Are People Risk-Vulnerable?

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Abstract

The paper reports on a within-subject experiment, with substantial monetary incentives, designed to test wether or not people are risk-vulnerable. In the experiment, subjects face a simple portfolio choice problem in which they have to invest part of their wealth in a safe and a risky asset. We elicit risk vulnerability by observing each subject's portfolio choice in two different contexts that only differ by the presence of a significant but actuarially neutral background risk. We find that most subjects, 78.3%, are risk-vulnerable. Precisely, 52.6% have invested less in the risky asset when exposed to background risk and 25.7% were indifferent. Thus only 21.7% of the subjects have invested strictly more in the risky asset when exposed to background risk.

I. Introduction

Most individuals are exposed to several risks simultaneously. While for some risks individuals can choose their preferred level, there are other risks to which individuals are simply exposed without control, i.e. risks that are non-diversifiable and/or non-insurable. The fundamental implication of this fact is that there is no risk-free situation for individuals. Diversification is limited because of systematic risk. Economic fluctuations caused by natural disasters, nuclear hazards, financial crisis, wars or popular uprisings, cannot be fully insured. Furthermore, because of informational asymmetries, non-transferability and/or transaction costs, there exists many idiosyncratic risks for which full insurance is unfeasible. In any event, some risks remain necessarily in the background. All such committed but unresolved risks constitute what is usually called the 'background risk'.

Depending on the structure of individuals' preferences, the presence of background risk may lead to more or less cautious behavior, impacting thereby the price of risk in the economy. Taking into account the background risk to which individuals are exposed can significantly improve our understanding of risk-taking behavior in many economic contexts. Examples include the demand for insurance (Doherty and Schlesinger, 1983; Eeckhoudt and Kimball, 1992), portfolio choices and asset prices (Mehra and Prescott, 1985; Weil, 1992; Finkelshtain and Chalfant, 1993; Franke, Stapleton and Subrahmanyam, 1998, 2004; Heaton and Lucas, 1997, 2000), and efficient risk-sharing (Gollier, 1996; Dana and Scarsini, 2007).

The fundamental conjecture upon which this literature rests is that risk-averse agents consider independent risks as substitutes rather than as complements. According to Gollier and Pratt (1996, p. 1109): *"Conventional wisdom suggests that independent risks are substitutes for each other. In particular, adding a mean-zero background risk to wealth should increase risk aversion to other independent risks. However, risk aversion is not sufficient to guarantee this"*. Hence, relying on von Neuman and Morgenstern (1944)'s expected utility theory (EU), Gollier and Pratt (1996) identified 'risk vulnerability' as the weakest restriction to impose on the Bernoulli utility function of a decreasingly risk-averse individual to guarantee that he/she would behave in a more cautious way if an actuarially neutral background risk is added to his/her initial wealth, be it random or not (see Gollier and Pratt, 1996, Proposition 2 and 4, p. 1112 and p. 1120). Since the seminal contribution of Pratt (1964) and Arrow (1971), it is well-known that the absolute risk aversion function governs the risk-taking behavior of individuals with *EU* preferences. Therefore, the comparative-static properties of *RV* are derived directly from the standard comparative-static properties of 'comparative risk-aversion' (Pratt, 1964, Theorem 1, p. 128).

In the framework of *EU*, *RV* fits nicely to commonly accepted restrictions that have important and desirable comparative statics properties: risk vulnerability implies decreasing absolute risk aversion (*DARA*) and is a consequence of more general notions of risk-aversion such as proper risk aversion (Pratt and Zeckhauser, 1987) and standard risk aversion (Kimball, 1993).[1] Since risk vulnerability is necessary to obtain desirable static-comparative properties in many economic contexts, the question of wether or not most individual's behavior actually exhibits risk vulnerability is of paramount interest for economic analysis under *EU*.[2]

But the empirical relevance of the risk vulnerability conjecture is beyond the scope of *EU* theory as it is a relevant issue for any decision-theoretic setting. In contrast with *EU*, Quiggin (2003) showed that for the wide class of risk-averse generalized expected utility preferences exhibiting constant risk aversion in the sense of Safra and Segal (1998) and Quiggin and Chambers (1998), independent risks are actually complementary.[3] An individual with such preferences who is exposed to background risk would therefore contradict the risk vulnerability conjecture by behaving in a more cautious way.

Since alternative theories have different predictions about the impact of background risk on risk-taking behavior, there is a need for empirical evidence about risk vulnerability in order to contrast predictions with data. In this paper we provide experimental evidence about the impact of an actuarially neutral background risk on individuals' risk-taking behavior, i.e. we question whether people are risk-vulnerable?

To the best of our knowledge few studies have attempted to answer this question. Using naturally occurring data, Guiso, Jappelli and Terlizzese (1996) found that investment in risky financial assets responds negatively to income risk, and Guiso and Paiella (2008) showed that individuals who are more likely to face income uncertainty or to become liquidity constrained exhibit a higher degree of absolute risk aversion.

Based on a framed field experiment Harrison, List and Towe (2007) found strong evidence in favor of risk vulnerability for numismatists. They relied on Holt and Laury (2002)'s multiple price list methodology to elicit traders' relative risk aversion ($CRRA$ parameter) under three alternative incentives: monetary prizes, graded coins, and ungraded coins which entailed background risk. Their estimates show that using ungraded coins in the lotteries increases sharply the level of risk-aversion of coin traders compared to the conditions where monetary prizes or graded coins were used[4]. They suggest that it would be worth to explore further the extent of their empirical findings on the basis of a controlled laboratory experiment aiming at isolating the impact of background risk on risk-taking behavior.

Lee (2008) reports experimental findings from a laboratory experiment whose aim was to compare the *random round payoff mechanism* ($RRPM$) to a system where all rounds are being paid, the *accumulated payoff mechanism* (APM). In each round subjects had to perform two tasks: task 1 was a risk-taking decision for which subjects had to trade off a higher (lower) probability of winning for a lower (higher) prize. Task 2 was identical except that the event of winning was not determined by a chance event but by the choice made by an opponent player. According to the author the $RRPM$ entails background risk because the subject has to take his decision for task 1 without knowing the outcome of task 2 ,while in the APM treatment the subject knows his accumulated wealth for task 1 and task 2. The main finding is that risk-averse subjects tend to behave in a more cautious way under $RPPM$ than under APM. But the data is scarce and the results are not clear-cut.

Our study is more closely related to Lusk and Coble (2008) who designed explicitly a laboratory experiment to test the risk vulnerability conjecture. Their experiment involved

130 subjects each one endowed with \$10. The experiment consisted in eliciting subjects' risk aversion based on Holt and Laury (2002)'s method in a between-subject design: 50 subjects faced no background risk, 27 subjects faced a zero-mean background risk $\left(-\$10, \frac{1}{2}; \$10, \frac{1}{2}\right)$ and 53 subjects faced an unfair background risk $\left(-\$10, \frac{1}{2}; \$0, \frac{1}{2}\right)$. The impact of background risk on risk aversion is measured by comparing subjects' number of safe choices across treatments. The authors found weak evidence of risk vulnerability: the median number of safe choices is identical in the three treatments (6 safe choices) and a slightly greater number of safe choices was observed in the zero-mean and unfair background risk treatments (5.89 and 5.68 safe choices, respectively) compared with the no background risk treatment (5.40 safe choices).[5]

In the present paper, we rely on a within-subject analysis and we use a different method to elicit subjects' risk-aversion. Instead of relying on Holt and Laury (2002)'s procedure, we adopt the simpler method proposed by Gneezy & Potters (1997) and Charness & Gneezy (2010) which relies on a standard portfolio choice problem in which the investor has to allocate his wealth between a safe and a risky asset. The safe asset secures the amount invested whereas the risky asset is a binary lottery which involves a random rate of return $k = \left(0, \frac{1}{2}; 3, \frac{1}{2}\right)$. In case of failure the return takes value 0 (the amount invested is lost) and in case of success the return takes value 3 (the amount invested is tripled). Failure and success are equally likely.

We report on two experiments, labelled *Exp.1* and *Exp.2*, both of which rely on basically the same portfolio choice problem. However, to allow for robustness check, we deliberately varied many aspects between the two experiments. *Exp.1* was run as a paper and pencil session involving 82 subjects while *Exp.2* was a computerized experiment involving 167 subjects. In both experiments each subject faced the portfolio choice task described above. In *Exp.2*, preliminary to the portfolio choice task, subjects had to *work* in order to accumulate wealth (€20) by performing a boring task. In contrast, in *Exp.1* subjects' wealth was a windfall endowment (€100) provided by the experimenter. In *Exp.1* which involved high stakes, only 10% of the participants (randomly selected) were paid out for real. In contrast, in *Exp.2* where stakes were much lower, all participants were paid according to their earnings. Despite these differences, we show that the results of the two experiments are almost exactly the same.

In both experiments, half of each subject's wealth was in a blocked account while the other half was available for the portfolio choice task. Moreover, each subject faced the portfolio choice task twice, in two different situations labelled situation A and situation B. Situation A involved no background risk. In situation B the investor had to face an independent additive and actuarially neutral background risk $\widetilde{y} = (-y, \frac{1}{2}; y, \frac{1}{2})$ on his blocked account. We chose the level of background risk such that subjects could eventually loose their whole wealth in the blocked account, i.e. $y =$ €50 in *Exp.1* or $y =$ €10 in *Exp.2*. Since the two situations A and B differ only by the presence or absence of a background risk, we unambiguously elicit RV by comparing for each subject his investment decision in situation A and in situation B. We control for a possible order effect by randomizing the sequence of situations: in each experiment half of the subjects faced situation A first, while the other half faced situation B first. Our results were not affected by the ordering of the treatments.

Our main finding is that 78.3% of our 249 subjects exhibit RV. Precisely, 52.6% of the subjects invested a strictly lower amount in the risky asset when exposed to background risk while 25.7% were indifferent. Only 21.7% of the subjects behaved in a less cautious way when exposed to background risk.

We contrast our experimental data with respect to the predictions of EU, Quiggin (1982)'s rank dependent utility theory (RDU), Yaari (1987)'s dual theory (DT) and Tversky and Kahneman (1992)'s cumulative prospect theory (CPT). These theories have contrasted predictions about the impact of background risk. in particular we show that DT predicts a more risky portfolio choice in the presence of background risk and, accordingly, that RDU can predict either more or less risky behavior. Predicting the behavior of a CPT-investor raises serious and previously unmentioned difficulties, even for the simple portfolio choice problem discussed in this paper.

The remainder of the paper is organized as follows. Section 2 describes the experimental design and provides the theoretical foundation for our elicitation procedure of risk vulnerability. Section 3 presents the predictions for our portfolio choice problem under various choice theories: *EU*, *DT*, *RDU* and *CPT*. Our experimental findings are reported in section 4. Section 5 concludes.

II. Portfolio choice and risk vulnerability

In the experiment subjects faced a simple portfolio choice problem in two situations, labelled situation A and situation B. Situation B only differed from situation A by the presence of an actuarially neutral background risk. We elicit RV by comparing each subject's investment decision with and without background risk. The theoretical framework described below fully mirrors our artefactual experiment.

We assume an investor with initial wealth level $x > 0$, half of which is in a blocked account. The other half can be allocated between a safe asset which secures the amount invested and a risky asset with a binary random rate of return $\widetilde{k} = \left(0, \frac{1}{2}; 3, \frac{1}{2}\right)$. We first consider the problem without background risk. Letting $\delta \in [0, 1]$ be the fraction of $\frac{1}{2}x$ invested in the risky asset, the endogenous discret probability distribution of the investor's wealth is written $\widetilde{x} = \left(x_1, \frac{1}{2}; x_2, \frac{1}{2}\right)$, where $x_1 = \frac{1}{2}x + [1 - \delta]\frac{1}{2}x = [2 - \delta]\frac{1}{2}x$ and $x_2 = \frac{1}{2}x + [1 - \delta]\frac{1}{2}x + 3\delta\frac{1}{2}x = [1 + \delta]x$ are the wealth levels in case of failure ($\widetilde{k} = 0$) and success ($\widetilde{k} = 3$) of the risky investment, respectively.

We assume that the investor maximizes a general preferences function $v(\cdot)$ defined over random wealth $\widetilde{x}$. Without background risk, the optimal portfolio is given by

$$\delta^A = \underset{\delta \in [0,1]}{\arg\max} v\left(\widetilde{x}\right). \tag{1}$$

Now suppose that the agent is forced to bear an independent additive and actuarially neutral background risk $\widetilde{y} = (-y, \frac{1}{2}; y, \frac{1}{2})$ on his blocked account. In the experiment, we chose the level of background risk such that $y = \frac{1}{2}x$, i.e. subjects could eventually loose their wealth in the blocked account[6]. Under background risk the random wealth of the agent becomes $\widetilde{x} + \widetilde{y} = \left\{x_{11}, \frac{1}{4}; x_{21}, \frac{1}{4}; x_{12}, \frac{1}{4}; x_{22}, \frac{1}{4}\right\}$, where $x_{i1} = x_i - y$ and $x_{i2} = x_i + y$ for $i = 1, 2$. The optimal portfolio is now given by

$$\delta^B = \underset{\delta \in [0,1]}{\arg\max} v\left(\widetilde{x} + \widetilde{y}\right). \tag{2}$$

In our framework, risk vulnerability means that $\delta^B \leq \delta^A$. An individual is risk-vulnerable if he/she chooses a less risky portfolio when he/she moves from situation A to situation B and conversely. Definition 1 below characterizes all the possible types an individual may exhibit:

Definition 1. *An individual is risk-vulnerable if $\delta^A \geq \delta^B$. He/she is strictly-risk-vulnerable if $\delta^A > \delta^B$, and he/she is indifferent if $\delta^A = \delta^B$. Otherwise, if $\delta^A < \delta^B$, he/she is non-risk-vulnerable.*

III. Theoretical predictions and numerical results

The preferences function $v(\cdot)$ can take various forms depending on the behavioral assumption (*EU*, *DT*, *RDU* or *CPT*). While predicting the impact of background risk within the *EU* framework is rather straightforward, the task becomes rather unobvious under alternative behavioral assumptions, even for the simple portfolio choice problem that we considered in our experiment. To the best of our knowledge, outside *EU*, there are surprisingly no theoretical results regarding the impact of background risk on portfolio choice. We illustrate, using our parametrized portfolio choice problem, that under conventional and/or empirically founded assumptions, alternative theories have opposite predictions. Moreover under *CPT* background risk affects the reference point in an ambiguous way, which raises a methodological issue that has not yet been addressed. We consider successively the predictions under *EU*, *DT*, *RDU* and *CPT*.

A. Expected utility theory

Under EU, the preferences function is linear in probability and takes the following form:

$$v(\widetilde{x}) = \mathbf{E}u(\widetilde{x}) = \frac{1}{2}u(x_1) + \frac{1}{2}u(x_2), \tag{3}$$

where u is a strictly increasing ($u' > 0$) and concave ($u'' < 0$) real-valued Bernoulli utility function defined over final wealth. Following previous literature (Kihlstrom and al., 1981; Nachman, 1982; Pratt, 1988; Pratt and Zeckhauser, 1987; Gollier and Pratt, 1996) it is convenient to define an indirect Bernoulli utility function as $U(s) = \mathbf{E}u(s + \widetilde{y})$. Thus, in the presence of background risk, the preferences function is written[7]

$$v(\widetilde{x} + \widetilde{y}) = \mathbf{E}U(\widetilde{x}) = \frac{1}{2}U(x_1) + \frac{1}{2}U(x_2). \tag{4}$$

The Kuhn-Tucker first-order conditions for situation A yield:

$$\delta^A \begin{cases} = 1 & \text{if } \frac{u'\left(x_1^A\right)}{u'\left(x_2^A\right)} < 2 \\ \in [0,1] & \text{if } \frac{u'\left(x_1^A\right)}{u'\left(x_2^A\right)} = 2 \\ = 0 & \text{if } \frac{u'\left(x_1^A\right)}{u'\left(x_2^A\right)} > 2. \end{cases} \tag{5}$$

where $x_1^A = \left[2 - \delta^A\right]\frac{1}{2}x$ and $x_2^A = \left[1 + \delta^A\right]x$. Substituting U for u gives the analogous conditions in the presence of background risk, i.e. for situation B. Since $x_1 \leq x_2$ for any level of investment, it is apparent from (5) that risk-loving and risk-neutral agents (with non-decreasing marginal utility) both choose the maximum possible investment, and that a zero investment cannot be an optimal choice under monotonic preferences. Moreover, observe that the optimal investment is a decreasing function of the ratio of marginal utilities, and that this ratio cannot be smaller than one under risk aversion. In addition, it cannot decrease as the individual becomes more risk-averse. Indeed, as observed by Pratt (1988, eq. 2, p. 398), U is at least as risk-averse than u if and only if $\frac{U'(x_1)}{U'(x_2)} \geq \frac{u'(x_1)}{u'(x_2)}$ for $x_1 \leq x_2$. According to

(5), this suggests that if U is at least as risk-averse than u, that is if u is risk-vulnerable, then $\delta^A \geq \delta^B$. Gollier and Pratt (1996, Def. 1, p.1112) equivalently defined risk vulnerability as the assumption that the background risk increases the individual's absolute risk aversion function:

$$r(x) = -\frac{u''(x)}{u'(x)} \leq -\frac{U''(x)}{U'(x)} = R(x) \quad \text{for all } x. \tag{6}$$

Thus, the Arrow-Pratt framework of comparative risk aversion fully applies as if u and U corresponded to the preferences of two different individuals. In particular the following well-known result applies (see Pratt, 1964, Theorem 1, p. 128; Gollier and Pratt, 1996, Proposition 1, p. 1112).

Proposition 2. *Under* EU *the following statements are equivalent:*

- $r(x) \leq$ *[*$<, =, >$*]* $R(x)$ *for all* x.
- *The individual is risk-vulnerable [strictly-risk-vulnerable, indifferent, non-risk-vulnerable].*

As mentioned by Gollier and Pratt (1996), all commonly used Bernoulli utility functions exhibiting (non-increasing) harmonic absolute risk aversion ($HARA$) exhibit risk vulnerability. Under constant absolute risk aversion ($CARA$), the introduction of an additive background risk has obviously no impact on absolute risk aversion. Therefore individuals with $CARA$ preferences are indifferent to the introduction of an additive background risk. On the other hand, under $DARA$, it is easy to show that the individual is here strictly-risk-vulnerable if:

$$\frac{r(x-y) - r(x)}{r(x) - r(x+y)} > \frac{u'(x+y)}{u'(x-y)} \quad \text{for all } x. \tag{7}$$

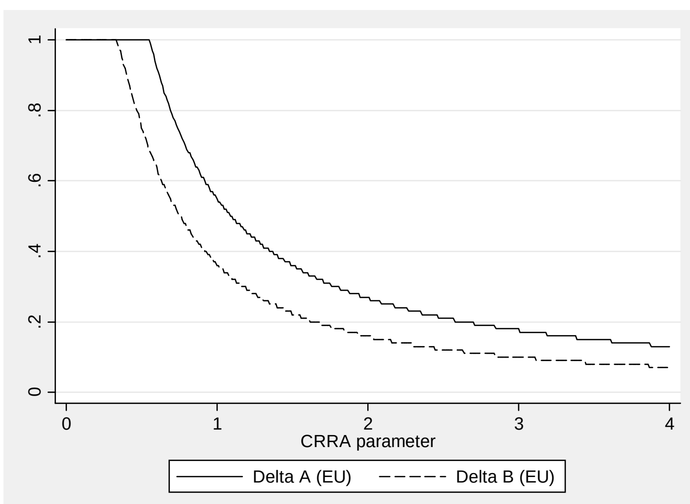
Under monotonic and risk-averse preferences, the ratio of marginal utilities in the r.h.s. of the inequality in (7) is strictly smaller than one. Therefore a sufficient condition for (7) to hold is that the ratio of changes in absolute risk aversion in the left hand side of the inequality in (7) is greater than one. Since the numerator (resp. denominator) of this latter ratio is the increase in absolute risk aversion due to a loss of $-y$ at wealth level x (resp. $x+y$), it follows that decreasing and convex absolute risk aversion is a sufficient condition for risk vulnerability, as observed by Gollier and Pratt (1996).

To illustrate, consider the widely used power utility function exhibiting constant relative risk aversion (*CRRA*) with parameter γ:

$$u(x) = \begin{cases} \frac{1}{1-\gamma}x^{1-\gamma} & \text{if} \quad \gamma > 0 \\ \ln x & \text{if} \quad \gamma = 1, \end{cases} \tag{8}$$

where $\gamma = r(x)\,x \geq 0$ for $x > 0$. The predicted impact of background risk on the portfolio choice is illustrated for a plausible range of the *CRRA* parameter (e.g. Holt and Laury, 2002) in Figure 1.

Figure 1. Predicted portfolio choice and risk vulnerability under EU



As the *CRRA* parameter increases, the optimal investment curve is first horizontal and then becomes strictly decreasing and convex. In the presence of background risk the curve simply translates to the left since, under risk vulnerability, the background risk increases risk aversion. The model also predicts stronger absolute falls in investment for weakly risk-averse agents (the difference $\delta^A - \delta^B$ is decreasing and convex in γ). This property fits nicely to our experimental data.

B. Dual theory

Suppose now that the individual behaves according to DT. Under DT, the preferences function is linear in monetary outcomes (rather than in probabilities as in EU). With no loss of generality, outcomes are ordered (from the smallest to the largest). Without background risk, only two outcomes, x_1 and x_2, are possible with $x_1 < x_2$ for $\delta^A > 0$ and $x_1 = x_2$ for $\delta^A = 0$. Furthermore, the cumulative distribution function $F_{\widetilde{x}}(x) = \Pr(\widetilde{x} \leq x)$ is distorted by a weighting function $w : [0,1] \rightarrow [0,1]$, with $w(0) = 0$ and $w(1) = 1$. The weight attributed to wealth x_i is written:

$$\pi(x_i) = w(\Pr(\widetilde{x} \leq x_i)) - w(\Pr(\widetilde{x} < x_i)). \tag{9}$$

Thus, the preferences function takes the following form:

$$v(\widetilde{x}) = \sum_{i=1}^{2} \pi(x_i)\, x_i = x + \delta\left[1 - \frac{3}{2} w\left(\frac{1}{2}\right)\right] x, \tag{10}$$

where $\pi(x_1) = w\left(\frac{1}{2}\right)$ and $\pi(x_2) = 1 - w\left(\frac{1}{2}\right)$. Since the preferences function is linear in δ, the optimal investment is typically a corner solution, i.e. $\delta^A = 0$ or $\delta^A = 1$. It follows that DT-investors exhibit '*plunging*' behavior: they stay put until plunging becomes justified (as observed by Yaari, 1987):

$$\delta^A \begin{cases} = 1 \quad \text{if} \quad w\left(\frac{1}{2}\right) < \frac{2}{3} \\ \in [0,1] \quad \text{if} \quad w\left(\frac{1}{2}\right) = \frac{2}{3} \\ = 0 \quad \text{if} \quad w\left(\frac{1}{2}\right) > \frac{2}{3}. \end{cases} \tag{11}$$

Thus, the investors' behavior is fully determined by the value of $w\left(\frac{1}{2}\right)$, i.e. the distorted probability assigned to an unsuccessful investment. If the individual is optimistic or (not too much) pessimistic with $w\left(\frac{1}{2}\right) < \frac{2}{3}$ then he/she chooses the maximum possible investment and, on the other hand, if the individual is strongly pessimistic with $w\left(\frac{1}{2}\right) > \frac{2}{3}$ then he/she chooses the minimum possible investment.

In situation B there are four possible outcomes the ordering of which depends on δ^B:

$$\begin{aligned} x_{11} &< x_{12} < x_{21} < x_{22} \quad \text{if} \quad \delta^B > \frac{2}{3} \\ x_{11} &< x_{21} = x_{12} < x_{22} \quad \text{if} \quad \delta^B = \frac{2}{3} \\ x_{11} &< x_{21} < x_{12} < x_{22} \quad \text{if} \quad \delta^B < \frac{2}{3}. \end{aligned}$$

The probability weight attributed to wealth x_{ij} is

$$\pi\left(x_{ij}\right) = w\left(\Pr\left(\widetilde{x} + \widetilde{y} \leq x_{ij}\right)\right) - w\left(\Pr\left(\widetilde{x} + \widetilde{y} < x_{ij}\right)\right). \tag{12}$$

The preferences function is then:

$$v\left(\widetilde{x} + \widetilde{y}\right) = \sum_{i=1}^{2} \sum_{j=1}^{2} \pi\left(x_{ij}\right) x_{ij}. \tag{13}$$

In the presence of background risk, the optimal portfolio is as follows:[8]

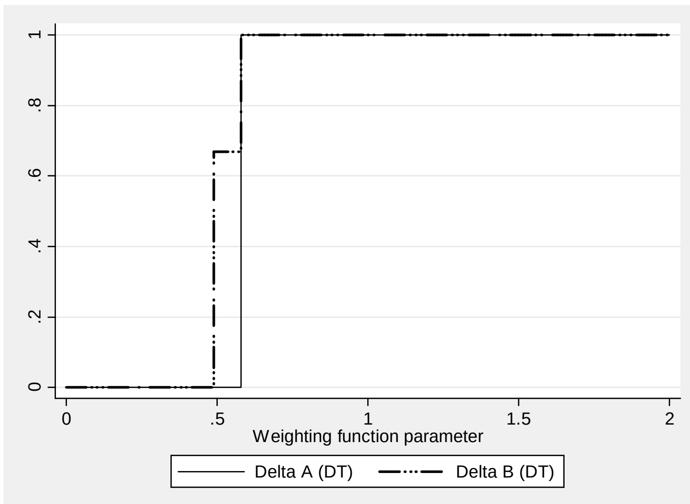
$$\delta^B = \begin{cases} = 1 \quad if \quad w\left(\frac{1}{2}\right) < \frac{2}{3} \\ \in \left(\frac{2}{3}, 1\right] \quad if \quad w\left(\frac{1}{2}\right) = \frac{2}{3} \\ = \frac{2}{3} \quad if \quad \frac{1}{2}w\left(\frac{1}{4}\right) - w\left(\frac{1}{2}\right) + \frac{3}{2}w\left(\frac{3}{4}\right) \leq \frac{2}{3} \leq 3w\left(\frac{1}{4}\right) + w\left(\frac{1}{2}\right) - w\left(\frac{3}{4}\right) \\ \in \left[0, \frac{2}{3}\right) \quad if \quad w\left(\frac{1}{4}\right) - w\left(\frac{1}{2}\right) + w\left(\frac{3}{4}\right) = \frac{2}{3} \\ = 0 \quad if \quad w\left(\frac{1}{4}\right) - w\left(\frac{1}{2}\right) + w\left(\frac{3}{4}\right) > \frac{2}{3}. \end{cases} \tag{14}$$

Whenever $w\left(\frac{1}{2}\right) < \frac{2}{3}$, DT predicts $\delta^A = \delta^B = 1$. Observe that, if $w\left(\frac{1}{2}\right) = \frac{2}{3}$ then $\delta^A \in [0, 1]$ and $\delta^B \in \left(\frac{2}{3}, 1\right]$. This suggests that a DT-investor cannot be strictly-risk-vulnerable. Let us illustrate with the simple power weighting function used in Safra and Segal (2008):

$$w\left(p\right) = p^{\alpha}, \text{ where } \alpha > 0. \tag{15}$$

This function is strictly concave (resp. convex) for $\alpha < 1$ (resp. $\alpha > 1$), so that relatively more (resp. less) weight is given to probabilities associated to bad outcomes. Thus, if $\alpha < 1$ (resp. $\alpha > 1$), the individual is '*pessimistic*' (resp. '*optimistic*'. The predicted impact of background risk on the portfolio choice of the DT-investor is illustrated in Figure 2.

Figure 2. Predicted portfolio choice and risk vulnerability under DT



C. Rank-dependent utility theory

Both EU and DT are special cases of RDU. The preferences function takes therefore the following form:

$$v(\widetilde{w}) = \sum_{i=1}^{2} \pi(x_i) u(x_i), \tag{16}$$

and the Kuhn-Tucker first-order conditions for situation A yield:

$$\delta^A \begin{cases} = 1 \quad if \quad w\left(\frac{1}{2}\right) < \left[1 + \frac{1}{2}\frac{u'\left(\frac{1}{2}x\right)}{u'(2x)}\right]^{-1} \\ \in [0,1] \quad if \quad w\left(\frac{1}{2}\right) = \left[1 + \frac{1}{2}\frac{u'\left(x_1^A\right)}{u'\left(x_2^A\right)}\right]^{-1} \\ = 0 \quad if \quad w\left(\frac{1}{2}\right) > \frac{2}{3}. \end{cases} \tag{17}$$

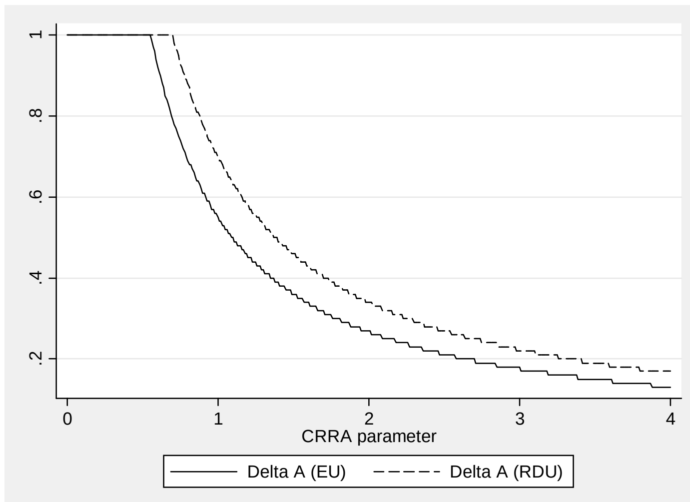
where $x_1^A = \left[2 - \delta^A\right] \frac{1}{2}x$ and $x_2^A = \left[1 + \delta^A\right] x$. For risk-loving and risk-neutral individuals (with non-decreasing marginal utility), we have $\left[1 + \frac{1}{2}\frac{u'\left(\frac{1}{2}x\right)}{u'(2x)}\right]^{-1} \geq \frac{2}{3}$ and, thus, $w\left(\frac{1}{2}\right) < \frac{2}{3}$ is sufficient for them to choose the maximum possible investment. Moreover, in contrast with EU, a zero investment can be an optimal choice under monotonic preferences if $w\left(\frac{1}{2}\right) > \frac{2}{3}$. But this latter prediction is highly implausible in light of empirical findings. In fact, empirical estimates indicate that the probability weighting function is asymmetric (with fixed point $p^* = w(p^*) \in [0.3; 0.45]$), regressive ($w(p) > p$ if $p < p^*$ and $w(p) < p$ if $p > p^*$) and inverse S-shaped (first concave, then convex). To illustrate, we use the power utility function in (8) with $CRRA$ parameter γ) together with Prelec (1998)'s single parameter probability weighting function

$$w(p) = \frac{1}{\exp\left\{-\left[-\ln(p)\right]^{\theta}\right\}}, \text{ for } p > 0 \text{ and } 0 < \theta \leq 1. \tag{18}$$

Using $\theta = 0.65$ we have $w\left(\frac{1}{2}\right) = 0.45 < \frac{1}{2}$. The smaller is the weighting function parameter θ, the smaller is the probability assigned to an unsuccessful investment in the risky asset.

Thus, optimism (small θ) moderates risk aversion (high γ), but the results are qualitatively the same than under EU: the optimal investment curve is first horizontal, for small values of γ and then becomes a strictly decreasing and convex function[9]. This is illustrated in Figure 3 below.

Figure 3. Predicted portfolio choice and risk aversion under EU and RDU



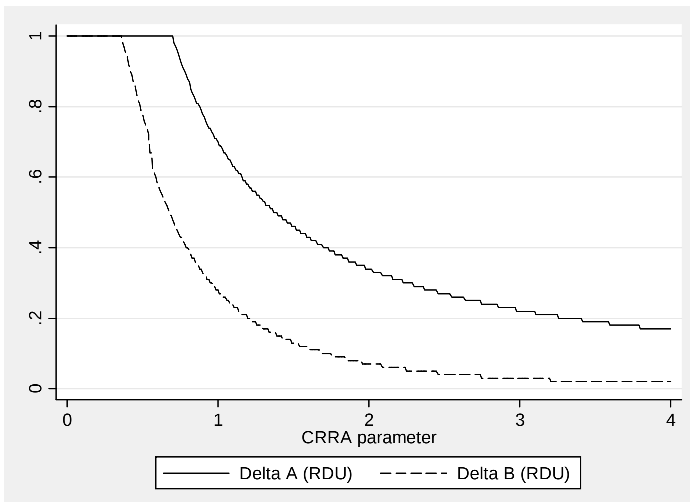
In situation B the preferences function is written[10]:

$$v\left(\widetilde{x}+\widetilde{y}\right)=\sum_{i=1}^{2}\sum_{j=1}^{2}\pi\left(x_{ij}\right)u\left(x_{ij}\right), \tag{19}$$

Because the ranking of outcomes is endogeneous (it depends on the level of investment), the Kuhn-Tucker first-order conditions are quite complicated (as under EU). To illustrate we rely on a numerical example. We choose $\theta = 0.65$, which yields: $\pi\left(x_{11}\right) = 0.29$, $\pi\left(x_{22}\right) = 0.36$, if $\delta^B < \frac{2}{3}$ then $\pi\left(x_{21}\right) = 0.16$ and $\pi\left(x_{12}\right) = 0.19$; if $\delta^B = \frac{2}{3}$, then $\pi\left(x_{12}\right) = \pi\left(x_{21}\right) = 0.18$; if $\delta^B > \frac{2}{3}$, then $\pi\left(x_{21}\right) = 0.19$ and $\pi\left(x_{12}\right) = 0.16$. Under this specification, the individual overweights $\pi\left(x_{11}\right)$ and $\pi\left(x_{22}\right)$, whereas he/she underweights $\pi\left(x_{12}\right)$ and $\pi\left(x_{21}\right)$. It is interesting to note that overweighted probabilities $\pi\left(x_{11}\right)$ and $\pi\left(x_{22}\right)$ are associated with an outcome that combine 'bad with bad' and 'good with good', respectively. Indeed, $\pi\left(x_{11}\right)$

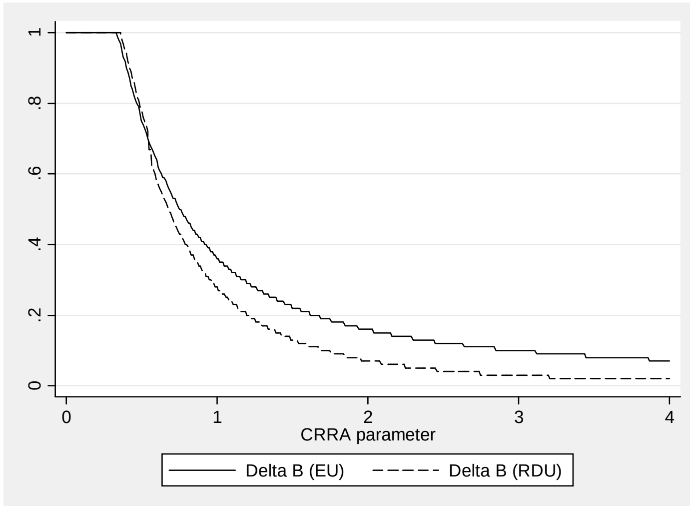
(resp. $\pi(x_{22})$) is the probability assigned to an unsuccessful (resp. successful) investment in the risky asset *and* a defavorable (resp. favorable) realization of the background risk. On the other hand, underweighted probabilities $\pi(x_{12})$ and $\pi(x_{21})$ are both associated with an outcome that combine 'bad with good'. Indeed, $\pi(x_{21})$ (resp. $\pi(x_{12})$) is the probability assigned to a successful (resp. unsuccessful) investment in the risky asset *and* a defavorable (resp. favorable) realization of the background risk. The impact of background risk on the portfolio choice is illustrated in Figure 4.

Figure 4. Predicted portfolio choice and risk vulnerability under RDU



In our numerical example the impact of background risk is stronger under *RDU* than under *EU*. This is so because without background risk the individual is more optimistic under *RDU* than under *EU*, whereas in the presence of background risk the individual is more pessimistic under *RDU* than under *EU*. This is can be viewed by observing Figure 3 and Figure 5 below.

Figure 5. Predicted portfolio choice and risk aversion under EU and RDU



D. Cumulative Prospect Theory

Under *CPT* the transformation function is the same as in *RDU*, but since asset integration does not apply, the carriers of value are not the final net wealth positions but the variations of wealth with respect to a reference point x^*. This is captured by the value function introduced by Tversky & Kahneman (1992) that is defined over gains and losses (above and below the reference point):

$$u(x) = \begin{cases} u^+(x - x^*) = [x - x^*]^\alpha & \text{if} \quad x \geq x^* \\ -u^-(x^* - x) = -\lambda [x^* - x]^\beta & \text{if} \quad x < x^* \end{cases} \tag{20}$$

where $u(x^*) = u^-(0) = u^+(0) = 0$, $0 < \alpha < \beta < 1$ and $\lambda > 1$. The value function is increasing, concave for gains and convex for losses[11]. Without background risk the reference point is clearly the initial wealth x, which is also the sure level of wealth that is obtained if the individual chooses to invest $\delta^A = 0$ in the risky asset (maximin). With this assumption, we have $x_1 = x^* = x_2 \Longleftrightarrow \delta^A = 0$, and $x_1 < x^* < x_2 \Longleftrightarrow 0 < \delta^A \leq 1$.

A second feature of *CPT* is that the probability weighting function also differs above and below the reference point. The probability weighting function is defined over the cumulative distribution function for losses

$$\pi^{-}(x_i) = w^{-}(\Pr(\widetilde{x} \leq x_i)) - w^{-}(\Pr(\widetilde{x} < x_i)) \tag{21}$$

and over the decumulative distribution function for gains

$$\pi^{+}(x_i) = w^{+}(\Pr(\widetilde{x} \geq x_i)) - w^{+}(\Pr(\widetilde{x} > x_i)). \tag{22}$$

Thus, we have $\pi^{-}(x_1) = w^{-}\left(\frac{1}{2}\right)$ and $\pi^{+}(x_2) = w^{+}\left(\frac{1}{2}\right)$ and the preferences function takes the following form:

$$v(\widetilde{x}) = w^{+}\left(\frac{1}{2}\right)u^{+}(X_2) - w^{-}\left(\frac{1}{2}\right)u^{-}(X_1), \tag{23}$$

where $X_1 = x - x_1 = \delta\frac{1}{2}x$ and $X_2 = x_2 - x = \delta x$.[12] Using $u^{+}(X) = X^{\alpha}$ and $u^{-}(X) = \lambda X^{\beta}$, where $0 < \alpha < \beta < 1$ and $\lambda > 1$, we have

$$\delta^{A} = \frac{1}{x}\left[\frac{w^{+}\left(\frac{1}{2}\right)}{w^{-}\left(\frac{1}{2}\right)}2^{\beta}\frac{\alpha}{\lambda\beta}\right]^{\frac{1}{\beta-\alpha}} \tag{24}$$

With background risk, it is unclear what the reference point might be. If we take the same definition than without background risk, i.e. the level of wealth that can be reached by the agent if he invests $\delta^{B} = 0$ in the risky asset, there are two possible candidates : $x_{i1} = \frac{1}{2}x$ and $x_{i2} = \frac{3}{2}x$. Taking $\frac{3}{2}x$ as the reference point under background risk can be interpreted as if the agent adopted an *optimistic* view about the outcome of the background risk, while if $\frac{1}{2}x$ is the reference point, the interpretation would be that the agent adopts a *pessimistic* view about the outcome of the background risk. Note that the optimistic and pessimistic reference points are respectively above and below the reference point without background risk, i.e. $\frac{1}{2}x < x < \frac{3}{2}x$. Observe that a reference point at x with background risk, i.e. insensitivity to the background risk, is another possibility but seems to us rather inconsistent with *CPT*.

In the presence of a background risk the derivation of Kuhn-Tucker first-order conditions for optimal portfolio choice is a particularly tedious task under *CPT*. Thus it is developed in an appendix.

Just note that in the case of a pessimistic reference point, i.e. $\frac{1}{2}x$, the possible gains and losses are:

$$\begin{aligned}
X_{11} &= \frac{1}{2}x - x_{11} \quad \text{(loss)} \\
X_{12} &= x_{12} - \frac{1}{2}x \quad \text{(gain)} \\
X_{21} &= x_{21} - \frac{1}{2}x \quad \text{(gain)} \\
X_{22} &= x_{22} - \frac{1}{2}x \quad \text{(gain)}
\end{aligned}$$

The probability weights are given by

$$\pi^{-}(X_{ij}) = w^{-}(\Pr(\widetilde{x} \leq x_{ij})) - w^{-}(\Pr(\widetilde{x} < x_{ij}))$$

for losses and by

$$\pi^{+}(X_{ij}) = w^{+}(\Pr(\widetilde{x} \geq x_{ij})) - w^{+}(\Pr(\widetilde{x} > x_{ij}))$$

for gains.[13] Thus, the preference functional takes the following form:

$$v(\widetilde{x} + \widetilde{y}) = \pi^{+}(X_{12})\,u^{+}(X_{12}) + \pi^{+}(X_{21})\,u^{+}(X_{21}) + \pi^{+}(X_{22})\,u^{+}(X_{22}) - \pi^{-}(X_{11})\,u^{-}(X_{11})\,.$$

On the other hand, in the case of a optimistic reference point, $\frac{3}{2}x$, the possible gains and losses are:

$$
\begin{aligned}
X_{11} &= \frac{3}{2}x - x_{11} > 0 \quad \text{(loss)} \\
X_{12} &= \frac{3}{2}x - x_{12} \geq 0 \quad \text{(loss)} \\
X_{21} &= \frac{3}{2}x - x_{21} \geq 0 \quad \text{(loss)} \\
X_{22} &= x_{22} - \frac{3}{2}x > 0 \quad \text{(gain)}
\end{aligned}
$$

Thus, the preference functional takes the following form:

$$
v(\widetilde{x} + \widetilde{y}) = \pi^+ (X_{22}) \, u^+ (X_{22}) - \pi^- (X_{21}) \, u^- (X_{21}) - \pi^- (X_{12}) \, u^- (X_{12}) - \pi^- (X_{11}) \, u^- (X_{11}) \, .
$$

To illustrate we use Prelec (1998)'s probability weighting function (18) with $\theta^+ = 0.5$ for gains and $\theta^- = 0.65$ for losses. Thus, we get the following transformed value for the cumulative distribution function:

p	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1
$\underset{\theta^+=0.5}{w^+(p)}$	0	0.31	0.43	0.58	1
$\underset{\theta^-=0.65}{w^-(p)}$	0	0.29	0.45	0.64	1

We also consider three cases for parameters characterizing the value function, i.e. α, β and λ. The results are summarized below.

Optimal portfolio predicted by CPT

$(x = 20, \theta^- = 0.65, \theta^+ = 0.5)$	$\underset{x^*=x}{\delta_A}$	$\underset{x^*=\frac{1}{2}x}{\delta_B}$	$\underset{x^*=\frac{3}{2}x}{\delta_B}$
Case a. $\{\alpha = 0.2, \beta = 0.40, \lambda = 1.00\}$	0.51%	$\underset{strictly-RV}{0.01\%}$	$\underset{Non\text{-}RV}{100\%}$
Case b. $\{\alpha = 0.6, \beta = 0.90, \lambda = 2.00\}$	0.88%	$\underset{RV}{0.09\%}$	$\underset{Non\text{-}RV}{100\%}$
Case c. $\{\alpha = 0.8, \beta = 0.88, \lambda = 2.25\}$	0.07%	$\underset{RV}{0.01\%}$	$\underset{Non\text{-}RV}{100\%}$

Without background risk CPT predicts a very small investment in the risky asset, less than 1% in all cases. With a pessimistic reference point in the presence of background risk, i.e. $x^* = \frac{1}{2}x$, the optimal investment is even weaker, and the individual behavior exhibits RV. On the contrary, with a optimistic reference point in the presence of background risk, i.e. $x^* = \frac{3}{2}x$, the optimal investment is full investment. Thus, depending on the reference point chosen in the presence of background risk, CPT may predict risk vulnerability or not. Here, predictions under CPT are quite similar to the ones obtained under DT. Thus, as DT, CPT is here hardly able to explain our experimental data.

IV. Experimental method and design

Our experimental design follows closely the theoretical framework described in section IV. A total of 279 student-subjects participated in our experiments. Participants were randomly selected from a large pool of over 3000 volunteers. Real monetary incentives were offered to all participants. Two different experiments based on a within-subject design were organized: a paper & pencil experiment (*Exp.1*) and a computerized experiment (*Exp.2*). Besides the experimental "technology", *Exp.1* and *Exp.2* differ on several key aspects: the incentive scheme, the subjects' endowments, the level of stakes and the subject pool. *Exp.1* involved 91 first year master students in economics belonging to the same group. "High stakes" were used (participants could win up to €250) but only 10% of the participants were randomly selected at the end of the experiment to be paid out for real. Finally in *Exp.1* participants could invest a windfall money endowment provided by the experimenter.

Exp.2 involved 188 participants randomly selected from a large subject pool of over 3000 volunteers from various disciplines. All participants were paid according to their earnings in the experiment, but stakes were much lower than in experiment 1 (the maximum was €50). Finally, participants had to "work" in a preliminary task to earn a money endowment of €20 that they could invest in a risky asset in the active part of the experiment. More details will be provided in the description of each experiment.

We chose to vary simultaneously several aspects of the portfolio task that are irrelevant from a theoretical perspective (except for the level of endowment) in order to check the robustness of our initial findings in *Exp.1.* Since many reasons could explain subjects' observed behavior in *Exp.1* (windfall money, random selection, participants' major, ...) *Exp.2* provides a strong test of the robustness of our findings in *Exp.1.* A translation of the instructions can be found in appendix 1.

A. Experiment 1

Exp.1 was a single session involving 91 first-year master students majoring in economics. Subjects were seated in a large room with numbered seats clearly separated and chose one of the numbered seats as they entered the room. Written instructions were available on each place, and participants were instructed to read them silently once seated. Once all participants had read the instructions, they were read again aloud by one of the assistants present in the room. Instructions specified that each participant had a potential endowment of $w =$€100 which was equally split between two independent accounts: account 1 and account 2. The decision task corresponded to the portfolio choice problem described in section II. Subjects were told that account 1 was blocked. They could only use their endowment in account 2, containing $x =$€50, for making the investment decision. Participants had to decide about their investment into the risky option by choosing an integer amount between €0 and €50: thus, $\delta \in \{0, 2\%, 4\%, ..., 100\%\}$. They had to answer the same questions in two situations (A and B) presented sequentially. Half of the participants had to answer situation A before situation B, while the other half was confronted to the reverse ordering. Situation B involved background risk $\widetilde{x_1} = (-€50, \frac{1}{2}; €50, \frac{1}{2})$ for account 1: €50 would be added or substracted from account 1 with equal probability, after the respondent made his investment decision for account 2. It was made clear that only one of the two situations would apply for real at the end of the experiment, the choice of the relevant situation being decided on a random basis. Furthermore, subjects were told that 10% of the participants would be randomly selected (by choosing randomly 10% of the numbered seats) to be paid out for real a the end of the session.

B. Experiment 2

Exp. 2 involved 188 participants and differed in many aspects from Exp.1. The experiment followed a two-stage procedure. In the first stage of the experiment, each subject can accumulate real money by accomplishing a simple, but tedious task during a limited period of time. The task consisted in reporting the number of times number "1" appeared in a matrix containing strings of "0"'s and "1"'s. Ten different matrixes with varying sizes had to counted this way. *A* substantial reward of €20 was paid for accomplishing the task without error. At the end of the first stage only subjects who had completed the task correctly for all ten matrixes received the flat reward of €20. Only 18 participants out of 188 $(8,5\%)$ failed to accomplish the task in time and/or without error. They were instructed that they could stay in the experiment until the end, but that they would play with fictitious money. Those who succeeded were instructed that they could use their earning from stage 1 to participate in second stage decisions. The reason for including this first stage was to control for a windfall money effect that could have affected the results of *Exp.1.*

At the end of the first stage participants were informed that their $x =$€20 reward would be split into two identical parts, credited on two separate personal accounts (account 1 and account 2) and that they would receive in cash the balance of both accounts at the end of the session.

In the second step of the experiment subjects were told that they could use the $\frac{1}{2}x =$€10 of their account 2 for making an investment decision. The investment decision corresponded to the portfolio choice problem described in section 2 with $\frac{1}{2}x =$ €10. Investment choice possibilities in the risky asset were restricted to integer amounts between €0 and €10. Thus in contrast to *Exp.1* there where fewer choice options for $\delta \in \{0, 10\%, 20\%, ..., 1\}$. As in *Exp.1*, in the second stage subjects were informed that they would be asked to choose δ

in two distinct situations labelled A and B, where situation B involved background risk $\widetilde{y} = (-10, \frac{1}{2}; 10, \frac{1}{2})$ for account 1. Therefore depending on the realized outcome the balance of account 1 is either 0€ or 20€. Subjects were told that at the end of the experiment one of the two situations would be chosen randomly to be paid out for real. To control for order effects half of the participants were assigned to situation A first, and the other half to situation B first.

One can argue that since we used the *RRPM* procedure, i.e. only one of the two situations (A or B) was randomly selected to be paid our for real, through a coin toss in both experiments and, in addition, only some of the participants were randomly selected to be paid out for real in *Exp. 1* - we actually induced a second background risk, as in Lee (2008)'s experiment. Even if this is true, it does not affect our conclusion because of our within-subject design. Such additional implemented background risk simply adds to the subjects' own background risk that they bring with them to the lab and which we do not control. With the exception of the order effect, the impact of background risk is therefore captured *all other things equal* in our experiment. Note also that under *EUT*, Gollier and Pratt (1996) have demonstrated that *RV* is necessary and sufficient for an increase in background risk to generate more risk aversion, if it takes the form of adding an independent actuarially neutral risk to the background random wealth, as in our experiment.

V. Data analysis and results

We rely on two categorizations: a "coarse" categorization which distinguishes between *RV* and *Non-RV* individuals and a "fine" categorization that adds a further distinction within the *RV* category between the *strict-RV* and the *Indifferent* individuals. We start by providing descriptive statistics about relative frequencies for each category. We next provide estimates for *RV* based on regression analysis.

A. Descriptive results

Among the 279 subjects, 10 have chosen $\delta_A = \delta_B = 0$ and 20 have chosen $\delta_A = \delta_B = 1$[14]. Such extreme investment decisions are consistent with *Indifference*, *Strict-RV* and *Non-RV*. For instance a subject for whom $\delta_A = \delta_B = 1$, could have preferred $\delta_A > \delta_B = 1$ but faced a binding constrain for his investment decision in situation A. Likewise a subject for whom $\delta_A = \delta_B = 0$, could have preferred $\delta_A = 0 > \delta_B$ but was censured at zero investment. In order to avoid an ambiguous categorization of our subjects we drop these 30 observations for the descriptive analysis.[15]

Table 1 summarizes the results of our categorization for the pooled data of the two experiments. 78.3% of the subjects invested an equal or lower amount in the risky asset in situation B and are therefore classified as *RV* for the coarse categorization, while 21.7% are *Non-RV*, i.e. $\delta_A < \delta_B$. According to the fine categorization 52.6% are *Strict-RV*, i.e. they invested strictly less in situation B than in situation A ($\delta_A > \delta_B$), while 25.7% are *Indifferent*, i.e. $\delta_A = \delta_B$. Our main result is therefore that the relative frequency of *RV* individuals is significantly larger than 50% (binomial test, 5%) in both experiments and for both treatments.

Table 1. Frequencies within the entire experimental population

RV ($\delta_B \leq \delta_A$)	*Strict-RV* ($\delta_B < \delta_A$)	*Indifferent* ($\delta_A = \delta_B$)	*Non-RV* ($\delta_B > \delta_A$)
78.3% ($n = 195$)	**52.6**% ($n = 131$)	**25.7**% ($n = 64$)	**21.7**% ($n = 54$)

Table 2 presents the data separately for each experiment. Despite the strong differences in design features, it is apparent from the frequencies of *RV* (versus *Non-RV*) reported in Table 2, that the two experiments provide exactly the same picture. The relative frequency of

RV individuals is equal in both treatments and in both experiments (Fisher exact test, 5%) as illustrated by Table 3 which compares the frequency of types according to the ordering of the investment decisions (treatment AB vs BA) for the pooled data. It is apparent that there is no order effect. The frequency of RV and *Non-RV* is exactly the same for the two orderings. There is however a slight difference in the frequencies of *Strict-RV* and Indifferent.

Table 2. Frequencies across experiments

	RV ($\delta_B \leq \delta_A$)	*Strict-RV* ($\delta_B < \delta_A$)	*Indifferent* ($\delta_A = \delta_B$)	*Non-RV* ($\delta_B > \delta_A$)
Exp.1 ($n = 82$)	**78.0%** ($n = 64$)	**57.3%** ($n = 47$)	**20.7%** ($n = 17$)	**22.0%** ($n = 18$)
Exp.2 ($n = 167$)	**78.4%** ($n = 131$)	**50.3%** ($n = 84$)	**28.1%** ($n = 47$)	**21.6%** ($n = 36$)

Table 3. Frequencies across treatments

	RV ($\delta_B \leq \delta_A$)	*Strict-RV* ($\delta_B < \delta_A$)	*Indifference* ($\delta_A = \delta_B$)	*Non-RV* ($\delta_B > \delta_A$)
Treatment AB ($n = 120$)	**78.3%** ($n = 94$)	**49.2%** ($n = 59$)	**29.2%** ($n = 35$)	**21.7%** ($n = 26$)
Treatment BA ($n = 129$)	**78.3%** ($n = 101$)	**55.8%** ($n = 72$)	**22.5%** ($n = 29$)	**21.7%** ($n = 28$)

The null hypothesis of equal distributions of percentages invested in experiments 1 and 2 cannot be rejected neither for situation A nor for situation B (KS test, two-sided, 5%). Furthermore the null hypothesis of equal distributions for $\delta_A - \delta_B$ in Exp.1 and Exp.2 cannot be rejected either (KS test, two-sided, 5%). We therefore conclude that Exp.1 and

Exp.2 produce the same results with respect to our categorizations (coarse and fine).

While the relative frequency of indifferent individuals is equal in both treatments and in both experiments (Fisher exact test, 5%), there exists a small difference among the *RV* with respect to the frequency of *Strict-RV* vs *Indifferent* individuals. We think that this difference is essentially due to the fact that the choice-space is richer in the classroom experiment than in the laboratory experiment. It is indeed conceivable that, in the laboratory experiment, some subjects would have been willing to reduce slightly their investment in situation B, for instance by 5%, but since such an option was not feasible (the minimum reduction possibility was 10%) they finally kept their investment at the same level as in situation A. A striking result is that the frequency of *Non-RV* is exactly the same in both experiments, a result that suggests that the richer choice-space of *Exp.1* only affected the frequency of *indifferent* subjects with respect to the fine categorization.

B. Analytical results

We first look at the determinants of the amount invested without background risk (δ_A), which we interpret as a measure of risk-tolerance. Table 4 summarizes the results. In accordance with other experiments, women are more risk-averse than men: on average women invest about 8% less than men although the coefficient for gender is only weakly significant. There is a also a weak effect of the variable *religion* which measures the number of days of worship per week. δ_A is also significantly affected by the two treatment variables: if situation A follows situation B (variable AB), subjects invest more in situation A compared to the treatment where the ordering of situation A and B are reversed. This could be possibly due to the fact that when situation A follows situation B, subjects have a reference point (the amount invested in situation B) while in the AB treatment such reference point is not available. Furthermore subjects invested a larger percentage in experiment 2 (paper & pencil experiment).

We next study the variables that affect the probability for a subject to be risk-vulnerable.

Table 5 shows that the only significant variable is δ_A: the larger δ_A the higher the probability for an individual to be *RV*. Recall that δ_A is an indirect measure of risk-tolerance. The fact that larger values δ_A increase the likelihood for an individual to be *RV* is consistent with the prediction of *EUT.* As shown in Figure 1 the lower the *CRRA* value the stronger the difference $\delta_A - \delta_B$, i.e. the stronger the risk-vulnerability. Note that if we substitute δ_B for δ_A in the regression reported in Table 5, consistently δ_B is the only significant variable and has a negative sign. Note also that if the dependent variable is Exp.1 and Exp.2 instead of *RV* the only significant variable is also δ_A, and the estimated coefficient of δ_A is of the same sign.

Table 4. Determinants of the amount invested in the risky asset without background risk (δ_A)

Tobit regression	Number of obs	=	276
	LR chi2(6)	=	23.66
	Prob > chi2	=	0.0006
Log likelihood = -1122.0284	Pseudo R2	=	0.0104

deltaA	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
gender	-8.544083	4.563487	-1.87	0.062	-17.52863	.4404596
labpp	10.40633	4.932733	2.11	0.036	.6948155	20.11784
AB	14.20874	4.544253	3.13	0.002	5.26206	23.15541
religion	-.3561126	.1986249	-1.79	0.074	-.747163	.0349379
experience	3.025013	4.859739	0.62	0.534	-6.542788	12.59281
siblings	13.37056	8.03722	1.66	0.097	-2.453035	29.19415
_cons	47.24831	5.450006	8.67	0.000	36.5184	57.97823
/sigma	36.06888	1.903768			32.32076	39.817

Obs. summary: 19 left-censored observations at deltaA<=0
209 uncensored observations
48 right-censored observations at deltaA>=100

We next study the variables that affect the probability for a subject to be risk-vulnerable. Table 5 shows that the only significant variable is δ_A: the larger δ_A the higher the probability for an individual to be *RV*. Recall that δ_A is an indirect measure of risk-tolerance. The fact that larger values δ_A increase the likelihood for an individual to be *RV* is consistent with the prediction of *EUT*. As shown in Figure 1 the lower the *CRRA* value the stronger the difference δ_A- δ_B, i.e. the stronger the risk-vulnerability. Note that if we substitute δ_B for δ_A in the regression reported in Table 5, consistently δ_B is the only significant variable and has a negative sign. Note also that if the dependent variable is Exp.1 and Exp.2 instead of *RV* the only significant variable is also δ_A, and the estimated coefficient of δ_A is of the same sign.

Table 5. Determinants of the probability of RV (δ_A)

Logistic regression	Number of obs = 277
	LR chi2(7) = 36.88
	Prob > chi2 = 0.0000
Log likelihood = -118.20935	Pseudo R2 = 0.1349

RV	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
deltaA	.0342302	.0066643	5.14	0.000	.0211684	.0472919
gender	-.2156711	.3293173	-0.65	0.513	-.8611212	.429779
birth	-.0707571	.0539034	-1.31	0.189	-.1764059	.0348916
labpp	-.3251627	.3677411	-0.88	0.377	-1.045922	.3955966
AB	-.1250736	.3289446	-0.38	0.704	-.7697932	.519646
religion	-.0047269	.0141545	-0.33	0.738	-.0324692	.0230155
siblings	.4305903	.6885043	0.63	0.532	-.9188533	1.780034
_cons	140.8337	107.1559	1.31	0.189	-69.18802	350.8553

VI. Conclusion

(to be completed)

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[1]In fact, $DARA$ is equivalent to vulnerability to sure losses, while properness and standarness are equivalent to vulnerability to background risks that reduce expected utility and increase expected marginal utility, respectively (see Gollier and Pratt, 1996). Thus, $DARA$ is necessary but not sufficient for risk vulnerability, while proper and standard risk aversion are sufficient but not necessary.

[2]As observed by Quiggin (2003, pp. 610-611): "*The use of terminology such as 'standard' and 'proper' in the expected-utility literature indicates the expectation that aversion to one risk will be enhanced in the presence of another, that is, that independent risks are substitutes rather than complements*". This quotation suggests that risk vulnerability is "expected" (by EU modeler) to be a clear empirical fact.

[3]An important special case of constant risk aversion is that of rank-dependent preferences with linear utility, namely Yaari (1987)'s *dual theory* (DT).

[4]A similar field experiment was carried out with farmers by Herberich & List (2012).

[5]Observe that theirs results from the unfair background risk treatment allows a test of $DARA$ rather then a test of risk vulnerability (since the unfair background risk which they have chosen exhibits non-positive monetary outcomes only).

[6]Observe that this additive background risk is equivalent to a multiplicative background risk $\widetilde{z} = (0, \frac{1}{2}; 2, \frac{1}{2})$ affecting the blocked account. Formally, we have $\widetilde{x} + \widetilde{y} = \widetilde{x} - \frac{1}{2}x + \frac{1}{2}x\widetilde{z}$.

[7]The rational for this formulation is that examining the effect of the introduction of background risk is equivalent to examining differences between preferences represented by u and U. An agent exposed to background risk and having preferences represented by u would act as a non-exposed agent with preferences represented by U. Observe that the signs of the successive derivatives of u and U are identical. Thus, monotonicity, risk aversion, prudence,

etc., are preserved after the introduction of a background risk.

[8]Whatever the level of investment, we have $\pi(x_{11}) = w\left(\frac{1}{4}\right)$ and $\pi(x_{22}) = 1 - w\left(\frac{3}{4}\right)$. On the other hand, if $\delta^B < \frac{2}{3}$ then $\pi(x_{21}) = w\left(\frac{1}{2}\right) - w\left(\frac{1}{4}\right)$ and $\pi(x_{12}) = w\left(\frac{3}{4}\right) - w\left(\frac{1}{2}\right)$. If $\delta^B = \frac{2}{3}$, then $\pi(x_{21}) = \pi(x_{12}) = \frac{1}{2}\left[w\left(\frac{3}{4}\right) - w\left(\frac{1}{4}\right)\right]$. If $\delta^B > \frac{2}{3}$, then $\pi(x_{21}) = w\left(\frac{3}{4}\right) - w\left(\frac{1}{2}\right)$ and $\pi(x_{12}) = w\left(\frac{1}{2}\right) - w\left(\frac{1}{4}\right)$. Substituting $x_{11} = [1-\delta]\frac{1}{2}x$, $x_{12} = [3-\delta]\frac{1}{2}x$, $x_{21} = \left[\frac{1}{2}+\delta\right]x$ and $x_{22} = \left[\frac{3}{2}+\delta\right]x$, we get:

$$\frac{v(\widetilde{x}+\widetilde{y})}{x} = \begin{cases} \frac{3}{2} - w\left(\frac{1}{4}\right) + w\left(\frac{1}{2}\right) - w\left(\frac{3}{4}\right) + \delta\left[1 - \frac{3}{2}w\left(\frac{1}{2}\right)\right] & if \quad \delta^B > \frac{2}{3} \\ \frac{13}{6} - w\left(\frac{1}{4}\right) - w\left(\frac{3}{4}\right) & if \quad \delta^B = \frac{2}{3} \\ \frac{3}{2} - w\left(\frac{1}{2}\right) + \delta\left[1 - \frac{3}{2}w\left(\frac{1}{4}\right) + \frac{3}{2}w\left(\frac{1}{2}\right) - \frac{3}{2}w\left(\frac{3}{4}\right)\right] & if \quad \delta^B < \frac{2}{3}. \end{cases} \tag{25}$$

[9]Basically, when θ decreases, the individual becomes more optimistic and the optimal investment curve translates to the east. The opposite is obtained when θ increases (i.e. pessimism reinforces risk aversion).

[10]

$$v(\widetilde{w}+\widetilde{y}) = \begin{cases} w\left(\frac{1}{4}\right)u(x_{11}) + \left[w\left(\frac{1}{2}\right) - w\left(\frac{1}{4}\right)\right]u(x_{12}) + \left[w\left(\frac{3}{4}\right) - w\left(\frac{1}{2}\right)\right]u(x_{21}) + \left[1 - w\left(\frac{3}{4}\right)\right]u(x_{22}) & if \\ w\left(\frac{1}{4}\right)u(x_{11}) + \frac{1}{2}\left[w\left(\frac{3}{4}\right) - w\left(\frac{1}{4}\right)\right]\left[u(x_{21}) + u(x_{12})\right] + \left[1 - w\left(\frac{3}{4}\right)\right]u(x_{22}) & if \quad \delta^B = \frac{2}{3} \\ w\left(\frac{1}{4}\right)u(x_{11}) + \left[w\left(\frac{1}{2}\right) - w\left(\frac{1}{4}\right)\right]u(x_{21}) + \left[w\left(\frac{3}{4}\right) - w\left(\frac{1}{2}\right)\right]u(x_{12}) + \left[1 - w\left(\frac{3}{4}\right)\right]u(x_{22}) & if \end{cases}$$

[11]

$$u'(x) = \begin{cases} u^{+\prime}(x - x^*) = \alpha\left[x - x^*\right]^{\alpha-1} \geq 0 & if \quad x \geq x^* \\ -u^{-\prime}(x^* - x) = \lambda\beta\left[x^* - x\right]^{\beta-1} > 0 & if \quad x < x^* \end{cases}$$

and

$$v''(x, \omega^*) = \begin{cases} u^{+\prime\prime}(x - x^*) = \alpha\left[\alpha - 1\right]\left[x - x^*\right]^{\alpha-2} \leq 0 & if \quad x \geq x^* \\ -u^{-\prime\prime}(x^* - x) = -\lambda\beta\left[\beta - 1\right]\left[x^* - x\right]^{\beta-2} > 0 & if \quad x < x^* \end{cases}.$$

[12]The Kuhn-Tucker first-order conditions yield:

$$\delta^A \begin{cases} = 1 \quad \text{if} \quad \frac{w^+\left(\frac{1}{2}\right)}{w^-\left(\frac{1}{2}\right)} > \frac{1}{2}\frac{u^{-\prime}(X_1)}{u^{+\prime}(X_2)} \\ \in [0,1] \quad \text{if} \quad \frac{w^+\left(\frac{1}{2}\right)}{w^-\left(\frac{1}{2}\right)} = \frac{1}{2}\frac{u^{-\prime}(X_1)}{u^{+\prime}(X_2)} \\ = 0 \quad \text{if} \quad \frac{w^+\left(\frac{1}{2}\right)}{w^-\left(\frac{1}{2}\right)} < \frac{1}{2}\frac{u^{-\prime}(X_1)}{u^{+\prime}(X_2)}. \end{cases} \tag{26}$$

[13]Whatever the level of investment, we have $\pi^-(X_{11}) = w^-\left(\frac{1}{4}\right)$ and $\pi^+(X_{22}) = w^+\left(\frac{1}{4}\right)$. On the other hand, if $\delta^B < \frac{2}{3}$ then $\pi^+(X_{21}) = w\left(\frac{3}{4}\right) - w\left(\frac{1}{2}\right)$ and $\pi^+(X_{12}) = w\left(\frac{1}{2}\right) - w\left(\frac{1}{4}\right)$. If $\delta^B = \frac{2}{3}$, then $\pi^+(X_{21}) = \pi^+(X_{12}) = \frac{1}{2}\left[w\left(\frac{3}{4}\right) - w\left(\frac{1}{4}\right)\right]$. If $\delta^B > \frac{2}{3}$, then $\pi^+(X_{21}) = w\left(\frac{1}{2}\right) - w\left(\frac{1}{4}\right)$ and $\pi^+(X_{12}) = w\left(\frac{3}{4}\right) - w\left(\frac{1}{2}\right)$.

[14]279 $(= 188^c + 91^{p\&p})$, 10 $(= 6^c + 4^{p\&p})$ and 20 $(= 15^c + 5^{p\&p})$.

[15]If situation A is first, $\delta_A = \delta_B = 0$ is inconsistent with *Non-RV* but is consistent with both *Indifference* and *Strict-RV*. If situation B is first, it becomes inconsistent with *Strict-RV* and consistent with *Non-RV* and *Indifference*. In the same way, if situation A is first, $\delta_A = \delta_B = 1$ is inconsistent with *Strict-RV* and *Indifference* but is consistent with *Non-RV*. But if situation B is first, it becomes inconsistent with *Non-RV* and consistent with *Strict-RV* and indifference.