract. Drivers could make a lump sum payment and buy a car, purchase a car on a loan, or lease. Families planning their vacations could purchase a vacation home, rent, or own a time-share. Prior empirical literature has shown that in situations like these consumers exhibit preferences that, on the surface, do not seem rational. For example,

1. **Sunk-cost effects**: Consumption decisions are affected by previously paid investment costs, even though only the future costs and benefits should matter at the time of consumption (Thaler, 1980). The sunk cost effect decreases if the investment is partially paid by someone else (Arkes and Blumer, 1985), or if the time distance between the investment date and today increases (Gourville and Soman, 1998).

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2. **Preference for upfront payment**: If a durable good is to be paid all at once, consumers prefer to make this payment just before consumption begins, rather than at an earlier or a later date (Prelec and Loewenstein, 1998). It makes more economic sense to pay later if one considers the time value of money.

3. **Flat-rate bias**: Consumers have a tendency to prefer flat-rate tariffs even when they could save money under pay-per-use tariffs. DellaVigna and Malmendier (2006) studied the contractual choices of consumers at gyms and found that 80 percent of consumers under the flat-rate contract would have saved money under the pay-per-visit contract. The extra-cost was as large as 70 percent for those consumers who chose a membership of over $70 per month. Similar evidence exists when consumers choose tariffs on internet services (Lambrecht and Skiera, 2006).

4. **Reference Price Effects**: The willingness to pay for an exact same product depends on the reference price of the product (Thaler, 1985).

In this paper, we explain these various anomalies using a utility model that evaluates the distinct hedonic benefits of various consumption and payment streams. In the single period version, the model predicts that the willingness to pay increases with the reference price (Thaler (1985)'s beer beach experiment). The model is particularly suited to evaluate repeated purchases and durable goods. Our model builds on reference effects and the psychology of (double) comparison and adaptation (Kahneman and Tversky, 1979; Thaler, 1985). Consumers keep a reference price in mind and perform two comparisons. First, the benefits of consumption are compared with the reference price, evaluating the desirability of consumption. The hedonic value of this comparison yields *acquisition utility*. Second, the reference price is compared with the actual price, evaluating the budgetary benefits of the deal. The hedonic value of this comparison yields *transaction utility*. Acquisition utility is received at the time of consumption, and transaction utility at the time of payment.

Keeping reference prices in mind is very functional, allowing consumers to initiate many purchase decisions without knowing the actual price (e.g. enter internet to book a flight or a restaurant). The purchase will be consummated if the actual price is not much higher than the reference price. For an accurate consumer, transaction utility should be an small correction due to price forecasting error, and acquisition utility should be an unbiased estimate of the rational cost and benefit of the purchase. For many consumers, however, reference prices may not be accurate forecasts, and follow instead simple adaptive rule based on the history of observed prices. This naïve adaptation rule may distort acquisition and transaction utility in a way that increases the role of reference prices in their utility.

Our model can be considered a dynamic extension of (a modified version of) Thaler (1985)'s double comparison model, one that includes reference price adaptation. Adaptation explains the flat rate bias: by using a service "for free", the naïve reference price decreases, and the acquisition utility necessarily increases. In the pay-as-you-go tariff, in contrast, the consumer is reminded of the price. Hence, the reference price, and hence the acquisition utility, is constant over time. A similar logic explains the preference for advance payment. The sunk cost effect can be purely explained using mental accounting and loss aversion, and adaptation explains payment depreciation. We analyze the flat-rate bias under valuation and demand uncertainty, and the dynamic effect when consumers try to switch tariff schemes.

Our model is primarily descriptive. For business decisions, taken on behalf of third party stakeholders, the emotional impact of reference price comparisons should not influence decisions, and one should avoid all the before-mentioned anomalies. As taught in business schools, the sunk cost effect and a preference for upfront payment are biases. A consumer that takes this business approach will be called *non-emotional*. Many consumers, however, may adopt the view that reference price effects are an integral part of the consumer experience. We call these consumers *emotional*. For emotional consumers, the model is normative and the flat rate bias, or the sunk cost effect, may be rational (although their hedonic value may be subject to framing effects).

A modification of discounted utility (DU) that does account for the flat-rate bias and the preference for advanced payment was introduced in Prelec and Loewenstein (1998). PL98 rests on three assumptions: prospective accounting, prorating, and coupling. Prospective accounting stipulates that utility from consumption is psychologically affected by *future* payments, and disutility from payment is psychologically affected by *future* consumption. The discrete nature of this assumption makes a consumer's utility discontinuous between prepayment and postpayment when we extend it to a continuous version. Discontinuity poses serious interpretation problems. Prorating assumes the process that one prorate the benefit of residual consumption to residual payments to buffer the pain of payment, and vice versa. Prorating is reasonable for simple decisions, but becomes psychologically implausible and mathematically intractable for large problems. Lastly, coupling refers to the degree to which consumption calls to mind thoughts of payment, and vice versa, and can be represented by coupling coefficients. Coupling coefficients are add-hoc parameters that render the model too flexible. Our model and PL98 are substitutes. We argue ours is more parsimonious.

Other modification of DU incorporate psychological factors such as mental accounting (Thaler, 1985, 1990, 1999), comparison with reference prices (Popescu and Wu, 2007), anticipated feelings of regret (Bell, 1982, 1985; Nasiry and Popescu, 2011), habit formation (Pollak, 1970; Wathieu, 1997; Rozen, 2010), satiation (Baucells and Sarin, 2007, 2010) or time-inconsistent preferences (DellaVigna and Malmendier, 2004). None of these models account for the type of anomalies we explain.

2 Double Comparisons in Single Purchase

Mental accounting, as proposed by Thaler (1985), postulates two sources of utility from consumption: acquisition utility and transaction utility. Acquisition utility refers to the hedonic net benefit a consumer obtains purely from consumption, as it is customary in economic models. In addition, a consumer obtains transaction utility, which refers to the hedonic benefit derived from the budgetary benefits of the deal. We adopt this single period model with a minor modification.

Let u be the benefit of consumption, i.e., the consumer's valuation of the good expressed in monetary units. Let $\hat{p}$ be a consumer's reference price and p be the actual price of the good. Also, let $v(\cdot)$ be an S-shaped value function satisfying the following properties: $v(0) = 0, v'(x) > 0$, and $v''(x) \leq 0$ for $x > 0$, and $v''(x) \geq 0$ for $x < 0$. As customary, v will exhibit loss aversion, in forms that we will define later (Tversky and Kahneman, 1991). The value function captures the emotional reaction to gains and losses. Given any such v, the consumer's utility is defined as

$$v(u - \hat{p}) + v(\hat{p} - p), \tag{1}$$

where the first term is acquisition utility and the second term is transaction utility. The consumer's valuation u admits two interpretations: a "local" willingness to pay, or a "global" consumption utility of the rational version of this consumer. In the latter case, we scale utility in such a way that the Lagrange multiplier associated with the budget constraint is one. In the normative interpretation of the model, (1) can be interpreted as the per-period experienced utility (Kahneman et al., 1997).

For future use, we define the following value function.

Definition 1. *The **power value function** is defined as follows:* $v(x) = x^\gamma$ *if* $x \geq 0$ *and* $v(x) = -\lambda v(-x)$ *if* $x < 0$*, where* $0 < \gamma \leq 1$ *and* $\lambda \geq 1$*.*

The carrier of acquisition utility is the comparison between the benefits of the purchase and the reference price. When the actual price is not available, the purchase decision will be initiated if $u > \hat{p}$, and completed if p falls below or is not too far from $\hat{p}$. The carrier of transaction utility is the comparison between the reference price and the actual price. If the actual price is equal to the reference price, a consumer perceives the transaction as a "fair deal" and she feels no additional pleasure or displeasure from the purchase and subsequent consumption. If the actual price is lower than the reference price, then she perceives that she is getting a "favorable deal" and experiences positive transaction utility. If the actual price is higher, she perceives it is being ripped-off and experiences negative transaction utility.

We define a consumer *non-emotional* iff her utility does not depend on the reference price, that is, $v(u - \hat{p}) + v(\hat{p} - p) = f(u - p)$ for some strictly increasing function f. As it turns out, the non-emotional consumer will necessarily have a linear value function and her utility simply collapses to $u - p$, the standard economic comparison of utility and cost.

Proposition 1. *The consumer is non-emotional if and only if* $v(x) = cx, c > 0$*.*

Reference prices are used to enter the purchased item in the mental account, and to take it out from the account at the moment of consumption. Eliminating the emotional component by setting $v(x) = x$, the accounted mental profit should be independent of the reference price. That the model admits a non-emotional consumer implies that the model respects a principle of proper accounting. When we extend the model to multiple periods, we will make sure the model has a non-emotional consumer as a particular case. The property is missing in Thaler (1985).[1]

[1] Thaler defines the acquisition utility as some function of u and p; and transaction utility as some function of $\hat{p}$ and p. One specification could be $v(u - p) + v(\hat{p} - p)$. This formulation is problematic, because when we rationalize the consumer with a linear value function we obtain $u-2p+\hat{p}$, which still depends on the reference price. Moreover, because $u \geq 0$, when $\hat{p} > 2p$, a consumer would always obtain positive per-period utility from the purchase. Formulation (1) possesses two important properties: in the linear version it becomes independent of reference prices *and* does not yield the implausible conclusion that by increasing the reference price one can induce consumers to buy any item.

We explore the implications of this double comparison model and the effect of reference prices on consumers' willingness-to-pay, consumption utility, and demand functions.

2.1 Willingness-to-Pay and the Beer Beach Experiment

The consumer's *willingness-to-pay* as the value of w_b that solves

$$v(u - \hat{p}) + v(\hat{p} - w_b) = 0.$$

In standard economics, the willingness to pay is equal to u, the valuation of the product. In our model, a moment's reflection reveals that $u = w$ iff $v(x) = -v(x)$. Clearly, a non-emotional consumer will satisfy this "no loss aversion" property. This property is compatible with diminishing sensitivity and a symmetric s-shaped v. For an emotional consumer with loss aversion and who is considering the purchase of a good, $u > \hat{p}$, the willingness to pay increases with the reference price.

Proposition 2. *Assume $u > \hat{p}$ and that loss aversion takes the form $v'(-x) > v'(x), x > 0$. Then, w_b is strictly increasing in $\hat{p}$.*

In the famous beer beach experiment, Thaler (1985) asked two similar groups of people about their willingness-to-pay for the exactly same beer. One group was told that they were going to buy the beer at a fancy resort hotel and the other group was told that they were going to buy it at a small run-down grocery store.[2] He found that the median willingness-to-pay of the first group was \$2.65 whereas that of the second group was \$1.50, a difference he argued was difficult to reconcile with standard economic models.

This difference is natural for an emotional consumer. People expect that a fancy resort hotel would charge much more, that is, $\hat{p}_{resort} > \hat{p}_{grocery}$. If the actual price falls between these two references, the buyer will perceive the deal is favorable if buying from the resort, but feel ripped off if buying from the grocery. As predicted, the reference prices influence willingness to pay.

To illustrate, assume $u = 3$, $\hat{p}_{resort} = 2.5$ and $\hat{p}_{grocery} = 1$. The utility is $v(3 - 2.5) + v(2.5 - p)$ for the resort, and $v(3 - 1) + v(1 - p)$ for the grocery. Acquisition utility is positive for both,

[2] The survey explained to respondents that their friends (who they trust) will buy the beer for them and the respondents will drink it on a beach, so the atmosphere of the supplier is irrelevant. Also, they were informed that bargaining is not possible. Hence, the author claims that the respondents' best strategy is to state their true reservation price for the beer.

and higher for the grocery. Transaction utility is higher for the resort. The consumer will surely buy from the resort if the price is less than 2.5. For the grocery, however, the transaction utility becomes negative if $p > 1$. Diminishing sensitivity for losses and loss aversion results in a lower willingness to pay for the grocery. Using a power value function with $\gamma = 0.6$ and $\lambda = 2.25$ matches the experimental result of $w_b^{resort} = 2.6$ and $w_b^{grocery} = 1.5$.

2.2 The Non-Emotional and the Emotional Consumer

Recall that the consumer will consider buying the product if $u > \hat{p}$, and will actually buy the product if $w_b > p$. The latter holds iff total utility is positive. For a non-emotional consumer, the purchase decision should be done iff $u > p$. An emotional consumer will be more cautious, as his willingness to pay is generally lower than u.

Proposition 3. *Assume loss aversion in the form $-v(-x) > v(x), x > 0$. If $\hat{p} \neq u$, then $w_b < u$.*

If $p > u$, then $p > w$ and both the emotional and the non-emotional consumer abstain from buying the product. If $u > p$, the non-emotional consumer buys, but the emotional consumer may incur negative utility and abstain from buying. There are two such cases.

1. **Rip-off effect**. $u > w_b > \hat{p}$. The emotional consumer may not buy because the price is higher than the reference price. The beer beach example gives the intuition for this case.

2. **Unjustified Expense**. $\hat{p} > u > w_b$. The emotional consumer may not buy because the acquisition utility is negative. Even if $u > p$, the good is perceived as "too expensive" to justify the expense. When low cost flights were introduced in Europe, consumers might have hesitated at the beginning to engage in a recreational weekend trip. The expense, while affordable, seemed unjustified. Over time, reference prices adjust, and consumers find it more ordinary to purchase these trips.

If the reference price is totally adaptive, then repeated exposure to the bargain offer will produce an adjustment of $\hat{p}$ towards the offered price, p. This implies that, eventually, the consumer will buy if and only if $u > p$. For purchasing decisions, reference prices may adjust downward more rapidly than upward. The unjustified expense effect will be short lived as $\hat{p}$ will drop below w_b, ensuring that the consumer will feel that the item has gotten cheaper to justify the purchase.

These two effects show that the history of prices influences the demand. In other words, demand curves are price-history dependent. The exploration of this important implications is outside the scope of this paper.

2.3 The Endowment Effect

One important anomaly in consumption-payment decisions is the *endowment effect*: the minimum selling price is generally higher than the maximum purchasing price (Thaler, 1985). The maximum purchasing price is our willingness-to-pay, w_b. In order to define w_s, the minimum selling price or willingness-to-accept, we need to define transaction utility and acquisition utility for a selling decision. For a non-emotional consumer, selling an item with valuation u at price p produces $p - u$. For an emotional consumer, selling such item produces:

$$v(p - \hat{p}) + v(\hat{p} - u). \tag{2}$$

The transaction utility is given by the comparison between the selling price and the reference price. The (dis)acquisition utility is the comparison between the reference price and the valuation of the product. If v is linear, the expression becomes $p - u$. Total utility is unambiguously positive if one sells at a higher price than expected, and the expected price is higher than the valuation.

Let w_s be the willingness to accept, or the hypothetical price that solves $v(w_s - \hat{p}) + v(\hat{p} - u) = 0$. We have that:

Proposition 4. *Assume loss aversion in the form $-v(-x) > v(x), x > 0$. If $\hat{p} \neq u$, then $w_s > u$.*

If $p < u$, then $p < w$ and both the emotional and the non-emotional consumer abstain from selling the product. If $p > u$, the non-emotional consumer sells, but the emotional consumer may incur negative utility and abstain from selling. There are two such cases.

1. **Cheap Offer**. $\hat{p} > w_s > u$. The emotional consumer does not sell because the price is lower than the reference price, even though the price may exceed the valuation. Reference prices may act as a search threshold.

2. **Not-for-Sale Effect**. $w_s > u > \hat{p}$. The emotional consumer may not sell because the acquisition utility is negative. Even if $p > u$, the good is perceived as "too valuable" to justify the sell. The price is too good to be true.

We collect all the previous results in this general proposition.

Proposition 5. *If $\hat{p} = u$, then $w_b = w_s = u$. If $-v(-x) > v(x), x > 0$ and $u \neq \hat{p}$, then $w_b < u < w_s$. If $u > \hat{p}$, then w_b increases with $\hat{p}$ and w_s decreases with $\hat{p}$. If $u < \hat{p}$, then w_b decreases with $\hat{p}$ and w_s increases with $\hat{p}$.*

To illustrate, assume v is power and let $\lambda^* = \lambda^{1/\gamma}$. We have that

$$w_b = \begin{cases} \left(1 - \frac{1}{\lambda^*}\right)\hat{p} + \frac{1}{\lambda^*}u, & \text{if } \hat{p} < u, \\ u - (\lambda^* - 1)(\hat{p} - u), & \text{if } \hat{p} \geq u; \text{ and} \end{cases} \tag{3}$$

$$w_s = \begin{cases} u + (\lambda^* - 1)(u - \hat{p}), & \text{if } \hat{p} < u, \\ \left(1 - \frac{1}{\lambda^*}\right)\hat{p} + \frac{1}{\lambda^*}u, & \text{if } \hat{p} \geq u. \end{cases} \tag{4}$$

If $\lambda = 1$ or, more in general, if $v(-x) = -v(x)$, then $w_b = w_s = u$ and the endowment effect disappears. Our focus is on the gap between w_s and w_b, which is proportional to $\lambda^{1/\gamma} - \frac{1}{\lambda^{1/\gamma}}$ and to the absolute difference between u and $\hat{p}$:

$$w_s - w_b = \left(\lambda^* - \frac{1}{\lambda^*}\right)|u - \hat{p}|. \tag{5}$$

$\lambda > 1$ is absolutely necessary for the endowment effect, and the effect is accentuated with the curvature of v and the gap between the valuation and the reference price.

In experiments, it is possible to elicit w_b and w_s, but one cannot observe $\hat{p}$ and u. If we know λ^*, and whether $\hat{p} > u$ or $\hat{p} < u$, then we can infer the valuation u as a convex combination of w_b and w_s:

$$u = \begin{cases} \frac{\lambda^*}{\lambda^*+1}w_b + \frac{1}{\lambda^*+1}w_s, & \text{if } \hat{p} < u, \\ \frac{1}{\lambda^*+1}w_b + \frac{\lambda^*}{\lambda^*+1}w_s, & \text{if } \hat{p} \geq u. \end{cases} \tag{6}$$

In the special case that $\lambda^* = 1/2$, then the valuation is the average between the willingness-to-pay and the willingness-to-accept, *independent of the reference price.* Taking the experimental averages of $\lambda = 2.25$ and $\gamma = 0.88$, produces $\lambda^* = 2.5$ and $u = 0.72w_b + 0.28w_s$ if $\hat{p} < u$ and $u = 0.28w_b + 0.72w_s$ if $\hat{p} > u$. The valuation is closer to the willingness to pay if the reference price is low, and closer to the willingness to accept if the reference price is large.

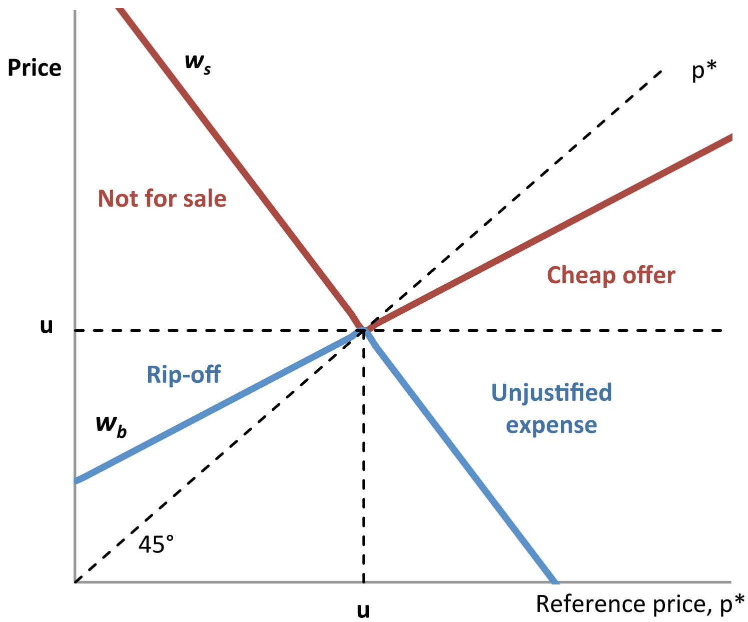

Figure 1: Willingness-to-pay, w_b, and willingness-to-accept, w_s, as a function of the reference price.

2.4 Demand Functions

Assume a continuum of consumers, with heterogeneous valuations for an indivisible product, say u is uniformly distributed between $[0,1]$. In a non-emotional world, the demand function, or fraction of consumers that will request one unit, is $d(p) = 1 - p$. What is the demand function when consumers are emotional?

Proposition 6. *Assume a continuum of consumers with valuations u uniformly distributed in $[0,1]$. Consumers make a discrete unit consumption choice and have the power value function. The demand $d(p)$ function, or fraction of consumers for which $w > p$, is given by*

$$d(p) = \begin{cases} 1 - \hat{p} + (\hat{p} - p)/\lambda^{1/\gamma}, & \textit{if } p < \hat{p}, \\ 1 - \hat{p} - (p - \hat{p})\lambda^{1/\gamma}, & \textit{if } p \geq \hat{p}. \end{cases} \tag{7}$$

Figure 2 shows the demand curves for three different reference prices. Except for non-emotional consumers, the demand curve has a kink at $p = \hat{p}$. For prices above $\hat{p}$, the transaction utility is negative, and the demand drop significantly. The acquisition utility is independent of the price, and is positive for a fraction $1 - \hat{p}$ of the population. The lower the reference price, the more consumers will experience positive acquisition utility. This explains why the population of consumers with lower reference price (i.e. $\hat{p} = 0.25$) show higher demand when the price is low. In contrast, the

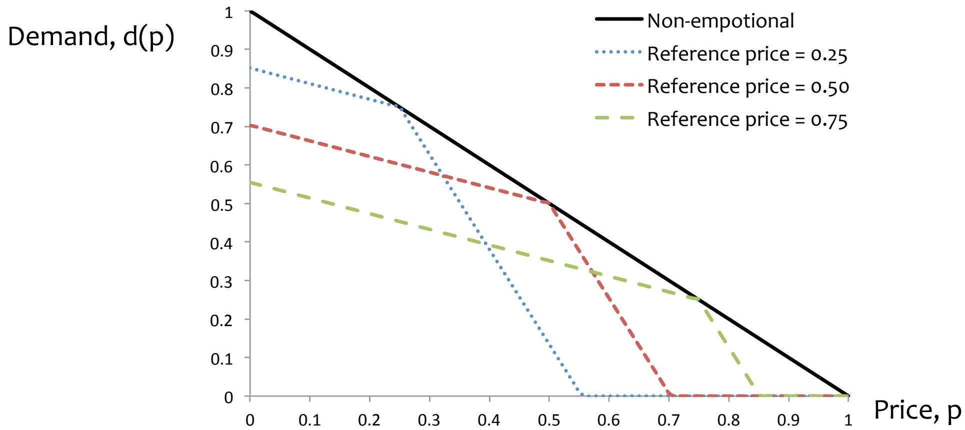

Figure 2: Demand Functions With Uniform Consumer Distribution and the Power Value Function [$\lambda = 2$ and $\gamma = 0.8$]. Non-emotional consumers exhibit $\lambda = \gamma = 1$.

population of consumers with a higher reference price (i.e. $\hat{p} = 0.75$) show higher demand when the price is high.

When facing a population consumers with a homogeneous reference price, $\hat{p}$, companies will tend to set prices at the current reference price. Below $\hat{p}$, the elasticity is low and the company will be tempted to increase prices. Above $\hat{p}$, the elasticity is high and the company is tempted to discount the price. This logic is correct if $\hat{p}$ is independent of p. Later, we will revisit this discussion in the light of the adaptive nature of reference prices.

3 Double Comparisons in Repeated Purchase

We now extend the one-period double comparison model to multiple periods. The goal is to evaluate repeated purchases, or the purchase of a durable good. A period is defined as a time window during which a single episode of consumption or payment (or both) occurs. The history of consumption and payment for a certain good or service can be represented as a repetition of this time window, and based on this we can construct a consumer's consumption and payment streams.

In a multi-period setting, consumers may make payments for items consumed in other periods. We introduce the following vectors, all in $\mathcal{R}_+^T$: The purchase quantity vector, $\boldsymbol{\theta}$, the payment vector, $\mathbf{y}$, the consumption quantity vector, $\mathbf{q}$, and the reference price vector, $\hat{\mathbf{p}}$.

The units purchases in period t, $\theta_t \geq 0$, may or may not be consumed during period t. If there is no purchase in period t, $\theta_t = 0$, then there is no payment, $y_t = 0$. The average price paid for the units purchased in period t can be obtained as $\bar{p}_t = y_t/\theta_t$ when $\theta_t > 0$. If $\theta_t = 0$, then we set $\bar{p}_t = 0$. If the price is constant, then $\mathbf{y} = \bar{p} \cdot \boldsymbol{\theta}$.

Clearly, total consumption is less or equal than total purchases, $\sum_{t=1}^{T} q_t \leq \sum_{t=1}^{T} \theta_t$, and we assume equality will hold in a deterministic demand setting. In period $t < T$, if $\sum_{i=1}^{t} q_i > \sum_{i=1}^{t} \theta_i$, then the consumer is in debt and will be making postpayment in later periods, and if $\sum_{i=1}^{t} q_i < \sum_{i=1}^{t} \theta_i$, then the consumer has pre-paid for units she may consume later.

The initial reference price, $\hat{p}_1$, is given. From period two and on, the reference price is endogenously determined. Our view is that reference updating rules are simple hard wired rules. These naïve learning rules can be overruled by explicit thinking (Kahneman, 2011). It is an empirical question to determine the particular form of the updating rule. For emotional consumers, the reference price $\hat{p}_t$ may change over time when consumption and payment occur in multiple periods. The reference price may be a function of past stimuli, and may also incorporate expectations consumers may form, and be influences by prices paid by oneself or peers on similar products. In order to provide a parsimonious model with limited degrees of freedom, we will assume the reference price is a weighted sum of past stimuli.

Given the initial reference price, $\hat{p}_1$, and the average prices in past periods, $\bar{p}_\tau, \tau = 1, ..., t-1$, ($\bar{p}_\tau = y_\tau/\theta_\tau$ if $\theta_\tau > 0$, and $\bar{p}_\tau = 0$ if $\theta_\tau = 0$), the reference price and the per-period utility V_t in period t is defined as follows:

$$\hat{p}_t = \alpha_{t,0}\hat{p}_1 + \sum_{\tau=1}^{t-1} \alpha_{t,\tau}\bar{p}_\tau, (t \geq 2), \tag{8}$$

$$V_t = v(\delta^{t-1}u(q_t) - q_t \cdot \hat{p}_t) + v(\theta_t \cdot \hat{p}_t - \delta'^{t-1}y_t). \tag{9}$$

The first term in V_t is acquisition utility and the second term is transaction utility. $0 < \delta \leq 1$ and $0 < \delta' \leq 1$ are the discount factors for valuation and payment, respectively. $\alpha_{t,\tau} \geq 0$ is the effect of the price on period $\tau, 1 \leq \tau \leq t-1$, on the reference price in period $t, 2 \leq t \leq T$; and $\alpha_{t,0} \geq 0$ is the effect of the initial reference price on the reference price in period $t, 2 \leq t \leq T$. $u_t(q_t)$ is the consumer's valuation for q_t units of consumption on period t. u is an increasing function, with $u(0) = 0$. Observe that if $q_t = 0$, then the acquisition utility is zero; and if $\theta_t = 0$, then the transaction utility is zero. The total utility for the entire periods is $V = \sum_{t=1}^{T} V_t$.

In this dynamic context, a consumer is *non-emotional* iff her utility does not depend on the reference price, that is, $V = \sum_{t=1}^{T} f(\delta^t u(q_t) - \delta'^t y_t)$ for some strictly increasing function f. We now show that, in addition to a linear value function, a non-emotional consumer will not change her reference prices over time.

Proposition 7. *The consumer is non-emotional if and only if $v(x) = cx, c > 0$, and the reference prices do not change, $\hat{\mathbf{p}} = \mathbf{1} \cdot \hat{p}_1$.*

To gain insight and tractability, we assume that if prices are constant, $\bar{p}_\tau = p, 1 \leq \tau \leq t$, and equal to the initial reference price, $\hat{p}_1 = p$, then the reference prices stay constant, $\hat{p}_{t+1} = p$. A simple way to ensure this property is to assume that $\sum_{\tau=0}^{t-1} \alpha_{t,\tau} = 1, t = 1, ..., T-1$.

Weight will exhibit *total adaptation* if $p_t \to p$ implies $\hat{p}_t \to p$. Weights will exhibit *minimal adaptation* if $\alpha = \min_{1 \leq t \leq T-1} \alpha_{t,t-1} > 0$. We call α the speed of adaptation. We consider on two specifications, each characterized by the speed of adaptation.

RA. **The recency-weighted average**: For $0 \leq \alpha \leq 1$, let

$$\hat{p}_{t+1} = (1-\alpha)\hat{p}_t + \alpha \bar{p}_t, \quad t = 1, ..., T-1.$$

AFL. **The average of the first and the last price**: For $0 \leq \alpha \leq 1$, let

$$\hat{p}_{t+1} = (1-\alpha)\hat{p}_1 + \alpha \bar{p}_t \quad t = 1, ..., T-1.$$

RA is widely used in marketing modeling (Mazumdar et al., 2005). The implicit weights are $\alpha_{t,\tau} = \alpha(1-\alpha)^{t-1-\tau}, 1 \leq \tau \leq t-1, 2 \leq t \leq T$; and $\alpha_{t,0} = (1-\alpha)^{t-1}$, $2 \leq t \leq T$. RA exhibits total adaptation if $\alpha > 0$.

AFL is simple and empirically plausible. Baucells et al. (2011) present a sequence of experiments showing that, in the formation of reference prices, the intermediate prices received a small weight compared to the first and last price. AFL captures this effect of primacy and recency, often found in psychology. The implicit weights are $\alpha_{t,0} = (1-\alpha)$, $\alpha_{t,t-1} = \alpha$, $2 \leq t \leq T$, and $\alpha_{t,\tau} = 0$ for $1 \leq \tau \leq t-2$. AFL exhibit minimal adaptation if $\alpha > 0$.

Henceforth, we assume the time span is sufficiently short so as to ignore discounting, and use $V_t = v(u(q_t) - q_t \cdot \hat{p}_t) + v(\theta_t \cdot \hat{p}_t - y_t)$. Throughout, we will compare contracts that a non-emotional consumer will deem indifferent.

4 Preference for Advance Payment

Prelec and Loewenstein (1998) argue that people generally like to pay first and consume later, a preference that is at odds with the most elementary notions of finance and the time value of money. For example, 60 percent of the people expressed a preferred to prepay for a one-week vacation to the Caribbean. To examine the difference between prepayment and postpayment in our model, we first define the payment schedules $\boldsymbol{\theta_n^a}$ and $\boldsymbol{\theta_n^b}$ as follows.

Definition 2. $\boldsymbol{\theta_n^a}$ *is defined as* $\theta_i = n$ *if* i *is a multiple of* n*, and* $\theta_i = 0$ *otherwise.*

Definition 3. $\boldsymbol{\theta_n^b}$ *is defined as* $\theta_i = n$ *if* $(i + n - 1)$ *is a multiple of* n*, and* $\theta_i = 0$ *otherwise.*

Under $\boldsymbol{\theta_n^a}$, consumers make postpayment for n units every nth period; and under $\boldsymbol{\theta_n^b}$, consumers prepay for n units every nth period. For example, $\theta_3^a = (0, 0, 3, 0, 0, 3, \ldots, 0, 0, 3)$ and $\theta_3^b = (3, 0, 0, 3, 0, 0, \ldots, 3, 0, 0)$. We will consider a constant consumption stream, $\mathbf{q} = \mathbf{1}$, for a known number of periods, $T \geq 2$, with constant prices, $\mathbf{y} = p \cdot \boldsymbol{\theta}$, and no discounting.

Recall that transaction utility occurs only at the moment of payment (at $t = 1$ for pre-payment and $t = T$ for post-payment). Assuming $\mathbf{y} = p \cdot \boldsymbol{\theta}$, $\mathbf{q} = \mathbf{1}$ and $u(1) = 1$, we have that

$$V(\boldsymbol{\theta_T^b}) = v(\hat{p}_1 T - pT) + v(1 - \hat{p}_1) + \sum_{t=2}^{T} v(1 - \alpha_{t,0}\hat{p}_1 - \alpha_{t,1}p), \text{ and} \tag{10}$$

$$V(\boldsymbol{\theta_T^a}) = v(\alpha_{T,0}\hat{p}_1 T - pT) + v(1 - \hat{p}_1) + \sum_{t=2}^{T} v(1 - \alpha_{t,0}\hat{p}_1). \tag{11}$$

If $\hat{p}_1 \geq p \geq \alpha_{T,0}\hat{p}_1$, then the transaction utility is positive for the pre-payment and negative for the post-payment. In both cases, the reference price decreases over time because $\theta_t = 0$, $2 \leq t \leq T - 1$. This has no hedonic effect if one pays *before* this adaptation occurs, but becomes painful to make the payment *after* one has been accustomed to use the service for free. If $p \geq \alpha_{t,0}\hat{p}_1$, $2 \leq t \leq T$, then the acquisition utility is higher when using a post-payment than a pre-payment. If weights satisfy a mild condition, then the transaction utility advantage of pre-payment offsets the acquisition utility disadvantage.

Proposition 8. *Assume reference price adaptation satisfies* $\alpha_{t,0} \geq \alpha_{t+1,0} + \alpha_{t+1,1}$, $2 \leq t \leq T - 1$*; loss aversion takes the form* $-v(-x) \geq v(x)$, $x > 0$*; consumption is constant,* $\mathbf{q} = \mathbf{1}$*, prices are* *constant,* $\mathbf{y} = p \cdot \boldsymbol{\theta}$*, fair or somewhat better,* $\hat{p}_1 \geq p \geq \alpha_{2,0}\hat{p}_1$*, and attractive,* $p \leq u(1)$*. If* $\alpha_{T,0} < 1$ *and* $T \geq 2$*, then the consumer strictly prefers pre-payment to post-payment.*

Both AFL and RA satisfy the adaptation condition. Under AFL, $\alpha_{t,0} = \alpha_{t+1,0} + \alpha_{t+1,1} = 1 - \alpha$, $2 \leq t \leq T-1$. Under RA, $\alpha_{t,0} = \alpha_{t+1,0} + \alpha_{t+1,1} = (1-\alpha)^{t-1}$, $2 \leq t \leq T-1$. Because the inequality is strict, the result holds in an open neighborhood, that is it holds if discounting is small, or the initial reference price is slightly below p. For a non-emotional consumer, reference prices are non-adaptive, $\alpha_{T,0} = 1$, and hence the result does not apply: he will find both contracts equally attractive, and prefer post-payment if some discounted is considered.

Shafir and Thaler (2006) showed that when people buy wines in advance, they think of it as an "investment," and later when they consume the wines, they feel as if the wines were "free." Reference price adaptation, and the assumption that $\bar{p}_t = 0$ if $\theta = 0$, implies that the acquisition utility becomes larger and larger as one gets used to not paying. Further evidence for this effect is provided by Gourville and Soman (1998), who showed that when a consumer makes payment in advance, her attention to this payment will gradually decrease over time. The flip side is a decrease in future transaction utility if one re-purchases a service at a cost that now seems expensive.

	Period	1	2	3	4	5	6	7	8	9	10	Total
	$u(q_t)$	1	1	1	1	1	1	1	1	1	1	12.00
	θ_t	4	0	0	0	4	0	0	0	4	0	12.00
	y_t	2	0	0	0	2	0	0	0	2	0	6.00
RA	Reference Price	0.50	0.50	0.25	0.13	0.06	0.28	0.14	0.07	0.04	0.07	
	Acquisition Utility	0.57	0.57	0.79	0.90	0.95	0.77	0.89	0.94	0.97	0.78	9.98
	Transaction Utility	0.00	0.00	0.00	0.00	-3.13	0.00	0.00	0.00	-3.28	0.00	-6.41
	Per-period Utility	0.57	0.57	0.79	0.90	-2.18	0.77	0.89	0.94	-2.31	0.78	3.56

Table 1: Comparison of Pre-payment with Post-payment. Consumption of $\mathbf{q} = 1$. $[\bar{p} = 0.5, u(1) = 1, \alpha = 0.5, \gamma = 0.8,$ and $\lambda = 2)$

Specifically, this is how our model captures the result from Shafir and Thaler (2006). Assume a consumer has bought 10 bottles of wine at the price of \$20 for each bottle, and will consume each bottle over 10 separate occasions (or periods). First, if the reference price does not change, then $\hat{p}_t = \hat{p} = \$20$ for all t. From an accounting viewpoint, the purchase of 10 bottles in period 1 is considered an investment having a book value of $10\hat{p} = \$200$. We pay $y_1 = \$200$ for such an investment, producing a capital gain of $10\hat{p} - \$200 = \$200 - \$200 = \$0$, whose psychological value is $v(10\hat{p} - \$200) = v(\$0)$, which is transaction utility. We amortize the bottles as we consume. In period 2, we consume one bottle, and realize a benefit of $u(1) = u$ (the revenue of selling the bottle to our stomach and getting utility in return) minus the book cost of $\hat{p} = \$20$. The total benefit is $u - \hat{p} = u - \$20$, and its psychological value is perceived as $v(u - \$20)$, which is acquisition utility. If $v(\cdot)$ is linear, then the gain per bottle is u minus the average cost; and the total gain is $10u - \$200$, as one would expect. On the other hand, if the reference price changes, say $\hat{p}_2 < \hat{p}_1 = \$20$, then there is a capital loss of $\hat{p}_1 - \hat{p}_2 = \$20 - \hat{p}_2$ that we fail to acknowledge. This produces a larger perceived benefit at the moment of consumption, as if the cost of such bottle were now cheaper. It produces, however, a larger capital loss at the moment of re-purchasing a set of bottles of the same price, as these will be perceived as more expensive.

5 Sunk Cost Effects

Thaler (1980) suggested that once people have made an up-front payment for future consumption, their consumption decision is affected by this sunk cost, even though traditional economic theory stipulates that only the marginal cost at the time of consumption should affect the decision. This is called the "sunk-cost effect."

If a consumption item, a durable good or an investment becomes less valuable or obsolete, people who have made a previous investment do not like to forego consumption, get rid of the good, or abandon the project.

The three features of our model, convexity of the value function for losses, loss aversion, and reference price adaptation, all contribute to explain the different versions of the sunk cost effect. We begin with a consumption item, the sports event ticket, that has become less valuable because of the snow storm. Here, we need to assume that u, the value of going to the game, has an average value of zero, but with some degree of uncertainty.

The prospect of not going to the game produces a sure loss of $v(-\hat{p})$. The prospect of going to the game produces $v(u - \hat{p})$. If u is uncertain, then Jensen's inequality drives the aversion to forego consumption.

Proposition 9. *Let $\tilde{u}$ be a mean preserving spread around 0. A non-emotional consumer would be indifferent between foregoing or not consumption. If v is strictly convex for losses, there is some uncertainty, $P(\tilde{u} = 0) < 1$, and the valuation stays below the reference price, $P(\tilde{u} \leq \hat{p}) = 1$, then $E[v(\tilde{u} - \hat{p})] > v(-\hat{p})$ and the consumer will strictly prefer to consume rather than forego consumption.*

Losing or giving up a pre-purchased item is painful. In the model, if some pre-purchase quantities are lost or become obsolete, then there is a mental amortization. Suppose we have pre-purchased a consumption quantity vector, $\mathbf{q}$, and the item becomes obsolete or unusable passed $\tau, 1 \leq \tau < T$. Formally, $\mathbf{q} = (q_1, ..., q_\tau, q_{\tau+1}, ..., q_T)$ is replaced by $\mathbf{q}' = (q_1, ..., q_\tau, 0, ..., 0)$. Clearly, the term $\sum_{t=\tau+1}^{T} v(u - \hat{p}_t)$ disappears from the utility, as no consumption will take place. Instead, the loss of $n_o = T - \tau$ units is written in the mental book as $v(-n_o \hat{p}_{\tau+1}) < 0$, and the total hedonic value lost by obsolescense is given

$$v(-n_o \hat{p}_{\tau+1}) - \sum_{t=\tau+1}^{T} v(u - \hat{p}_t) < 0.$$

To gain insight, let v be power and set $\gamma = 1$. Assume AFL and $\tau \geq 2$, so that $\hat{p}_{\tau+1} = (1 - \alpha)p$. Then, the hedonic loss due to obsolescence is given by $-n_o \left[u + (1 - \alpha)(\lambda - 1)p_1\right]$, which increases with λ. Reference price adaptation mitigates the emotional loss, as one thinks the item is not worth much. For a non-emotional consumer, obsolescence produces a loss of $-n_o u$.

Gourville and Soman (1998) showed that the intensity of the sunk cost effect decreases with the passage of time. They analyzed the attendance records of a gym, where all customers had one-year memberships and made payments twice a year. They found out that the attendance was the highest in the month when payment was made, and it has steadily decreased over time afterwards. Similarly, in the sports ticket and the snowstorm situation, the probability of going to the event decreased with the time elapsed between the purchase of the ticket and the game.

In our model, two mechanisms explain this *Payment Depreciation*. For durable goods, the passage of time may produce usage of the good, and part of it may be mentally amortized.

Proposition 10. *Assume loss aversion takes the form $v'(-x) > v'(x), x > 0$ and reference prices are totally adaptive. The hedonic loss due to obsolescence of n_o units, $v(-n_o \hat{p}_{\tau+1}) - \sum_{t=\tau+1}^{\tau+n_o} v(u - \hat{p}_t) < 0$, strictly decreases with τ.*

There is a second mechanism by which the passage of time reduces the sunk cost effect, even if the durable good is not in use. The passage of time reduces the sunk cost effect because it lowers reference prices. This applies to durable goods once they have become obsolete, as well as single purchase consumption items such as the sports ticket. It also explains why, if the item has been received as a gift, the tendency to forego consumption is higher.

Proposition 11. *Let u be any given random variable. The difference between consuming the item, $E[v(\tilde{u} - \hat{p})]$, and foregoing consumption, $v(-\hat{p})$, decreases as $\hat{p}$ decreases.*

Let $\tilde{u}$ be a mean preserving spread around 0. A non-emotional consumer would be indifferent between foregoing or not consumption. Let v be strictly convex for losses, strictly concave for gains, and exhibit loss aversion. Then, for some $\epsilon > 0$, $E[v(\tilde{u} - \hat{p})] \leq v(-\hat{p})$ if $\hat{p} < \epsilon$. That is, the individual prefers to forego consumption if the reference price is sufficiently low.

In the beginning of a flat-rate contract, for example, the large up-front payment saliently remains on consumers' mind and consumers feel significant mental pain when they have to forgo consumption for some external reasons. However, if the reference price strictly decreases over time (e.g. by RA), people gradually adapt to using the service for free and they do not feel as much pain from not using the service as they did in the beginning. For example, when people sign up for a gym with a one year contract, they may be eager to go to the gym as frequently as possible in the beginning. Since the big payment they made when they signed up still remains saliently in their mind, it feels painful when they cannot go to the gym. But as time passes, the salience of the initial payment gradually dissipates (i.e., the reference price decreases), and it feels less painful not to go to the gym. This may be one explanation why people prefer to choose a flat-rate contract when signing up for a gym, but later they do not go to the gym that much, so they actually could have saved money under a pay-as-you-go contract (DellaVigna and Malmendier, 2006; Gourville and Soman, 1998).

Sunk cost depreciation is difficult to incorporate in a model where u is compared to p, and not to $\hat{p}$ as in our case.

5.1 Replacement of Durable Goods

Suppose you bought an smart phone, anticipating a per-period benefit of u during T periods. The decision was made because the lifetime benefit exceeds the price, $u > p/T = \bar{p}_1$. Suppose that after having paid for the phone, and before using it, you learn that a new model costing also p gives you θu, $\theta > 1$. The old model cannot be sold because is obsolete. Would you buy the new model? If $(\theta - 1)u/\bar{p}_1 > 1$, then a non-emotional consumer will surely replace the old model with the new one. Because $uT > p$, this necessarily holds if $\theta \geq 2$.

How would an emotional consumer react? Buying and keeping the old model produces:

$$v(T\hat{p}_1 - p) + \sum_{t=1}^{T} v(u - \hat{p}_t) > 0.$$

Buying the old model and switching to a new one at $\tau = 1$ produces:

$$v(T\hat{p}_1 - p) + v(-T\hat{p}_1) + v(T\hat{p}_1 - p) + \sum_{t=1}^{T} v(\theta u - \hat{p}_t).$$

For an emotional consumer, the mental amortization of the first model, $v(-T\hat{p}_1)$, is painful. Assume that the price is fair, $p = T\hat{p}_1$, which eliminates the transaction utility terms. To gain insights, assume the value function is piecewise linear, $\gamma = 1$. Moreover, assume reference prices stay constant, $\alpha = 0$. The emotional consumer will switch to the new model iff

$$\lambda < (\theta - 1)u/\bar{p}_1.$$

The cost/benefit ratio of the new model needs to be $\lambda + 1$ times as good the original one to justify the switch.

6 Tariff Choices in Repeated Consumption

Lambrecht and Skiera (2006) analyzed the transactional data of 10,882 customers of an Internet service provider. They show that, over 5 months, 46.4% of consumers under a with a high fixed-fee (and allowance) would have been on average better off under an alternative tariff with lower fixed-fee (and lower allowance). More than half of the consumers with the flat-rate bias paid at least 100% more than they would have on the least costly pay-per-use tariff.

We first identify the flat-rate bias with deterministic demand and valuation. As before, there is a firm providing a service to consumers for T periods and a consumer makes a discrete consumption choice in each period. We consider a consumer that purchases n units at a time. In the case of $n = 1$, this corresponds to a pay-as-you-go scheme. In the case of $n = T$, and certainty about T, this corresponds to a flat-rate fee. The case of $1 < n < T$ corresponds to a pre-purchase card of n uses (see Table 2). Formally, we will explore preferences over purchase quantity vectors, $\boldsymbol{\theta}^{\boldsymbol{b}}_{\boldsymbol{n}}$. Recall that, for example, $\boldsymbol{\theta}^{\boldsymbol{b}}_{\boldsymbol{2}} = (2, 0, 2, 0, ...)$ and that $\boldsymbol{\theta}^{\boldsymbol{b}}_{\boldsymbol{3}} = (3, 0, 0, 3, 0, 0, ...)$.

Throughout, we assume unit consumption, $\mathbf{q} = \mathbf{1}$, constant price, $\mathbf{y} = p\boldsymbol{\theta}$, and no discounting.

For ease of comparison, we set the initial reference price at the average price, $\hat{p}_1 = p$, and normalize $u(1) = 1$. We consider the case when the consumer knows with certainty the number of uses, T, and the case of uncertainty on how often she will use the service because the per-use valuation is uncertain, or the number of periods is uncertain.

	Period	1	2	3	4	5	6	7	8	9	10	11	12	Total
	$u(q_t)$	1	1	1	1	1	1	1	1	1	1	1	1	12.00
	θ_t	4	0	0	0	4	0	0	0	4	0	0	0	12.00
	y_t	2	0	0	0	2	0	0	0	2	0	0	0	6.00
AFL	Reference Price	0.50	0.50	0.25	0.25	0.25	0.50	0.25	0.25	0.25	0.50	0.25	0.25	
	Acquisition Utility	0.57	0.57	0.79	0.79	0.79	0.57	0.79	0.79	0.79	0.57	0.79	0.79	8.65
	Transaction Utility	0.00	0.00	0.00	0.00	-2.00	0.00	0.00	0.00	-2.00	0.00	0.00	0.00	-4.00
	Per-period Utility	0.57	0.57	0.79	0.79	-1.21	0.57	0.79	0.79	-1.21	0.57	0.79	0.79	4.65

Table 2: Hedonic Value of a Repeated Purchase of a Card of Valid for $n = 4$ uses. [$\mathbf{q} = \mathbf{1}$, $\bar{p} = 0.5, u(1) = 1, \alpha = 0.5, \gamma = 0.8,$ and $\lambda = 2$)

6.1 Flat-rate Bias under Certainty

First, we compare a consumer's utility under a pay-as-you-go tariff and a flat-rate tariff. Under a pay-as-you-go tariff, a consumer makes payment in every period. If $\sum_{\tau=0}^{t} \alpha_{t,\tau} = 1$, then the reference price $\hat{p}_t$ remains at the unit price p throughout the periods, and the transaction utility is zero. Because $u(1) = 1$ and $\hat{p}_1 = p$, the hedonic value of a simple pay-as-you-go scheme is

$$V(\boldsymbol{\theta}_{\mathbf{1}}^{\boldsymbol{b}}) = \sum_{t=1}^{T} v(1 - \hat{p}_t) = n \cdot v(1 - p). \tag{12}$$

Under a flat-rate tariff, a consumer pays everything in the first period and nothing thereafter, so the reference price $\hat{p}_t$ decays throughout the periods (recall that if $\theta_i = 0$, then $\bar{p}_i = 0$). Transaction utility occurs only in periods of payment, that is, in the first period. Because $\hat{p}_1 = p$, we have that $\theta_1 \hat{p}_1 = Tp = y_1$. Therefore, transaction utility is zero and

$$V(\boldsymbol{\theta}_{\boldsymbol{T}}^{\boldsymbol{b}}) = \sum_{t=1}^{T} v(1 - \hat{p}_t). \tag{13}$$

For a consumer with no changes in reference prices, such as the non-emotional type, $V(\boldsymbol{\theta}_{\mathbf{1}}^{\boldsymbol{b}}) = V(\boldsymbol{\theta}_{\boldsymbol{T}}^{\boldsymbol{b}}) = nv(1 - p)$. If reference prices are minimally adaptive, however, a flat-rate tariff will be strictly preferred to a pay-as-you-go tariff.

Proposition 12. *If $T > 1$ and $\alpha_{t,t} > 0$, then $V(\boldsymbol{\theta}_{\boldsymbol{T}}^{\boldsymbol{b}}) > V(\boldsymbol{\theta}_{\mathbf{1}}^{\boldsymbol{b}})$.*

PL98 explain the flat-rate bias using a complicated mechanism of coupling and prospective accounting. Our explanation is based on a natural mechanism of double comparison and adaptation. The use of adaptation, a general psychological principle, to explain the flat rate bias is novel.

In our numerical exploration of pre-purchase cards, we find that $V(\boldsymbol{\theta_n^b})$ increases with n under a broad set of parameter values and adaptation rules, provided n is a divisors of T. The result is violated under RA and n is small (see Table 3). Under AFL, the more uses a pre-purchase card has, the more hedonic value it has. Consequently, flat-rate is preferred to a pre-pruchase card, and a pre-pruchase card is preferred to pay-per-use. Formally,

Proposition 13. *Under AFL, the value of a pre-purchase card of n uses is given by*

$$V(\boldsymbol{\theta_n^b}) = \left(1 + \frac{T}{n}\right) v(1-p) + \left(T - 1 - \frac{T}{n}\right) v(1-(1-\alpha)p) + \left(\frac{T}{n} - 1\right) v(-\alpha n p).$$

Let n and n' be divisors of T. If $\alpha > 0$ and $1 \leq n < n' \leq T$, then $V(\boldsymbol{\theta_{n'}^b}) > V(\boldsymbol{\theta_n^b})$.

	n	1	2	4	5	8	10	16	20	40	80
RA	γ=0.8	45.9	6.9	8.6	11.9	20.7	25.2	34.9	39.7	56.4	78.7
	γ=0.5	56.6	5.4	22.4	28.6	40.9	46.0	55.5	59.4	69.9	79.2
AFL	γ=0.8	45.9	9.7	20.9	23.9	29.8	32.4	38.0	40.7	50.3	63.1
	γ=0.5	56.6	7.6	27.9	33.0	42.1	45.7	52.3	55.1	62.5	69.0

Table 3: Hedonic value of pre-purchase cards, $V(\boldsymbol{\theta_n^b})$, as a function of n and the adaptation process. [$T = 80, u(1) = 1, \alpha = 0.5$, and $\lambda = 2$]

6.2 Tariff Choice under Demand Uncertainty

Existing literature have shown consumers to have a biased preference for flat-rate tariffs even if they face demand uncertainty (Lambrecht and Skiera, 2006; Narayanan et al., 2007; DellaVigna and Malmendier, 2006; Grubb, 2009; Miravete, 2002, 2003; Goettler and Clay, 2009). We will show that reference price adaptation successfully explains this finding.

At the moment of signing a flat rate contract, or buying a pre-purchase card, consumers often do not now the exact use they will make of the service. Two sources of uncertainty seem relevant. One is the valuation of the service as time progresses. For example, the value of going to the gym may vary over time because of unforeseeable time availability constraints, or tiredness and satiation effects. The second effect, which we consider in the next subsection, is the risk of product obsolescence.

We first examine the effect of uncertainty in the valuation. Let T be fixed and known, and assume the consumers' valuation, $u_t(q_t)$, is uncertain. The consumer will hold some (joint) probability distribution about the realization of $u(q_t)$, and the actual realization will be known at t, before deciding whether to use the service or not. We restrict ourselves to independent realizations, the value of $u(q_t)$ has not forecasting value, and the consumer will choose q_t so as to maximize the per-period utility.

For a simple comparison, we compare a flat-rate tariff and a pay-as-you-go tariff. We consider a discrete choice problem, let $\hat{p}_1 = p$ and assume $\sum_{\tau=0}^{t} \alpha_{t,\tau} = 1$. Under a flat-rate tariff, the payment schedule is $\boldsymbol{\theta_T^b}$ as before. The per-period utility of using the service is always higher than that of not using the service because of the up-front payment. Consumption, however, may not bring positive feelings if $u_t < \hat{p}_t$. The acquisition utility is negative because one is using a service whose assigned cost is higher. One feels such consumption is not justified, and would not have been done under a pay-as-you-go scheme. Under AFL, these feelings will stay over time as long as $u < (1-\alpha)p$. In the RA model, these feelings will disappear over time, as $\hat{p} \to 0$, and one will act as if the service were truly for free. In any case, the consumer will always use the service because $v(u_t - \hat{p}_t) > v(-\hat{p}_t)$ and $\mathbf{q} = \mathbf{1}$.

Under a pay-as-you-go tariff, a consumer would skip the service if the willingness to pay is below the price. Note that if we skip the purchase, there is no acquisition and no transaction, and by our definition of period, there is no reference price adaptation. As in the case of certainty, pay-as-you-go leads to $\hat{p}_t = p, t = 1, ..., T$ and transaction utility is zero. Therefore, $w_t = u_t$, and the payment schedule is $\tilde{\boldsymbol{\theta}}_{\mathbf{1}}^{\boldsymbol{b}}$, where $\tilde{\theta}_t(u_t) = 1$ if $u_t \geq p$ and $\tilde{\theta}_t(u_t) = 0$ if $u_t < p$.

Let $V(\boldsymbol{\theta_T^b})$ and $V(\tilde{\boldsymbol{\theta}}_{\mathbf{1}}^{\boldsymbol{b}})$ denote the total utility under a flat-rate and a pay-as-you-go tariff, respectively. The expected total utility is given by:

$$E[V(\boldsymbol{\theta_T^b})] = E_u[\sum_{t=1}^{T} v(u - \hat{p}_t)],$$
$$E[V(\tilde{\boldsymbol{\theta}}_{\mathbf{1}}^{\boldsymbol{b}})] = E_u[\sum_{t=1}^{T} \max\{v(u - p), 0\}].$$

In the pay as you go case, $E[V(\tilde{\boldsymbol{\theta}}_{\mathbf{1}}^{\boldsymbol{b}})]$ involves a maximum operator, because consumers can skip consumption whenever $v(u_t - p) < 0$.

It would seem that, under uncertainty, the pay-as-you-go tariff is superior, as it gives more flexibility. The flat rate bias, however, retains the hedonic value of a higher acquisition utility due to reference-price adaptation. Moreover, once the consumer gets adapted to using the service for free, she will use the service more often, further increasing the experienced utility. If reference prices were not to change, the buyer of a flat-rate contract would feel negative acquisition utility every time it uses the service when its valuation is low, $u_t < \hat{p}$. Because of reference price adaptation, these negative feeling vanish over time. Hence, if these negative feelings are under control because u does not fall too much below p, or T is sufficiently large, the flat-rate tariff will be preferred over the pay-as-you-go tariff.

Without loss of generality, assume u is continuously distributed on $[0, 1]$ (any other distribution can be approximated by such family). Because $\hat{p}_1 = p$, there is no transaction utility in the flat-rate and, as argued, pay-as-you-go does not have transaction utility either. Moreover, $\hat{p}_1 = \hat{p}_2 = p$ and, for $t \geq 3$, $\hat{p}_t \leq (1-\alpha)p < p$. The difference between the two contracts, $E[V(\boldsymbol{\theta_T^b})] - E[V(\boldsymbol{\tilde{\theta}_1^b})]$, is given by

$$\sum_{t=1}^{T} \int_{p}^{1} f(u)[v(u-\hat{p}_t) - v(u-p)]du + \sum_{t=3}^{T} \int_{\hat{p}_t}^{p} f(u)v(u-\hat{p}_t)du + \sum_{t=1}^{T} \int_{0}^{\hat{p}_t} f(u)v(u-\hat{p}_t)du. \quad (14)$$

The first and second terms are positive, strictly so if $T \geq 3$ and $\alpha > 0$. In general, the utility of the flat rate is higher, except for periods in which the valuation if below the reference price. If valuations satisfy $P(u_t < \hat{p}_t) = 0$ and $T \geq 2$, then we have that flat-rate is strictly preferred to pay-as-you-go. This requires $u_1, u_2 > p$, a condition that may fail. A more reasonable approach is to assume that T is sufficiently large, so that we compensate for this negative expected utility. We now show sufficient conditions that ensure that the flat-rate preference generalized to the case of uncertain valuation.

Proposition 14. *Consider a discrete choice repeated purchase with uncertain valuation, with u_t iid and $E[u_t] > p$. The consumer prefers a flat-rate tariff over a pay-as-you-go tariff if reference prices are minimally adaptive, T is sufficiently large and*

$$\sum_{t=1}^{\infty} v(-\hat{p}_t)P(u_t \leq \hat{p}_t) > -\infty. \quad (15)$$

It is not difficult to find sufficient conditions that ensure (15) is bounded:

1. $P(u < (1-\alpha)p) = 0$.

2. Reference prices are totally adaptive and for some $\epsilon > 0$, $P(u < \epsilon) = 0$.

3. Reference prices follow RA, $\alpha > 0$, v is power, and u_t is iid.

The result stresses again the crucial role that adaptation processes have on the flat-rate bias. In the case of uncertain valuation, however, T has to be large to justify this preference. Not surprisingly, flat-rate tariffs are often offered for relatively large time periods (two years for many phone contracts, or lifetime club memberships or lifetime ownership of time-share contracts). For durable goods, and uncertainty over the valuation of the good in any given period, the preference for ownership over rental will increase with the duration of the good.

The result holds for important examples such as u_t being a Bernoulli random variable with $P(u = 1) > \pi$ and $P(u = 0) = 1 - \pi > 0$. For example, the value of u_t could be determined by the time availability to use the service. If one is able to use the service, then the valuation is $u_t = 1$, and 0 otherwise.

6.3 Tariff Choice under Risk of Obsolescence

A tourist in a city usually faces the choice of buying a single-use ticket or a weekly/monthly pass for transportation, but they are rarely certain about how much they will use transportation. Also, when signing up for a gym, consumers do not know how often and for how long they will end up using the facilities. Consumers quite often face the decision of having to choose a tariff not knowing for how long they need the service.

Consumers choose a tariff between the two options based on the demand distribution prior to consumption. After choosing a tariff, consumers start using the service from period 1 and on, until the product becomes obsolete passed $t = \tau$. Rather than assuming T is random, let $T = \infty$ and let the demand be given by $\mathbf{q}_\tau = (q_1, \ldots, q_{\tau-1}, q_\tau, q_{\tau+1} \ldots) = (1, \ldots, 1, 1, 0, \ldots)$, i.e., the first τ components are ones, and the rest are zeros. $\tau \geq 1$ will be random, and we let $\eta = E[\tau] \geq 1$ be the expected consumed units.

Again, we compare a flat-rate tariff and a pay-as-you-go tariff. Under a flat-rate tariff, consumers consumers pay a fixed amount up-front and expect to consume η units. To simplify notation, we assume η is an integer. We conveniently denoted this tariff by $\tilde{\boldsymbol{\theta}}^{\boldsymbol{b}}_{\boldsymbol{\eta}}$. We can always express the total price as $\bar{p}^f_1\eta$, where $\bar{p}^f_1$ is the expected unit price under a flat rate tariff. Naturally, if a consumer's realized demand is less than η, she would incur negative utility for having paid more than what she would actually consume, and incurs an amortization cost of $v(-(\eta-\tau)\hat{p}_{\tau+1})$ at $t=\tau+1$. In the same manner, and keeping with accounting principles, if a consumer's realized demand is larger than η, she would feel that the units consumed over and above η are for free. Therefore, he will cease to subtract the reference price from the acquisition utility associated with these units. This procedure assures that for a non-emotional consumers the reference prices disappear from the evaluation. Conditional on τ, the total utility under a flat-rate tariff is given by:

$$V(\tilde{\boldsymbol{\theta}}^{\boldsymbol{b}}_{\boldsymbol{\eta}}|\tau) = v(\eta(\hat{p}_1-\bar{p}^f_1)) + \begin{cases} \sum_{i=1}^{\tau} v(1-\hat{p}_i) + v(-(\eta-\tau)\hat{p}_{\tau}), & \text{if } \tau<\eta, \\ \sum_{i=1}^{\eta} v(1-\hat{p}_i) + (\tau-\eta)v(1), & \text{if } \tau\geq\eta. \end{cases} \tag{16}$$

Under a pay-as-you-go tariff $\boldsymbol{\theta_1}$, consumers pay $\bar{p}_t$ in period t. Conditional on τ, the total utility under a pay-as-you-go tariff is given by:

$$V(\boldsymbol{\theta}^b_1|\tau) = \sum_{t=1}^{\tau} v(1-\hat{p}_t) + v(\hat{p}_t-\bar{p}_t). \tag{17}$$

Taking expectations, the expected hedonic value of each tariff is given by:

$$V(\boldsymbol{\theta}^b_\eta) = \sum_{t=1}^{\infty} V(\tilde{\boldsymbol{\theta}}^{\boldsymbol{b}}_{\boldsymbol{\eta}}|\tau)\cdot Pr(\tau=t), \tag{18}$$

$$V(\boldsymbol{\theta}^b_1) = \sum_{t=1}^{\infty} V(\boldsymbol{\theta}^b_1|\tau)\cdot Pr(\tau=t). \tag{19}$$

Proposition 15. *If the expected prices under a pay-as-you-go are the same as the average expected price under a flat-rate tariff, $\sum_{t=1}^{\infty}\bar{p}_t\cdot Pr(\tau=t)=\bar{p}^f_1$, then a non-emotional consumer is indifferent between choosing a flat-rate tariff and a pay-as-you-go tariff.*

In particular, if prices of the pay-as-you-go are constant, then the proposition requires $\bar{p}_1 = y^f_1/\eta = \bar{p}^f_1$. To help the comparison, we set the reference price is equal to the actual price, which we assume constant over time. The terms $v(\eta(\hat{p}_1-\bar{p}_1))$ disappears in the evaluation of the flat-rate tariff. The pay-as-you-go tariff becomes much simpler because $\hat{p}_t=\bar{p}_t=\bar{p}_1$. Hence, $V(\boldsymbol{\theta_1}|\tau)=\tau v(1-\bar{p}_1)$.

Table 4 shows examples of total expected utility for the non-emotional and the emotional consumer. For the non-emotional consumer, the expected utility under a pay-as-you-go tariff is

flat-rate (y_1)	Non-emotional	$\alpha = 0.3$	$\alpha = 0.5$
1	9.50	9.57	9.57
2	9.00	9.20	9.27
3	8.50	8.87	9.05
4	8.00	8.58	8.86
5	7.50	8.32	8.69
6	7.00	8.08	8.54
7	6.50	7.85	8.40
8	6.00	7.64	8.26
9	5.50	7.43	8.13
10	**5.00***	7.23	8.00
11	4.50	7.04	7.87
12	4.00	6.85	7.75
13	3.50	6.67	7.63
14	3.00	6.49	7.52
15	2.50	6.31	7.40
16	2.00	6.14	7.29
17	1.50	5.97	7.18
18	1.00	**5.80***	7.06
19	0.50	5.63	6.96
20	0.00	5.47	6.85
21	-0.50	5.31	6.74
22	-1.00	5.15	6.64
23	-1.50	4.99	6.53
24	-2.00	4.84	6.43
25	-2.50	4.68	6.33
26	-3.00	4.53	6.23
27	-3.50	4.38	6.13
28	-4.00	4.23	6.03
29	-4.50	4.08	5.93
30	-5.00	3.93	**5.83***
pay-as-you-go	5.00	5.74	5.74

Table 4: Expected total utility under demand uncertainty and RA for the non-emotional $[\lambda = \gamma = 1 - \alpha = 1]$ and the emotional consumer $[\gamma = 0.8$ and $\lambda = 2]$. We fix $\eta = E[\tau] = 10$.

higher than the expected utility under a flat-rate tariff if and only if the price is fair given the expected usage.

However, when consumers are emotional, the flat-rate tariffs become more attractive than the pay-as-you-go tariff, even if the prices are not fair given the expected usage ($n > 10$). For example, Case 1 shows that $V(\boldsymbol{\theta^f_{19}}) \leq V(\boldsymbol{\theta^p}) \leq V(\boldsymbol{\theta^f_{18}})$, which means that consumers would prefer a flat-rate tariff $\boldsymbol{\theta^f_{18}}$ to a pay-as-you-go tariff $\boldsymbol{\theta^p}$. Hence, consumers are willing to pay more to choose a flat-rate tariff and this can be considered a flat-rate bias as well. Also, the preference for a flat-rate tariff becomes stronger as the reference prices adapt more rapidly. Case 2 has a higher α, and therefore the reference prices adapts faster, and we can see that consumers would even prefer $\boldsymbol{\theta^f_{30}}$ to a pay-as-you-go tariff.

If the seller bears the risk of obsolescence, then the expected unit payment of a flat-rate is lower. In this case, consumers generally prefer a flat-rate tariff under demand uncertainty because of reference price adaptation. Under a flat-rate tariff, consumers have the risk of paying more when the realized demand is less than the units they have paid for, but as they start consuming the item, the feeling of using it for "free" offsets the disutility of having paid more. In addition, as discussed before, the pain of having paid for more units than the ones use gets mitigated by reference price adaptation as τ increases.

If the buyer bears the risk of obsolescence, then two forces intervene. The risk of paying for unused units shifts preference for pay-as-you-go. However, the hedonic benefits of paying in advance and enjoying higher acquisition utility is still at work, favoring a flat-rate tariff. In general, a pre-purchase card with n units will be the optimal payment scheme.

6.4 Tariff Switch

Lambrecht and Skiera (2006) observed that many consumers who were under a flat-rate tariff for Internet access could have saved money if they had switched to a pay-per-use tariff. However, their data showed that many of those people stuck to the flat-rate tariff persistently without switching to a pay-per-use tariff. In our model, consumers stay under a flat-rate tariff over multiple contract renewals. Reference price adaptation creates a "trap" and as consumers get more used to a flat-rate tariff, they get less attracted to a pay-as-you-go tariff.

The reference price decays over time under a flat-rate tariff, whereas it remains at the unit price under a pay-as-you-go tariff. Therefore, when consumers renew their contract, consumers that used a flat-rate tariff exhibit lower reference prices than consumers that used a pay-as-you-go tariff. This reduces the appeal of a pay-as-you-go tariff. Let $V(\boldsymbol{\theta}^b|\hat{p}_r)$ denote the utility during the T periods starting at $t = r$. The utility of the first contract is given by $V(\boldsymbol{\theta}^b|\hat{p}_1)$, and the utility of the renewal is $V(\boldsymbol{\theta}^b|\hat{p}_T)$.

The follows result shows that the appeal of the pay-as-you-go tariff increases with the consumer's initial reference price.

Proposition 16. *Assume $v'(-x) \geq v'(x)$ for all $x > 0$, and that $0 \leq \hat{p}_1 \leq p$. Then $V(\boldsymbol{\theta}_1^b|\hat{p}_1)$ is increasing in $\hat{p}_1$.*

A consumer that chooses a flat-rate finds that $\hat{p}_T < \hat{p}_1$. Therefore, the appeal of the pay-as-you-go contract decreases, $V(\boldsymbol{\theta}_1^b|\hat{p}_T) < V(\boldsymbol{\theta}_1^b|\hat{p}_1)$.

The appeal of the flat-rate tariff also decreases as $\hat{p}_T$ decreases. However, if T is sufficiently large, consumers will still exhibit the flat-rate bias.

Proposition 17. *Assume $\lim_{x\to-\infty} v'(x) = 0$, $v'(-x) \geq v'(x)$ for all $x > 0$, and that reference prices are minimally adaptive, and that $0 \leq \hat{p}_1 < p$. If the consumer is under a flat-rate tariff, and T is sufficiently large, then*

$$V(\boldsymbol{\theta}_T^b|\hat{p}_T) > V(\boldsymbol{\theta}_1^b|\hat{p}_T).$$

Previously, we showed that when $\hat{p}_1 = p$ consumers will always choose a flat-rate tariff over a pay-as-you-go tariff, $V(\boldsymbol{\theta}_T^b|\hat{p}_1) > V(\boldsymbol{\theta}_1^b|\hat{p}_1)$. Proposition 17 shows that the flat-rate bias still exists even when the initial reference price is lower than p, as long as the total period T is large enough. This result helps explain why consumers persistently stayed under a flat-rate tariff over multiple contract renewal opportunities even when they could have saved money by switching to a pay-as-you-go tariff (Lambrecht and Skiera, 2006).

7 Conclusion

Many studies have identified the existence of the flat-rate bias (Train, 1991) and tried to explain the phenomenon from different perspectives, such as prospective accounting (Prelec and Loewenstein, 1998; Lambrecht and Skiera, 2006) or demand uncertainty (Lambrecht and Skiera, 2006; Narayanan et al., 2007; DellaVigna and Malmendier, 2006; Grubb, 2009; Miravete, 2002, 2003; Goettler and Clay, 2009). We have introduced a model of double comparisons and reference price adaptation that offers a parsimonious and integrative account of the flat-rate bias, both under certainty and under uncertainty. The model predicts several other anomalies, all important and well documented, such as strong preference for advance payment, sunk-cost effect, payment depreciation, and tariff switching behaviors.

Our model can be considered a dynamic extension of the acquisition and transaction utility model of Thaler (1985). We adopt the concept of acquisition and transaction utility with minor modification. In the multi-period case, the reference price is intrinsically calculated, and the decision maker cannot help but use what his internal "adaptive" calculator proposes as a reference price. Hence, these consumers will experience hedonic feelings of "cheap or expensive" that are associated with payment methods.

Our model is parsimonious, in the sense that the value function and the reference price rule is "universal" to any reference-dependent model and can be externally validated. Given the wide acceptance of reference-dependence in psycho-economic models, it is an inescapable task of social sciences to estimate how reference points are determined and evolve over time.[3]

There are several directions of future research. First, we could explore the contractual implications of this model from a firm's perspective. A hint of the implications has been given when discussing demand curves in the single period case. A more explicit investigation of a firm's optimal dynamic pricing policies over multiple periods is a direct application, which we leave for future research.

Once the consumer is adapted to it, it becomes difficult to change to a pay-as-you-go tariff. An emotional consumer that foresees all this will choose a flat rate if she does not predict changing tariffs. Actual consumers, however, experience a common failure to anticipate adaptive proces. They falsely believe that the future reference prices will be closer to current reference prices than they will actually be (Loewenstein et al., 2003; Baucells and Sarin, 2010). Put simply, they assume α to be smaller than its actual value. Such consumers will not foresee the increase in switching

[3]Parsimony is lacking in PL98. The coupling coefficients of are proper to the model. Moreover, PL98 fails to distinguish many different streams of consumption and payments. For example, their model would predict that a prepayment of one hour and a prepayment of one year would provide the same hedonic benefit to consumers. In our case, because of adaptation, there would be a difference between these two.

cost as they get adapted to use the service for free. Projection bias then will find consumers trap in flat-rate contracts.

The model can be extended to new domains, such as supplier selection. Compare the option of choosing multiple suppliers (buy the plane ticket, book the hotel, rent a car) vs. one integrated supplier (a travel package provider). According to double accounting, people will exhibit a preference for one integrated supplier. With multiple suppliers, some prices will be above the reference price and others below. With an integrated supplier, there is a single comparison. Due to loss aversion, the integrated supplier will, on average, provide a higher hedonic value, even though the final cost may be more expensive.

Further experimental validation of the exact mechanism of double comparisons and reference price adaptation would strengthen the theoretical value of this model. Moreover, some of the new effects predicted by the model should be better substantiated.

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A Proofs

Proof of Proposition 1 If $v(x) = cx, c > 0$, then $v(u-\hat{p})+v(\hat{p}-p) = c\cdot(u-\hat{p})+c\cdot(\hat{p}-p) = c\cdot(u-p)$, which does not depend on the reference price $\hat{p}$. Conversely, set $x = u - \hat{p}$ and $y = \hat{p} - p$, and observe that f satisfies Pexider's equation:

$$f(x + y) = v_1(x) + v_2(y). \tag{20}$$

Because f is continuous at one point, necessarily, $f(x) = cx+a+b$, $v_1(x) = cx+a$ and $v_2(x) = cx+b$, where c, a, and b are arbitrary constants (Aczél, 1966). Because $v_1(0) = v_2(0) = 0$, $a = b = 0$, and $f(x) = v_1(x) = v_2(x) = v(x) = cx$. Because f is strictly increasing, $c > 0$.

3 implies 2. If $v(x) = cx$, then w solves $c(u - \hat{p}) + c(\hat{p} - w) = 0$, or $w = u$.

2 implies 1. If $w = u$, then $\square$

Proof of Proposition 2 First, note that if the value function satisfies $v'(-x) > v'(x)$ for all $x > 0$, then it also satisfies that $-v(-x) > v(x)$ for all $x > 0$.

Define $V(\hat{p}, p) = v(u - \hat{p}) + v(\hat{p} - p)$ for a given u. By definition, the willingness to pay, w, satisfies $V(\hat{p}, w) = 0$. V is strictly decreasing with p. Moreover $V(\hat{p}, \hat{p}) = v(u - \hat{p}) + v(0) > 0$, and, because of loss aversion, $V(\hat{p}, u) = v(u - \hat{p}) + v(\hat{p} - u) < 0$. Hence, there is a unique $\hat{p} < w < u$ solving $V(\hat{p}, w) = 0$. Using the convexity of v for losses and loss aversion, respectively,

$$v'(\hat{p} - w) \geq v'(\hat{p} - u) > v'(u - \hat{p}).$$

By the implicit function theorem,

$$\frac{\partial w}{\partial \hat{p}} = -\frac{\partial V / \partial \hat{p}}{\partial V \partial w} = 1 - \frac{v'(u - \hat{p})}{v'(\hat{p} - w)} > 0. \quad \square$$

Proof of Proposition 3 The non-emotional consumer buys iff $u > p$. For the emotional consumer, we distinguish all the possible cases (the proposition omits some for simplicity):

If $u > p$ and $u \geq \hat{p} \geq p$, then $v(u - \hat{p}) \geq 0$ and $v(\hat{p} - p) \geq 0$, and one of the inequality is strict. Hence $w > p$.

If $\hat{p} < p < u$, then $v(u - \hat{p}) > 0$ and $v(\hat{p} - p) < 0$. If $p < u < \hat{p}$, then $v(u - \hat{p}) < 0$ and $v(\hat{p} - p) > 0$. In both cases, the sign of the sum is undecided.

If $u = p \neq \hat{p}$, then $v(u - \hat{p}) + v(\hat{p} - p) < 0$, because $|u - \hat{p}| = |\hat{p} - p|$ and loss aversion.

If $u = p = \hat{p}$, then $v(u - \hat{p}) = 0$ and $v(\hat{p} - p) = 0$, so the total utility is zero.

Let $u < p$. If $u \leq \hat{p} \leq p$, then $v(u - \hat{p}) \leq 0$ and $v(\hat{p} - p) \leq 0$, one of the two inequalities is strict. If $\hat{p} < u < p$, then $v(u - \hat{p}) > 0$ and $v(\hat{p} - p) < 0$. Because $u - \hat{p} < |\hat{p} - p|$ and loss aversion, $v(u - \hat{p}) < |v(\hat{p} - p)|$. If $u < p < \hat{p}$, then $v(u - \hat{p}) < 0$ and $v(\hat{p} - p) > 0$. Because $|u - \hat{p}| > \hat{p} - p$ and loss aversion, $|v(u - \hat{p})| > v(\hat{p} - p)$. In all three cases, the total utility is strictly negative. $\square$

Proof of Proposition 6 Define $V(u, p) = v(u - \hat{p}) + v(\hat{p} - p)$ for a given $\hat{p} > 0$. Let $\bar{u}(p)$ be a function such that $V(\bar{u}(p), p) = 0$ for any p. $\bar{u}(p)$ is well defined because V is strictly increasing in u, $V(0, p) < 0$, and $V(\infty, p) > 0$. Then, given a price p, consumers with $u \geq \bar{u}(p)$ would obtain nonnegative utility from consumption. Therefore, the fraction of consumers who obtain nonnegative utility when the price is p can be calculated as $d(p) = 1 - \bar{u}(p)$, since u is uniformly distributed in $[0, 1]$.

If $p < \hat{p}$, then it has to be the case that $\bar{u}(p) < \hat{p}$, and therefore $V(\bar{u}(p), p) = v(\bar{u}(p) - \hat{p}) + v(\hat{p} - p) = -\lambda(\hat{p} - \bar{u}(p))^\gamma + (\hat{p} - p)^\gamma = 0$. Hence, $\hat{p} - p = \lambda^{1/\gamma}(\hat{p} - \bar{u}(p))$ and $\bar{u}(p) = \hat{p} - (\hat{p} - p)/\lambda^{1/\gamma}$.

If $p \geq \hat{p}$, then it has to be the case that $\bar{u}(p) \geq \hat{p}$, and therefore $V(\bar{u}(p), p) = v(\bar{u}(p) - \hat{p}) + v(\hat{p} - p) = (\bar{u}(p) - \hat{p})^\gamma - \lambda(p - \hat{p})^\gamma = 0$. Hence, $\bar{u}(p) - \hat{p} = \lambda^{1/\gamma}(p - \hat{p})$ and $\bar{u}(p) = \hat{p} + (p - \hat{p})\lambda^{1/\gamma}$. $\square$

Proof of Proposition 7 If $v(x) = cx, c > 0$ and $\hat{p}_t = \hat{p}_1$, then

$$V = \sum_{t=1}^{T} \left[\delta^t u(q_t) - q_t \cdot \hat{p}_1 + \theta_t \cdot \hat{p}_1 - \delta'^t \cdot y_t\right] = \sum_{t=1}^{T} \left[\delta^t u(q_t) - \delta'^t \cdot y_t\right] + \hat{p}_1 \left(\sum_{t=1}^{T} \theta_t - \sum_{t=1}^{T} q_t\right).$$

Because, $\sum_{t=1}^{T} q_t = \sum_{t=1}^{T} \theta_t$, the result follows. For the converse, fix $\delta = \delta' = 1$. Let $T = 1$. By

Proposition 1 we conclude that $f(x) = v(x) = cx, c > 0$. Let $T = 2$, $\theta_1 = 0$ and $\theta_2 = q_1 + q_2$. Then

$$\begin{aligned} V &= [u(q_1) - \hat{p}_1 q_1 - y_1] + [u(q_2) - \hat{p}_2 q_2 + \hat{p}_2(q_1 + q_2) - y_2] \\ &= u(q_1) - y_1 + u(q_2) - y_2 + (\hat{p}_2 - \hat{p}_1) q_1. \end{aligned}$$

If V is to be independent of the reference price, then $\hat{p}_2 = \hat{p}_1$. Assume $\hat{p}_t = \hat{p}_1, t = 1, ..., \tau - 1$. Let $T = \tau$, $\theta_t = 0, t = 1, ..., T-1$, and $\theta_\tau = \sum_{t=1}^{T} q_t$. Then,

$$\begin{aligned} V &= \sum_{t=1}^{\tau-1} [u(q_t) - q_t \hat{p}_1 - y_t] + [u(q_\tau) - \hat{p}_\tau q_\tau + \hat{p}_\tau \sum_{t=1}^{\tau} q_t - y_\tau] \\ &= \sum_{t=1}^{\tau} [u(q_t) - y_t] + (\hat{p}_\tau - \hat{p}_1) \sum_{t=1}^{\tau-1} q_t. \end{aligned}$$

If V is to be independent of the reference prices, then $\hat{p}_\tau = \hat{p}_1$. By induction, $\hat{\mathbf{p}} = \mathbf{1} \cdot \hat{p}_1$. □

Proof of Proposition 8 Note that $\alpha_{2,0} \geq \alpha_{3,0} \geq \cdots \geq \alpha_{T,0}$, so that $\hat{p}_1 \geq p \geq \alpha_{t,0}\hat{p}_1$, $2 \leq t \leq T$. Let $\Delta = V(\boldsymbol{\theta_T^b}) - V(\boldsymbol{\theta_T^a})$. Then,

$$\begin{aligned} \Delta &= v(\hat{p}_1 T - pT) - v(\alpha_{T,0}\hat{p}_1 T - pT) + \sum_{t=2}^{T} v(1 - \alpha_{t,0}\hat{p}_1 - \alpha_{t,1} p) - \sum_{t=2}^{T} v(1 - \alpha_{t,0}\hat{p}_1) \\ &\geq v(\hat{p}_1 T - pT) + v(pT - \alpha_{T,0}\hat{p}_1 T) + \sum_{t=2}^{T} v(1 - \alpha_{t,0}\hat{p}_1 - \alpha_{t,1}\hat{p}_1) - \sum_{t=2}^{T} v(1 - \alpha_{t,0}\hat{p}_1) \\ &\geq v(\hat{p}_1 T - \alpha_{T,0}\hat{p}_1 T) + v(1 - \hat{p}_1) + \sum_{t=2}^{T-1} [v(1 - \alpha_{t+1,0}\hat{p}_1 + \alpha_{t+1,1}\hat{p}_1) - v(1 - \alpha_{t,0}\hat{p}_1)] - v(1 - \alpha_{T,0}\hat{p}_1) \\ &\geq v(\hat{p}_1 T - \alpha_{T,0}\hat{p}_1 T) - v(\hat{p}_1 - \alpha_{T,0}\hat{p}_1) > 0. \end{aligned}$$

The first step follows from loss aversion, $-v(\alpha_{T,0}\hat{p}_1 T - pT) \geq v(pT - \alpha_{T,0}\hat{p}_1 T)$ and $\hat{p}_1 \geq p$. The second step follows from concavity, $v(\hat{p}_1 T - \alpha_{T,0}\hat{p}_1 T) - v(pT - \alpha_{T,0}\hat{p}_1 T) \leq v(\hat{p}_1 T - pT)$, and $\alpha_{2,0} + \alpha_{2,1} = 1$. The third step follows from $1 - \alpha_{t+1,0} - \alpha_{t+1,1} \geq 1 - \alpha_{t,0}$, $2 \leq t \leq T-1$, and concavity, $v(1 - \alpha_{T,0}\hat{p}_1) - v(1 - \hat{p}_1) \leq v(\hat{p}_1 - \alpha_{T,0}\hat{p}_1)$. The final step holds if $T \geq 2$ and $\alpha_{T,0} < 1$. □

Proof of Proposition 11.

Proof of Proposition 12 As argued, $V(\boldsymbol{\theta_1^b}) = n \cdot v(1-p)$ and $V(\boldsymbol{\theta_T^b}) = \sum_{t=1}^{T} v(1 - \hat{p}_t)$. If $\alpha_{t,t} > 0$ and $t \geq 2$, then $\hat{p}_t < p$, $v(1 - \hat{p}_t) > v(1-p)$, and the result follows. □

Proof of Proposition 13 Under AFL, $\hat{p}_{t+1} = (1-\alpha)\hat{p}_1 + \alpha \bar{p}_t$. Recall that $\hat{p}_1 = p$, and $\bar{p}_t = p$ if $\theta_t = 1$ and $\bar{p}_t = 0$ otherwise. The reference price is p in the first period and on periods $t = ni + 1, i = 1, ..., T/n$ following payment. On these $(1 + T/n)$ periods, the acquisition utility is $v(1-p)$, and there is only the transaction utility in the first period, which is zero. On the remaining $T - 1 - T/n$ periods $\hat{p}_t = (1-\alpha)p$. The acquisition utility is $v(1 - (1-\alpha)p)$. Transaction utility, which occurs in $T/n - 1$ such periods, is $v((1-\alpha)np - np) = v(-\alpha np)$. Hence,

$$\begin{aligned}
V(\boldsymbol{\theta}_{\boldsymbol{n}}^{\boldsymbol{b}}) &= \sum_{t=1}^{T} v(1-\hat{p}_t) + \sum_{i=1}^{T/n} v(\hat{p}_{(i-1)n+1} - np) \\
&= \left(1 + \frac{T}{n}\right) v(1-p) + \left(T - 1 - \frac{T}{n}\right) v(1-(1-\alpha)p) + \left(\frac{T}{n} - 1\right) v(-\alpha np) \\
&= Tv(1-(1-\alpha)p) - v(-\alpha np) - \left(1 + \frac{T}{n}\right)[v(1-(1-\alpha)p) - v(1-p)] + \frac{Tv(-\alpha np)}{n}.
\end{aligned}$$

The first term does not depend on n. If $\alpha > 0$, the second and third terms are strictly increasing in n. The last term increases with n because, by the convexity of v for losses,

$$v(-\alpha np) = v\left(-\frac{n}{n+1}\alpha(n+1)p\right) \leq \frac{n}{n+1} v(-\alpha(n+1)p). \quad \square$$

Proof of Proposition 14 Considering the third term in (14), we have that

$$\sum_{t=1}^{T} \int_0^{\hat{p}_t} f(u) v(u - \hat{p}_t) du \geq \sum_{t=1}^{T} v(-\hat{p}_t) P(u_t \leq \hat{p}_t) \geq -M.$$

The term M is independent of T. If reference prices are minimally adaptive, the first two terms are strictly positive and non-decreasing with t. If T sufficiently large, their sum will exceed M.
If $P(u < (1-\alpha)p) = 0$, then the third term is zero for $t \geq 3$, and hence (15) bounded. If prices are totally adaptive, then $\hat{p}_t$ will eventually be below ϵ. Because $P(u < \epsilon) = 0$, (15) is bounded. If RA holds, then $\hat{p}_t = (1-\alpha)^{t-2}, t \geq 2$. If v is power, then

$$\sum_{t=1}^{T} v(-\hat{p}_t) P(u_t \leq \hat{p}_t) \geq \sum_{t=1}^{T} v(-\hat{p}_t) = -\sum_{t=1}^{T} \lambda (1-\alpha)^{\gamma(t-2)} \geq -\frac{\lambda}{(1-\alpha) \ln \frac{1}{1-\alpha}}. \quad \square$$

Proof of Proposition 15 Assume that the value function is $v(x) = x$ for all x, and the reference price $\hat{p}_t$ is equal to the actual price p for all t. Then, when the realized demand is x, a consumer's utility under a flat-rate tariff is obtained as follows:

$$V(\boldsymbol{\theta}_{\boldsymbol{n}}^{\boldsymbol{f}}, \mathbf{q_x}) = \begin{cases} \sum_{i=1}^{x} v(1-\hat{p}_i) + v(-(n-x)\hat{p}_x) = x(1-p) - (n-x)p = x - np, & \text{if } x \leq n, \\ \sum_{i=1}^{n} v(1-\hat{p}_i) + (x-n)v(1) = n(1-p) + (x-n)\cdot 1 = x - np, & \text{if } x > n. \end{cases} \tag{21}$$

Hence, the total utility is always $V(\boldsymbol{\theta}_{\boldsymbol{n}}^{\boldsymbol{f}}, \mathbf{q_x}) = x - np$, and the expected total utility is $V(\boldsymbol{\theta}_{\boldsymbol{n}}^{\boldsymbol{f}}) = E_x[V(\boldsymbol{\theta}_{\boldsymbol{n}}^{\boldsymbol{f}}, \mathbf{q_x})] = E_x[x - np] = E[x] - np$. Also, the total utility under a pay-as-you-go tariff is $V(\boldsymbol{\theta}^{\boldsymbol{p}}(\mathbf{q_x})) = x \cdot v(1-p) = x(1-p)$, and therefore the expected total utility is $V(\boldsymbol{\theta}^{\boldsymbol{p}}) = E_x[V(\boldsymbol{\theta}^{\boldsymbol{p}}(\mathbf{q_x}))] = E_x[x(1-p)] = E[x](1-p)$. Therefore, we can observe that when $n = E[x]$, $V(\boldsymbol{\theta}_{\boldsymbol{n}}^{\boldsymbol{f}}) = V(\boldsymbol{\theta}^{\boldsymbol{p}})$.

Proof of Proposition 16 The total utility under a pay-as-you-go tariff when $\hat{p}_1 < p$ is obtained as follows:

$$V(\boldsymbol{\theta_1}, \hat{p}_1) = \sum_{t=1}^{T} \{v(u - \hat{p}_t) + v(\hat{p}_t - p)\}. \tag{22}$$

The derivative of this utility with respect to $\hat{p}_1$ is

$$\frac{\partial V(\boldsymbol{\theta_1}, \hat{p}_1)}{\partial \hat{p}_1} = \{-v'(u - \hat{p}_1) + v'(\hat{p}_1 - p)\} + \sum_{t=2}^{T} \{-v'(u - \hat{p}_t) + v'(\hat{p}_t - p)\} \cdot \frac{\partial \hat{p}_t}{\partial \hat{p}_1}. \tag{23}$$

Since we assumed $v'(-x) \geq v'(x)$ for all $x > 0$, we can observe that $-v'(u - \hat{p}_t) + v'(\hat{p}_t - p) > 0$ for all $1 \leq t \leq T$, because $u - \hat{p}_t > |\hat{p}_t - p|$, and $v(x)$ is strictly convex when $x < 0$ and strictly concave when $x > 0$. Also, the reference price is defined as a weighted sum of past stimuli including $\hat{p}_1$, where all weights are nonnegative, it must be the case that $\frac{\partial \hat{p}_t}{\partial \hat{p}_1} \geq 0$ for all t. Hence,

$$\begin{aligned}\frac{\partial V(\boldsymbol{\theta_1}, \hat{p}_1)}{\partial \hat{p}_1} &= \{-v'(u - \hat{p}_1) + v'(\hat{p}_1 - p)\} + \sum_{t=2}^{T} \{-v'(u - \hat{p}_t) + v'(\hat{p}_t - p)\} \cdot \frac{\partial \hat{p}_t}{\partial \hat{p}_1} \\ &\geq \{-v'(u - \hat{p}_1) + v'(\hat{p}_1 - p)\} > 0. \end{aligned} \tag{24}$$

Therefore, $\frac{\partial V(\boldsymbol{\theta_1}, \hat{p}_1)}{\partial \hat{p}_1} > 0$.

Proof of Proposition 17

i) Case 1: Assume the reference price is updated by AFL. Then, the total utilities of a pay-as-you-go and a flat-rate tariff are obtained as follows:

$$V(\boldsymbol{\theta_1}, \hat{p}_1) = \sum_{t=1}^{T} \{v(u - \hat{p}_t) + v(\hat{p}_t - p)\} \tag{25}$$

$$= \{v(u - \hat{p}_1) + v(\hat{p}_1 - p)\} + (T-1)\{v(u - ((1-\alpha)\hat{p}_1 + \alpha p)) + v(((1-\alpha)\hat{p}_1 + \alpha p) - p)\}, \tag{26}$$

$$V(\boldsymbol{\theta_T}, \hat{p}_1) = \sum_{t=1}^{T} v(u - \hat{p}_t) + v(T\hat{p}_1 - Tp) \tag{27}$$

$$= v(u - \hat{p}_1) + v(u - ((1-\alpha)\hat{p}_1 + \alpha p)) + (T-2)v(u - (1-\alpha)\hat{p}_1) + v(T(\hat{p}_1 - p)). \tag{28}$$

The difference between the two utilities is

$$V(\boldsymbol{\theta_T}, \hat{p}_1) - V(\boldsymbol{\theta_1}, \hat{p}_1) = (T-2)\{v(u - (1-\alpha)\hat{p}_1) - v(u - ((1-\alpha)\hat{p}_1 + \alpha p))\} \tag{29}$$

$$+ v(T(\hat{p}_1 - p)) - v(\hat{p}_1 - p) - v(((1-\alpha)\hat{p}_1 + \alpha p) - p). \tag{30}$$

We can substitute the following: $\epsilon_1 = \{v(u - (1-\alpha)\hat{p}_1) - v(u - ((1-\alpha)\hat{p}_1 + \alpha p))\} > 0$ and

$\epsilon_2 = -\{v(\hat{p}_1 - p) + v(((1-\alpha)\hat{p}_1 + \alpha p) - p)\} > 0$. Note that ϵ_1 and ϵ_2 are both positive. Then, the difference can be simply represented as

$$V(\boldsymbol{\theta_T}, \hat{p}_1) - V(\boldsymbol{\theta_1}, \hat{p}_1) = (T-2)\epsilon_1 + v(T(\hat{p}_1 - p)) + \epsilon_2. \tag{31}$$

Note that the first term $(T-2)\epsilon_1$ is positive and linearly increasing in T, and the second term $v(T(\hat{p}_1 - p))$ is negative and decreasing in T. Since we assumed $\lim_{x\to-\infty} v'(x) = 0$, we can observe that $\lim_{T\to\infty} v'(T(\hat{p}_1 - p)) = 0$, and hence there exists T^* such that for all $T \geq T^*$, $(T-2)\epsilon_1 \geq |v(T(\hat{p}_1 - p))|$. Therefore, there exists T^* such that for all $T \geq T^*$, $V(\boldsymbol{\theta_T}, \hat{p}_1) > V(\boldsymbol{\theta_1}, \hat{p}_1)$.

ii) Case 2: Assume the reference price is updated by RA. In this proof, for ease of exposition, we define $\boldsymbol{\theta_{n,T}}$ as $\boldsymbol{\theta_n}$ where the total period is T. Then, $\boldsymbol{\theta_{T,T}}$ and $\boldsymbol{\theta_{1,T}}$ are a flat-rate tariff and a pay-as-you-go tariff, respectively, when the total period is T. Also, $V(\boldsymbol{\theta_{T,T}})$ and $V(\boldsymbol{\theta_{1,T}})$ are the total utility under each tariff, respectively. Now, define $D(T) = V(\boldsymbol{\theta_{T,T}}) - V(\boldsymbol{\theta_{1,T}})$, which represents the difference of utility between a flat-rate tariff and a pay-as-you-go tariff. For convenience, let $D(0) = 0$. Also, define $\Delta_T = D(T) - D(T-1)$. Let $\hat{p}_t$ and $\hat{p}'_t$ be the reference prices with a flat-rate tariff and a pay-as-you-go tariff, respectively. Then,

$$\begin{aligned}
\Delta_T &= D(T) - D(T-1) && (32)\\
&= \{V(\boldsymbol{\theta_{T,T}}) - V(\boldsymbol{\theta_{T-1,T-1}})\} - \{V(\boldsymbol{\theta_{1,T}}) - V(\boldsymbol{\theta_{1,T-1}})\} && (33)\\
&= \{v(u - \hat{p}_T) + v(T(\hat{p}_1 - p)) - v((T-1)(\hat{p}_1 - p))\} - \{v(u - \hat{p}'_T) + v(\hat{p}'_T - p)\} && (34)\\
&> \{v(u - \hat{p}_T) + v(T(\hat{p}_1 - p)) - v((T-1)(\hat{p}_1 - p))\} - v(u - p), && (35)
\end{aligned}$$

since $v(u - \hat{p}'_T) + v(\hat{p}'_T - p)$ is increasing in $\hat{p}'_t$ by Proposition 16 and $\hat{p}'_t < p$. Rearranging the inequality above, we obtain

$$\Delta_T > \{v(u - \hat{p}_T) - v(u - p)\} + \{v(T(\hat{p}_1 - p)) - v((T-1)(\hat{p}_1 - p))\}, \tag{36}$$

and we call the RHS the lower bound of Δ_T. Note that $v(u - \hat{p}_T) - v(u - p) > 0$ and this is (weakly) increasing in T, because $\hat{p}_T < p$ and $\hat{p}_T$ is (weakly) decreasing under a flat-rate tariff. Also, the second term is negative but it is increasing in T due to diminishing sensitivity. Therefore the lower bound of Δ_T is a sum of two increasing functions and hence it is also increasing. In addition, since we assume $\lim_{x\to-\infty} v'(x) = 0$, for any $\delta > 0$, there exists T_0 such that for all $T \geq T_0$, $|v(T(\hat{p}_1 - p)) - v((T-1)(\hat{p}_1 - p))| < \delta$. Therefore, there exists T_1 such that for all $T \geq T_1$, the lower bound of Δ_T is positive. Let ϵ be the lower bound of Δ_{T_1}. Then, for all $T \geq T_1$, $\Delta_{T_1} > \epsilon$.

For any $T > T_1$,

$$D(T) = \sum_{t=1}^{T} \Delta_t = \sum_{t=1}^{T_1 - 1} \Delta_t + \sum_{t=T_1}^{T} \Delta_t > D(T_1 - 1) + (T - T_1)\epsilon. \tag{37}$$

Since $D(T_1 - 1)$ is a finite number and $(T - T_1)\epsilon > 0$ is linearly increasing in T, there exists T_2 such that for all $T \geq T_2$, $D(T) > D(T_1 - 1) + (T - T_1)\epsilon > 0$. Since $D(T) = V(\boldsymbol{\theta_{T,T}}) - V(\boldsymbol{\theta_{1,T}}) > 0$, when $T \geq T_2$, $V(\boldsymbol{\theta_{T,T}}) > V(\boldsymbol{\theta_{1,T}})$.