Preference for Diversification across Time

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The interplay between risk and time preference has been extensively examined. In an experimental setting, we investigate inter-temporal preference when risks across time are correlated. We find strong evidence in support of preference for diversification across time, when comparing perfectly negatively correlated risks and uncorrelated risks with perfectly positively correlated risks and certainty. This observation could not be explained by models with separable additivity including discounted expected utility, prospect theory, and those with preference for uncertainty resolution. We also propose a simple mean-variance model in dynamic environment with behavioral foundations to capture preference for diversification across time. Our study also provides a novel interpretation for the recent puzzle in Andreoni and Sprenger (2012a) on certainty effect in time.

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1. Introduction

One of the common assumptions in models involving time preferences is separable additivity, which states that the valuation of a stream of consumptions equals the summation of the discounted valuations of consumption in each period along the path. It is natural to extend this criterion to the environment where risks are involved, and to obtain a separable additive expected utility for a stream of contingent consumptions (discounted) as the following

$$\sum_t \beta^t E_t (u_t)$$

Many theories have been proposed under this category. The most conventional one is the recursive (discounted) expected utility, which imposes additional consistency requirements

$$V(z_t) = E_t(u(c_t) + \beta V(z_{t+1}))$$

Essentially, separable additivity is shared by a number of established theories, including Kreps and Porteus (1978), Chew and Epstein (1989), Epstein and Zin (1989) and Halevy (2008). Kreps and Porteus (1978) work on finite horizon and permit different expected utility functions in different time periods. Subsequently, Chew and Epstein (1989) extend their work to allow for general non-expected utilities. Epstein and Zin (1989), instead, provide a framework to explore preferences on consumption plans on infinite horizon, and provide some examples with recursive specifications. Halevy (2008) analyzes hyperbolic discounting by assuming uncertainty in the future payoffs. However, the additivity property has also been challenged by a number of observations, such as preference for improving
sequences of consumption and preference for spread in consumption (See review in Fredericke, Loewenstein, O'Donoghue, 2002).

Here, we challenge the separable additivity assumption with the following behavioral intuition. Consider two allocations in the domain of contingent plans as follows:

- Option A: 50% chance of getting $100 today; otherwise getting $100 7 days later.
- Option B: 50% chance of getting $100 today and $100 7 days later; otherwise getting nothing in both days.

Intuitively, people would prefer Option A to Option B, since the risks are diversified across time in Option A, while in Option B they are not. In other words, the risks in two periods are perfectly negatively correlated in Option A while they are perfectly positively correlated in Option B. Presumably, preference for diversification would lead people to choose Option A over Option B.

Put in a different way, preference for option A over Option B seems to suggest that the inter-temporal substitution may not be constant between different periods. To be more specific, the substitution rate for consumptions between today and 7 days later in the same state $s$ and across different states $s$ and $s'$ are not the same,\(^2\) thus making the correlation of consumptions between different time periods matter. However, we will point out later that separable additive utility functions usually cannot account for this particular preference.

One of the purposes in our current study is to test this intuition in an experimental setting. We adopt the experimental design of Convex Time Budgets (CTB) as in Andreoni and

\(^2\) State $s$ and $s'$ are equally likely, as in the example provided.
Sprenger (2012a; 2012b), where subjects are to allocate 100 points between a sooner and smaller payment and a later and larger payment with different interest rates. There are four treatments. In the first treatment, both sooner and later payments will be received for sure (SURE treatment). In the rest of three treatments, there are risks in receiving both payments with different correlation patterns. In treatment with perfectly positively correlated risks (POSITIVE treatment), there is a 50% chance that both payments will be received, otherwise no payment. In treatment with perfectly negatively correlated risks (NEGATIVE treatment), there is a 50% chance that sooner payment will be received, otherwise later payment will be received. In treatment with uncorrelated risks (UNCORRELATED treatment), there is a 50% chance of receiving the sooner payment, and independently a 50% chance of receiving the later payment. All the uncertainties are resolved at the end of the experiment to control the preference for uncertainty resolution.

The traditional recursive expected utility predicts that the utility from the latter 3 treatments are the same, and hence the optimal allocation should also be identical. In addition, due to “common ratio” effect across time, the optimal allocation in SURE treatment should also coincide with that in the latter 3 treatments. However, we find that the allocations are similar in treatments SURE and POSITIVE, and similar in NEGATIVE and UNCORRELATED, while the allocations in SURE/POSITIVE treatments differ substantively from that in NEGATIVE/UNCORRELATED treatments. In addition, there is a cross-over between SURE/POSITIVE treatments and NEGATIVE/UNCORRELATED treatments. By which we mean that, relative to NEGATIVE/UNCORRELATED treatments, subjects allocate more money to sooner payment when the interest rate is low while allocate more money to later

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3 See Andreoni (2011) for the detailed explanation.
payment when the interest rate is high in treatments SURE/POSITIVE. The intuition behind, as we argue, is that there is need for diversification in NEGATIVE/UNCORRELATED treatments, and in contrast no need for diversification in SURE/POSITIVE treatments. Overall, our data suggests that correlation of consumptions between different time periods matters for decision makers. This observation also provides an interpretation to Andreoni and Sprenger (2012), in which they include treatments SURE and UNCORRELATED, and attribute the cross-over observation to preference for certainty. Here our evidence suggest that it could be due to preference for diversification over time.

The other purpose of current study is to propose a behavioral model with axiomatic foundations to account for the observed behavioral patterns in our data. Our model is essentially a mean-variance model in the dynamic environment that incorporates the correlation between different time periods, which has the following form

$$V(w) = \sum_i \gamma_i E(\tilde{w}_i) - \phi \text{var} \left( \sum_i \tilde{w}_i \right)$$

where $w$ is an stochastic allocation over the “Risk × Time” plane, and $\tilde{w}_i$ represents the contingent consumption in every period $t$. Intuitively, this representation accommodates the correlation across time through the variance term, and can explain the differences observed in our experiment. We will discuss the implications of this model for our treatments in details later.

Our model is related to some decision theoretical models in other domains. For example, Chew and Sagi (2011) apply the quadratic utility (Chew, Epstein and Segal, 1991) to evaluate preference for stochastic allocations of wealth in a society, and obtain a generalized Gini index that can exhibit the preference for “shared destiny” (positive
correlation between the allocations in different cohorts). Chew and Sagi (2011) provide us a framework for the axiomatization of our model.

The rest of this paper is organized as follows. We propose our model and discuss distinct implications in comparison with existing models in section 2, and detail experimental design in Section 3. Section 4 reports our experimental results, and in section 5 we discuss the implications for related literature. Section 6 concludes. The axiomatization of our model is appended.

2. Model

In this section, we propose a behavioral model that captures the idea of diversification across time, and compare the predictions of our proposed model with some existing models. The model we propose is essentially a mean-variance model

\[ V(w) = \sum \delta^t E(\tilde{w}_t) - \phi \text{var} \left( \sum \tilde{w}_t \right) \]

If we separate the variance term into variances of \( \tilde{w}_t \) in every period \( t \) and the covariance between \( \tilde{w}_t \) and \( \tilde{w}_{t'} \) for different \( t, t' \), the behavioral implication of this model would be that the agent has diversification need in every period due to the variance of \( \tilde{w}_t \), as well as across different time periods due to the covariance term. We will leave the axiomatization of this utility function into appendix and first explore the exact implications of our model on the four treatments.
Consider our experimental settings, for SURE treatment, the optimization problem for the decision maker is the following:

\[
\begin{align*}
\max_{x,y} & \quad x + \delta y \\
\text{s.t.} & \quad rx + y = 100
\end{align*}
\]

As there is no risk involved, decision maker only cares about the relation between the discounting factor and the interest rate. The optimal solution will be in the corner. Hence the agent should show certain type of “switching point” in this treatment according to our model, by which we mean that as long as the relative price of sooner payment is low, people tend to allocate all their money in the sooner payment.

For UNCORRELATED treatment, due to the independence of risks in the two periods, the utility maximization problem is the following

\[
\begin{align*}
\max_{x,y} & \quad 0.5x + 0.5\delta y - \phi \left(0.25x^2 + 0.25y^2\right) \\
\text{s.t.} & \quad rx + y = 100
\end{align*}
\]

The optimal solution is

\[
x^* = \frac{(1-\delta)\phi + 100r}{1 + r^2}
\]

If there is no discounting (\(\delta = 1\)) and price is the same (\(r = 1\)), we shall have \(x^* = y^* = 50\). It is not straightforward to see the change of \(rx^*\) with respect to \(r\). To explore the comparative statics, we can calculate the

\[
y^* = \frac{100 - (1-\delta)r}{1 + r^2} \frac{\phi}{\phi}, \text{ which is an increasing function of } r.
\]

For POSITIVE treatment, we have the following optimization problem

---

\(^4\) One can check that \(\delta r^2 - r\) is an increasing function when \(r > 1\). This makes the overall value an increasing function of \(r\).
\[
\begin{align*}
\max_{x,y} & \quad 0.5x + 0.5\delta y - \phi \left(0.25(x+y)^2\right) \\
\text{s.t.} & \quad rx + y = 100
\end{align*}
\]

The optimal solution is $x^* = \frac{(1-\delta)r}{\phi + 100(r-1)}$. \footnote{Notice that we may have corner solution here, as we will point out later.} Similarly, for NEGATIVE treatment, the maximization problem becomes

\[
\begin{align*}
\max_{x,y} & \quad 0.5x + 0.5\delta y - \phi \left(0.25(x-y)^2\right) \\
\text{s.t.} & \quad rx + y = 100
\end{align*}
\]

The optimal solution is given by $x^* = \frac{(1-\delta)r}{\phi + 100(r+1)}$.  

It is not easy to examine the effects of change in $r$ on different optimal solutions. However, we can still have some sensitivity analysis. When $r = 1$, we have the following

\[
\begin{align*}
x^\ast_{\text{UNCORRELATED}} &= \frac{(1-\delta)}{\phi + 200}, \quad x^\ast_{\text{POSITIVE}} = 100 \quad \text{and} \quad x^\ast_{\text{NEGATIVE}} = \frac{(1-\delta)}{\phi + 200}.
\end{align*}
\]

Hence we have $x^\ast_{\text{NEGATIVE}} \approx x^\ast_{\text{UNCORRELATED}} < x^\ast_{\text{POSITIVE}}$ if $1-\delta$ is close to $\phi$.

Moreover, it is not difficult to observe that the effects of an increase in $r$ in the numerators of these three optimal solutions are the same, while the effects of such an increase in the denominators are different. Indeed, the effects on $x^\ast_{\text{POSITIVE}}$ is the most sensitive, followed by $x^\ast_{\text{UNCORRELATED}}$, and the effect on $x^\ast_{\text{NEGATIVE}}$ is the most insensitive.

This actually suggests a “cross-over” effect of the optimal solutions. Specifically, the optimal allocation to sooner payment in POSITIVE treatment is the highest when the relative
price is low, and the marginal effect of a change in relative price on the change of optimal allocation is also the greatest in POSITIVE treatment, hence making it possible to have a greater allocation in sooner payment in POSITIVE treatment when the relative price is low, but a lower allocation in sooner payment in POSITIVE treatment when the relative price is high, relative to the allocations in sooner payment in the other two treatments. Indeed, we have numerical example which shows that under some specific parameter values, there is a cross over between the optimal allocations $rx^*$ in POSITIVE treatment and NEGATIVE/UNCORRELATED treatment, and allocations in NEGATIVE treatment and UNCORRELATED treatment are similar in the example.  

We shall also consider some other prevailing models. The first one is the conventional separable additive expected utility. The utility for SURE treatment is $u(x) + \delta u(y)$, while the utilities for the other 3 treatments are the same: $0.5u(x) + 0.5\delta u(y)$. Hence, given the same budget constraint, the optimal solutions in all four treatments would be the same. This is the so-called common ratio property in time as in Andreoni and Sprenger (2012a).

Moveover, this result also holds for a broader class of utility functions, like Kreps and Porteus (1979), Chew and Epstein (1989), and Halevy (2008). Epstein and Zin (1991) generalize Kreps and Porteus (1979) to have the following recursive expected utility function

$$V(z_t) = E_t \left( c_t^\alpha + \beta E \left( z_{t+1}^{\nu} \right)^{\alpha/\rho} \right)$$

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6 See Appendix B.
The key difference between this representation and the traditional one is that it permits different expected utility functions in every stage, thus leading to a preference for different timing of resolution of temporal risk. However, the optimal solutions for four of our experiments would still be the same if we were to use this recursive utility function. The reason is that all the risks revealed in stage 1 in our experiments, hence Kreps and Porteus (1979) cannot generate differences among the latter three treatments. Moreover, one can check that common ratio effect also holds.

Besides recursive expected utility and Kreps and Porteus, Epstein Zin (1991) gives another class of recursive additive utility functions, which is recursive non-expected utility. Technically speaking, the functional forms in this class has no restrictions, utility functions in every stage could be Weighted Utility (Chew, 1983), Rank Dependent Utility (Quiggin, 1982) or other general non-expected utility functions (Starmer, 2003 for a review). Suppose we have recursive rank dependent utility, we will have the following

\[
V_{UNCORRELATED} = f(0.5)u(x) + f(0.5)\delta u(y) \\
V_{NEGATIVE} = f(0.5)u(x) + (1 - f(0.5))\delta u(y) \text{ if } u(x) > \delta u(y) \\
V_{POSITIVE} = f(0.5)(u(x) + \delta u(y)) \\
V_{SURE} = u(x) + \delta u(y)
\]

Hence it will imply that the optimal allocations in treatments UNCORRELATED, POSITIVE and SURE are the same. The specification of recursive weighted utility is not easy; hence we skip the detailed analysis here. However, similar to recursive rank dependent utility, the class of recursive non-expected utilities may exhibit the desire for diversification.

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7 Note that we may have a difference between Negative and the other 3 if probability weighting function \( f \) is not the identical mapping.
across time \((V_{\text{NEGATIVE}} > V_{\text{POSITIVE}})\), but through a different mechanism, which is the distortion of probability weights.

Halevy (2008) observes that the certainty effect in Kahneman and Tversky (1979) in a static setting could be extended to the time setting to account for the present bias. Halevy (2008) then applies the idea of “uncertain” future together with a probability weighting function to evaluate contingent consumption plans. His representation is the following

\[
V(c_i, r) = \sum g \left( \left( 1 - r^t \right)^\gamma \right) \beta^t E(\mu(c_i))
\]

where \(r\) represents the “uncertainty” of the future and \(g\) is a probability weighting function as in Quiggin (1982). Halevy (2008) points out that this function can exhibit hyperbolic discounting through the desire for “certain” consumptions in the current period. However, applying his representation to evaluate our treatments would deliver the same predictions as recursive expected utility.

There are some other models violating separable additivity that could be considered here (see Fredericgke, Loewenstein, O’Donoghue, 2002 for review). Basically most of them allow for history dependent utilities. For example, habit formation models assume a history dependent utility function in which the cross derivative with respect to current consumption and past consumption is positive. However, such a utility function will predict preference for option B over A in our motivating example, which is inconsistent with our intuition. One can consider the other models, like reference point models, models incorporating anticipation, etc, which provide different intuitions compared to our model in this study.
3. **Experimental Design**

In this section we describe in details the design of our experiments. We adopt the Convex Time Budgets (CTB) design proposed in Andreoni and Sprenger (2012a; 2012b). In each CTB decision, subjects are given a budget of experimental tokens to be allocated across a sooner payment at time $t$, and a later payment at time $t + k$. We include 2 time menus; and four risk-related treatments. For each treatment, subjects are to make 7 choices with different interest rates. This gives rise to 56 experimental decisions ($2 \times 4 \times 7$).

We include two time menus. In the first menu, sooner payments are one week from the experiment date, and the later payments are five weeks from the experiment date. We use this front-end-delay to avoid factors associated with “present” such as transaction cost (Holcomb and Nelson, 1992). In the second menu, sooner payments are 16 week from the experiment date, and the later payments are 20 weeks from the experiment date. We do not fix the early date the same as the first menu, as we are to control sub-additivity in eliciting time preference (Read 2001). The choice of time is set to avoid public holidays, weekends, and examination weeks.

In each CTB decision, subjects are given a budget of 100 tokens. Tokens allocated to the sooner date have a value of $a_t$ while tokens allocated to the later date have a value of $a_{t+k}$. In all cases, $a_{t+k}$ is $0.20$ per token and $a_t$ varies from $0.20$ to $0.14$ per token. The daily net interest rates in the experiment varied considerably across the basic budgets, from 0 to 1.28 percent, implying annual interest rates of between 0 and 2116.6 percent (compounded quarterly), which is the same for both menus as the time gap is the same.

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8Harrison and Swarthout (2011) point out that this income-indifference method is proposed in Cubitt and Read (2007), which have been popular in the Fill-In-the-Blank (FiB) literature.
We include a certainty condition (SURE), in which both payments will be delivered for sure, and three risk conditions, in which each payment will be delivered with a 50% chance. Within the three risk condition, there are three possible correlations in risks across time. In treatment with perfectly positively correlated risks (POSITIVE), there is a 50% chance that both payments will be received, otherwise no payment for either sooner or later. In the treatment with perfectly negatively correlated risks (NEGATIVE), there is a 50% chance that sooner payment will be received, and otherwise later payment will be received. In the treatment with uncorrelated risks (UNCORRELATED), there is a 50% chance that sooner payment will be received, otherwise it will be not received; independently, there is a 50% chance that later payment will be received, and otherwise it will be not received. The uncertainty is resolved at the end of the experiment, to avoid preference for timing of uncertainty resolution.

One difficulty in eliciting time preference with monetary incentive is the credibility of payment. To control for this, we use post-dated cheques issued by a local bank for payment (Chark, Chew, and Zhong, 2012). Subjects are paid with post-dated cheques which were not honored by the local bank when presented prior to the date indicated. To further control the transaction cost difference in time, we borrow the design in Andreoni and Sprenger (2012b), in which subjects are told that they would receive a $12 minimum payment for participating, to be received in two payments: $6 sooner and $6 later. All experimental earnings are added to these $6 minimum payments.

At the end of the experiment, one choice out of the 56 choices is randomly drawn by dices, the so-called Random Lottery Incentive Method (RLIM). Subjects are told to treat each decision as if it were to determine their payments. Starmer and Sugden (1991) study the
RLIM and test the ROCL axiom. They find that the behaviors of the subjects are inconsistent with ROCL. In a recent study by Harrison, Martínez-Correa and Swarthout (2011), they test the violation of ROCL in RLIM by comparing it with the 1 in 1 method. The evidence is mixed evidence for supporting the validity for RLIM. Nevertheless, it is simple to understand and efficient to collect data, which makes it is very popular in experimental economics.

Forty six undergraduate students were recruited as participants using advertisement posted in Integrated Virtual Learning Environment in the National University of Singapore. The experiment was done using paper and pencil at the lab of Center for Behavioral Economics at National University of Singapore. The experiment was conducted by the authors and one research assistant. The experiment consisted of 2 sessions. After subjects arrived at the experimental venue, they were given the consent form approved by the institutional review board at National University of Singapore. Subsequently the general instructions were read to the subjects, and we demonstrated one example before they started making decisions. The experimental instructions follow closely that in Andreoni and Sprenger (2012b) (See Appendix B for Experimental Instructions). Finally, in order to avoid the possible order effect, we randomized the order of decision sheets (8 decision sheets) for each of the subjects. Most of the subjects finished the decision making tasks within 30 minutes. At the end of the experiment, they came to the experimenters one by one, tossed the dices and got paid based on their choice and luck. On average, subjects were paid SGD22 including SGD12 show-up fee.
4. Results

Figure 1 summarizes the mean of allocation to earlier payment and the standard errors of aggregate behavior for the four treatments across the interest rates and time menus. The graph shows apparent distinction between UNCORRELATED/NEGATIVE treatment and SURE/POSITIVE treatment. Note that we also replicate the cross-over behavior between SURE and UNCORRELATED, as reported in Andreoni and Sprenger (2012a). Moreover, the allocation is similar for treatment UNCORRELATED treatment and NEGATIVE treatment, and it is also similar for treatment SURE treatment and POSITIVE treatment.

![Figure 1. Aggregate behavior across four conditions. The figure presents the allocation to sooner payment for four treatments: UNCORRELATED in blue; NEGATIVE in red; POSITIVE in green; SURE in purple. The error bars are standard errors of the mean.](image)

We test the differences across treatments including SURE versus POSITIVE, UNCORRELATED versus NEGATIVE, and UNCORRELATED/NEGATIVE versus SURE/POSITIVE. We regress allocation on treatment (binary variable), interest rate (7 ordered category variable), and time menu (binary variable), and the all interactions of these three variables. The results are reported in Table 1 below. We use F-tests to examine the null hypothesis.
that the treatment related terms have zero slopes. No significant difference was found between treatments UNCORRELATED and NEGATIVE ($p > 0.184$), and marginal significant difference was found between treatments SURE versus POSITIVE ($p < 0.068$). In contrast, the difference between treatments UNCORRELATED/NEGATIVE and SURE/POSITIVE is highly significant ($p < 0.001$). Under discounted expected utility, preference for uncertainty resolution models, and prospect theory, the allocations across the four treatments should be identical. Our results reject these models while our proposed model predicts the overall patterns.

We further test the effect of time using the similar F-test. Should there be a significant difference in allocation in the two time menus, the decision makers would exhibit hyperbolic discounting. Marginal significant difference in allocation would find for SURE/POSITIVE treatment ($p < 0.099$), but not for UNCORRELATED/NEGATIVE treatment ($p > 0.505$) (Table 1). We separate treatments SURE and POSITIVE to test the difference between the two time menus, and we find significant difference for the SURE treatment ($p < 0.010$), and marginal significant for the POSITIVE treatment ($p < 0.123$). These suggest the evidence for hyperbolic discounting for the SURE treatment, in sharp contrast with Andreoni and Sprenger (2012a; 2012b).
<table>
<thead>
<tr>
<th></th>
<th>NEGATIVE versus UNCORRELATED</th>
<th>SURE versus POSITIVE</th>
<th>UNCORRELATED/NEGATIVE versus SURE/POSITIVE</th>
</tr>
</thead>
<tbody>
<tr>
<td>interest rate</td>
<td>351.339*** (102.786)</td>
<td>761.235*** (76.655)</td>
<td>710.863*** (68.771)</td>
</tr>
<tr>
<td>time</td>
<td>1.526 (13.732)</td>
<td>-5.387 (9.921)</td>
<td>-3.770 (8.423)</td>
</tr>
<tr>
<td>treatment x interest rate</td>
<td>-122.842* (61.394)</td>
<td>-100.744* (59.276)</td>
<td>-543.787*** (79.921)</td>
</tr>
<tr>
<td>treatment x time</td>
<td>-1.852 (8.711)</td>
<td>3.233 (10.076)</td>
<td>2.517 (10.672)</td>
</tr>
<tr>
<td>interest rate x time</td>
<td>-28.646 (82.504)</td>
<td>-21.429 (56.880)</td>
<td>-18.489 (49.920)</td>
</tr>
<tr>
<td>treatment x interest rate x time</td>
<td>23.289 (19.428)</td>
<td>5.878 (61.465)</td>
<td>24.777 (62.497)</td>
</tr>
<tr>
<td># observations</td>
<td>1344</td>
<td>1344</td>
<td>2688</td>
</tr>
<tr>
<td># clusters</td>
<td>48</td>
<td>48</td>
<td>48</td>
</tr>
<tr>
<td>R-square</td>
<td>0.033</td>
<td>0.149</td>
<td>0.207</td>
</tr>
<tr>
<td>F-test: risk (p-value)</td>
<td>0.184</td>
<td>0.068</td>
<td>0.001</td>
</tr>
<tr>
<td>F-test: time (p-value)</td>
<td>0.505</td>
<td>0.099</td>
<td>0.254</td>
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<tr>
<td>F-test: interest rate (p-value)</td>
<td>0.005</td>
<td>0.001</td>
<td>0.001</td>
</tr>
</tbody>
</table>

Table 1. Regression Results. The table reports regression coefficient with clustered standard errors. In the last three rows, we test the effect of risk, time and interest rate using F-test, and the p-values are reported in respective row.

5. Discussion

5.1. Certainty Effect and Common Ratio Effect in Time

It has long been recognized the shared similarity between inter-temporal choice and risky decision (e.g., Prelec and Loewenstein, 1991). In particular, the uncertainty effect could be a cause for the present bias. For example, Keren and Roelofsam (1995) observe that
when uncertainty is introduced to the inter-temporal decision making setting, e.g., 90% chance of getting $100 now versus 90% chance of getting $120 one week later, the present bias disappears. Halevy (2008) incorporates this intuition into the prospect theory, and provides a theoretical foundation linking the present bias and the certainty effect.

Subsequently, Andreoni and Sprenger (2011) test the common ratio effect of separable additive expected utility function together with models with timing of uncertainty resolution and the model in Halevy (2008). In their design, they use 'front-end-delay' to avoid the impact of immediacy/certainty on decisions. They originated the Convex Time Budget design as we detailed in the experimental design section. They compare allocation decision between a \( p \) chance of receiving the earlier payment and a \( q \) chance of receiving later payment, and allocation decision between \( 0.5p \) chance of receiving the earlier payment and \( 0.5q \) chance of receiving later payment. (denoted as \( (p, q) \) treatment) They have three treatments with \((1, 1)\), \((1, 0.8)\), and \((0.8, 1)\). The key difference from ours is that risks between the two periods are always independent. Notice that \((1, 1)\) treatment is the same as our SURE/POSITIVE treatment. The separable additive expected utility function predicts that the optimal allocations are the same for \((p, q)\) and \((0.5p, 0.5q)\). However, they observe a similar cross-over for \((1, 1)\) treatment, and identify a preference to allocate more to the certain payments regardless of \((0.8, 1)\) or \((1, 0.8)\), relative to common ratio counterparts.

They interpret their results as being driven by the certainty effect, similar to that in static environment, similar as the observations in Allais (1953) and Kahneman and Tversky (1979) in the risk setting. Our model offers a distinct perspective: their observation could be due to preference for diversification across time. In our set-up, the agents have needs for
diversification whenever there is uncorrelated risk involved in both periods, e.g., (0.5, 0.4) or (0.4, 0.5), and while the need for diversification is reduced when the payment in one time point is certain, e.g., (1, 0.8), or (0.8, 1). This leads to allocate more to the certain payments in the case of (1, 0.8) or (0.8, 1), relative to the case of (0.5, 0.4) or (0.4, 0.5).

If we were to include (1, 0.8) and (0.5, 0.4) in our experiment, we would add positive correlated risk and a negative correlated risk treatments. We can generate a positive correlated risk for (0.5, 0.4) as follows: there is a 10-sided dice numbered from 1 to 10, and one tosses the dice once. The sooner payment will be received if the number drawn is between 1 and 5; and later payment will be received if the number drawn is between 1 and 4. Similarly, we can generate a negative correlated risk for (0.5, 0.4) as follows: The sooner payment will be received if the number drawn is between 1 and 5; and later payment will be received if the number drawn is between 7 and 10. Based on our model as well as the observations in the experiment, the allocation would be similar for the (1, 0.8) treatment and the positive correlated risk treatment; and similar for the negative correlated risk treatment and the uncorrelated risk treatment.

5.2. Hyperbolic Discounting

Another interesting observation in our study is the supporting evidence for hyperbolic discounting. This is in contrast with increasingly experimental studies with careful control and yet suggesting the lack of evidence for hyperbolic discounting (see Andersen et al., 2011 for a review). In particular, Andreoni and Sprenger (2012b) propose the CTB design with three sooner payment dates, t = (0, 7, 35), and three later dates, t = (30, 70, 98), while they do not find support for hyperbolic discounting. They hypothesize that this may be the result of careful controls that they took to equate transaction costs of
sooner and later payment and to increase confidence of receiving future payments. However, we used the similar experimental controls, and yet we do observe hyperbolic discounting.

The key difference between our design and theirs is that the time gap is always 4 weeks for both early menu and later menu in ours, while it varies in Andreoni and Sprenger (2012b), which in principle would decrease the chance of observing hyperbolic discounting if we consider the sub-additivity in time as proposed in Read (2001). In addition, Chew, Chark and Zhong (2011) observe hyperbolic discounting in a large sample from China, Hong Kong and Singapore, when the same delays are posited in the near future (7 days against 35 days) and further future (6 months and 7 months). Another factor which may have contributed to the distinct findings between ours and Andreoni and Sprenger (2012b) finding may have something to do with cultural differences. In a large-scale survey on time discounting conducted in 45 countries, Wang, Rieger, and Hens (2010) found strong evidence for cultural differences. Future studies are called for to understand the different results across studies.

Moreover, we observe evidence of hyperbolic discounting in the SURE and POSITIVE treatments, but not in the NEGATIVE and UNCORRELATED treatments. We posit that this could be due to the reason that the need for diversification leads to insensitivity to interest rate in the NEGATIVE and UNCORRELATED treatments. Takeuchi (2011) proposes a novel experimental design, in which the amount of payment is always fixed, while there is uncertainty on the time point for delivering the payment. Under discounted expected utility, his design allows calibration of the curvature of time discounting function. His design essentially corresponds to our NEGATIVE treatments. Interestingly he observes the
discounting curve as being initially convex and concave subsequently. It would be of interest to study the time discounting function in his design with different correlations across time. Hayashi (2003) and Olea and Strzalecki (2011) give characterizations of quasi-hyperbolic discounting, which is not within the specifications of our model in this current study. We are reluctant to do so since the findings in experimental literature have not yet been consolidated, and the interaction between discounting and desire for diversification across time would complicate the results in our treatments. We certainly are interested to explore further this interaction in our future studies.

5.3. Primary reward and monetary reward.

In most of the experiments about intertemporal choice, monetary reward is used to elicit time preference, which appears to be incoherent with the consumption-based theories. In some other works, Cubitt and Read (2007) observe that if subjects have access to external capital market, they can reschedule consumption spending relative to income. Therefore the discount rates inferred from monetary reward could cause misleading inferences about time preferences for consumption. Following up this argument, Harrison and Swarthout (2011) propose a direct experimental test, and show that these configurations of preferences are empirically implausible although theoretically sensible. Reuben, Sapienza and Zingales (2010) argue that if the discount rates inferred from monetary rewards and consumption goods are significantly correlated, the measurement through monetary reward might be ecologically valid. In this experiment, they have elicited discounting rate for both monetary rewards and primary reward (chocolate), and find a positive and statistically significant relation between short-term discount rates elicited with a monetary and a primary reward. Moreover, neuroeconomic studies show similar neural mechanism for time
preference over monetary reward and primary reward (McClure et al. 2007). These evidence overall support that preference inferred from intertemporal experiments using monetary reward could be a good proxy for preference for consumption goods.

However, the model we propose in this study is based on monetary reward and it may be subject to the concern that decision maker do not have enough incentives for smoothing monetary payoffs over time. Indeed, the data we observe in treatments SURE and POSITIVE tends to suggest that decision maker are almost neutral over money between different periods. Nevertheless, smoothing and diversification across time are separated in our experimental study. One can think of the effects of smoothing in treatments POSITIVE and NEGATIVE, they should be the same while the reason that causes the differences is actually preference for diversification across time. To sum up, the key point we are trying to argue here is that the correlation matters, not only for monetary payoffs, maybe also for consumption goods, and it will certainly be interesting to explore the effects of correlation between consumptions in future works.⁹

At last, we also would like to point out that our model essentially is similar to the following mean-variance model in finance for uncertain flows of payoffs:

\[ V(w) = E\left(\sum_i \delta^i \tilde{w}_i\right) - \phi \text{var}\left(\sum_i \delta^i \tilde{w}_i\right), \]

or a model with the following specification where \( u \) is concave

\[ V(w) = Eu\left(\sum_i \delta^i \tilde{w}_i\right). \]

---

⁹ One can interpret the monetary rearwards as certainty equivalents of consumption, and hence extending this current model to consumption goods. However, as we point out in appendix, extending the current work to the domain of infinite time horizon contingent consumption plans is non-trivial.

¹⁰ We thank Matthew Rabin for pointing out the second model. The mean-variance model could be seemed as one particular specification of the latter model. However, the latter model assumes additivity across time while risk aversion ex-ante, which we think may not be appealing.
6. Conclusion

In this paper, we investigate preference for diversification across time. We provide a behavioral model with axiomatic foundations that can explain this preference. We later test our model through experimental instruments. In our experiment, subjects are asked to allocate payments in two periods with variants of correlations patterns for risks. When we compare perfectly negatively correlated or uncorrelated risks, to perfectly positively correlated risks or sure, we find that subjects tend to allocate less money to sooner payment when interest rate is low and to allocate more money to sooner payment when the interest rate is high. This suggests that when risks are negatively correlated or uncorrelated, there is a stronger need for diversification. Our paper contributes to the understanding of the interplay between risk and time preference. We would like to explore this issue in more details using consumption goods, both theoretically and experimentally.

Reference.


Appendix A: Axiomatization Foundation of the Model

Let $Z$ be a compact subspace of $\mathbb{R}_+$. $T$ is a finite number which represents the periods of time, and let $Z^T$ be the product space. The set of states is a continuous probability space $(\Theta, \Sigma, \mu)$, where $\mu$ is convex-valued on a sigma algebra of events $\Sigma$. A contingent consumption plan $f$ is a mapping from $\Theta \times T$ to $Z$. The domain of preference is the set of all consumption plans $\mathcal{F}$.

For every $f$, define $w(f, \theta) = \sum_t f(t, \theta)$ be the aggregate wealth allocated in state $\theta$, while $\bar{w}_\theta$ the corresponding vector of allocations. And let $w(f, t) = \int f(t, \theta) d\mu(\theta)$ be the aggregate wealth allocated in period $t$, while $\bar{w}_t$ the corresponding random variable of allocations in period $t$. Moreover, let $x(t, \theta) = f(t, \theta)/w(f, \theta)$ be period $t$’s share of total wealth in state $\theta$, while $\bar{x}_t$ the corresponding random variable of shares in period $t$.

Chew, Epstein and Segal (1991) show that if the preference relationship $\succeq$ satisfies Continuity, Monotonicity, proper Quasiconcavity (convexity) and Mixture Symmetry, there exists two regions of $\mathcal{F}$, one in which the indifference surface is linear, and the other strictly concave (convex) and quadratic. Later, Chew and Sagi (2011, henceforth CS) apply this framework for quadratic utility to evaluate stochastic allocations over a population and they obtain generalized-Gini index as a special case where there is not mixture symmetry. Their final representation form not only contains the generalized-Gini part, but also a variance-like part that captures the idea that people prefer “shared” destiny when facing uncertain payoffs. The domain in their paper the set of mappings from $\Theta \times N$ to $\mathbb{R}_+$ and their representation allows for the following preference relations for distributions on $\Theta \times N$.
space, where the rows represent two equally-likely events and the columns represent two equally-sized cohorts.

\[
\begin{pmatrix}
\frac{z}{2} & \frac{z}{2} \\
\frac{z}{2} & \frac{z}{2}
\end{pmatrix} \succeq \begin{pmatrix}
z & z \\
0 & 0
\end{pmatrix} \succeq \begin{pmatrix}
z & 0 \\
0 & z
\end{pmatrix} \succeq \begin{pmatrix}
z & 0
\end{pmatrix}
\]

Note that the second preference is due to what they named “preference for shared destiny”. Their representation is the following

\[
V(f) = \sum \gamma_i E(\hat{w}_i) - \phi \sum E(\hat{x}_i \hat{w}_i)
\]

Applying this to the time setting, we see that what we need is actually “aversion for shared destiny” to describe the diversification needs over time cross risk plane.

By changing the Quasiconcavity to Quasiconvexity, we can partially tender our purpose and obtain the following representation

\[
V(f) = \sum \gamma_i E(\hat{w}_i) + \phi \sum E(\hat{x}_i \hat{w}_i)
\]

However, there are a few things to note about this representation. The key one is that this representation is still linear in states due to Axiom 3 in CS, which would imply the following

\[
\begin{pmatrix}
z & 0 \\
0 & 0
\end{pmatrix} \sim \begin{pmatrix}
0 & 0 \\
z & 0
\end{pmatrix} \sim \begin{pmatrix}
z/2 & 0 \\
z/2 & 0
\end{pmatrix}
\]

This is obviously not desirable in our setting since we believe the agents should have needs for diversification over risk alone. The other one is linked the first point, as the agent normally is risk averse, we actually do not have Quasiconvexity over the whole domain.
Hence, in the following, we give our simple characterization of a mean-variance model in the domain of all uncertain allocations.

First we impose the following basic conditions.

B1 (Ordering) $\succeq$ is complete, transitive and continuous.

B2 (Null Allocations) If $f$ and $g$ differ only in a set of $\mu$-measure 0, then $f \sim g$.

B3 (State Independence) Suppose $f$ and $g$ have the same joint distribution over $\Theta \times T$, then $f \sim g$.

B4 (Time Independence) Let $f$ be an allocation such that $f_t = f_{t'}$, and let $f'$ be obtained by swapping $f_t$ and $f_{t'}$, we have $f \sim f'$.

B5 (Discounting) Let $f$ be a deterministic allocation $(C_1, C_2, ..., C_T)$ and let $g$ be obtained by transferring a positive amount from every state in period $t$ to $t + 1$, then $f \succeq g$.

Additionally, we impose the following behavior axioms.

Monotonicity: $\succeq$ is increasing in the sense of first degree stochastic dominance.

Mixture Symmetry: For every $f$ and $f'$, and any $\alpha \in (0, \frac{1}{2})$, $f \sim f' \Rightarrow \alpha f + (1 - \alpha)g \sim \alpha f' + (1 - \alpha)g$.

Equal-Mean Quasiconcavity: For every $f$ and $f'$ such that $w(f, t) = w(f', t)$ for every $t$, $f \sim f' \Rightarrow \alpha f + (1 - \alpha)f' \succeq f$. 
Equal-Mean Time Independence: For every \( i \) such that \( w(f,t) = w(f,t') \) for some \( t \) and \( t' \), then permuting \( \tilde{w}_t \) and \( \tilde{w}_{t'} \) to get \( f' \) and we have \( f \sim f' \).

Certainty Independence: For every \( i \) and \( f' \) such that they allocation constant payments in all periods, \( f \sim f' \Rightarrow \alpha f + (1 - \alpha)f' \sim f \). We have the following theorem.

Theorem: A preference relationship \( \succsim \) satisfies B1-B4, Axiom 1-4 iff it can be represented by the following

\[
V(f) = \sum \gamma_i E(\tilde{w}_i) - \phi \text{var} \left( \sum \tilde{w}_i \right)
\]

where \( \gamma_1 > \gamma_2 \ldots > \gamma_T > 0 \), and \( \phi \geq 0 \).

Proof: First we deal with the simple case, where the state space is divided into \( S \) equal measure events. Denote this simplified domain as \( X(N) \) and elements of it as \( x_i \), also label the payoff in state \( u \) and time \( t \) as \( x_{st} \). Given Ordering, State Independence, Continuity and Mixture Symmetry, CS show that the preference restricted to \( X(N) \) can be represented by the following

\[
\sum \gamma_i \left( \frac{1}{S} \sum x_{st} \right) + \sum \phi_i \left( \frac{1}{S} \sum x_{st} x_{st}' \right) + \sum \phi_i' \left( \frac{1}{S} \sum x_{st} \right) \left( \frac{1}{S} \sum x_{st}' \right) + \rho
\]

We would like to note that CS obtain the above representation in a more restricted domain named mean-commontonic cone. When restricted to this domain, our axiom State Independence coincides with the axiom Symmetry and State Independence in CS, hence making it possible to apply Proposition A.1 in CS. Moreover, we ignore the discussions of different regions (quasi-concave, quasi-convex and linear) in this current study, actually our
representation result holds generally in the quadratic region, while the linear region is of no interest to this current study.

Now, since we have independence in the dimension of time when the allocations are certain, hence we have $\phi_{tt} + \varphi_{tt} = 0$ for all $t, t'$. Hence the representation in the quadratic region becomes

$$\sum_t \gamma_t \left( \frac{1}{S} \sum_s x_{st} \right) + \sum_{st} \phi_{st} \left( \frac{1}{S} \sum_s x_{st} x_{s't} - \frac{1}{S^2} \sum_s x_{st} \sum_s x_{s't} \right) + \rho$$

Next consider an allocation $\mathbf{x}$ which allocates a constant $w$ in all states and periods except for $st, s't$. Also let $x_{st} + x_{s't} = 2w$, and permute $\mathbf{x}$ by changing the allocations in state $s$ and $s'$ to get $x'$. By State Independence, we have $\mathbf{x} \sim \mathbf{x}'$, and we have $\frac{x}{2} + \frac{x'}{2} \geq x$ by Equal-Mean Quasiconcavity. Subtracting the utility of $\mathbf{x}$ from that of $\frac{x}{2} + \frac{x'}{2}$, we get the following

$$\phi_{st} \left( \frac{1}{S} \left( 2w^2 - x_{st}^2 - x_{s't}^2 \right) \right)$$

For this to be always greater than 0, we have $\phi_{tt} < 0$.

Similarly, consider an allocation $\mathbf{x}$ which allocates a constant $w$ in all states and periods expect for $st, s't, st'$, $s't'$. Also let $x_{st} + x_{s't} = x_{str} + x_{s'tr} = 2w$, $x_{st} = x_{str}$, and $x_{s't} = x_{s'tr}$. Then permute $\mathbf{x}$ by changing the allocations in state $s$ and $s'$ to get $x'$. By State Independence, we have $\mathbf{x} \sim \mathbf{x}'$, and we have $\frac{x}{2} + \frac{x'}{2} \geq x$ by Equal-Mean Quasiconcavity. Subtracting the utility of $\mathbf{x}$ from that of $\frac{x}{2} + \frac{x'}{2}$, we get the following
\[
2\phi_{tt}\left(\frac{1}{S}\left(2w^2 - x_{st}x_{s't'} - x_{st}'x_{s't'}\right)\right) + \phi_{tt'}\left(\frac{1}{S}\left(2w^2 - x_{st}^2 - x_{s't'}^2\right)\right) + \phi_{t't'}\left(\frac{1}{S}\left(2w^2 - x_{st}^2 - x_{s't'}^2\right)\right)
\]

For this to be always greater than 0, one must have \(\phi_{tt} + \phi_{tt'} + 2\phi_{tt'} \leq 0\).

At last, consider and allocation \(\mathbf{x}\) such that pays 1 in all periods except \(t\) and \(t'\). In addition, assume \(w(f, t) = w(f, t')\) and permute \(\mathbf{x}\) by swapping \(\tilde{w}_t\) and \(\tilde{w}_{t'}\) to get \(\mathbf{x}'\). By Equal-Mean Independence Axiom, we have \(\mathbf{x} \sim \mathbf{x}'\). The difference between the utility of \(\mathbf{x}\) and \(\mathbf{x}'\) is the following

\[
(\phi_{tt} - \phi_{t't'})\left(\frac{1}{S}\left(\sum_{t}x_{st}^2 + x_{st'}^2\right)\right) + \sum_{t'\neq t, s, t'}(\phi_{tt'} - \phi_{t't'})(\frac{1}{S}\sum_{t}(x_{st} + x_{st'}))
\]

For this to always be 0, we must have \(\phi_{tt} = \phi_{t't'}\) and \(\phi_{tt'} = \phi_{t't'}\). Hence we have the following representation

\[
\sum_{t'} \gamma_{t'} \left(\frac{1}{S}\sum_{t}x_{st}\right) + \sum_{t}\phi\left(\frac{1}{S}\sum_{t}x_{st}x_{st'} - \frac{1}{S^2}\sum_{t}s\sum_{t'}x_{st}\right)
\]

Finally, due to the fact that \(X(N)\) is dense in \(\mathcal{F}\), we have the following representation in \(\mathcal{F}\)

\[
V(f) = \sum_{t} \gamma_{t} E(\tilde{w}_t) + \phi \sum_{t} (E\tilde{w}_t^2 - (E\tilde{w}_t)^2) - \phi \sum_{t'} (E\tilde{w}_{t'}\tilde{w}_t - E\tilde{w}_t E\tilde{w}_{t'})
\]

Simple rearrangement delivers

\[
V(f) = \sum_{t} \gamma_{t} E(\tilde{w}_t) - \phi \text{var}\left(\sum_{t} \tilde{w}_t\right)
\]
where $\phi \geq 0$. By Discounting, we have $\gamma_1 > \gamma_2 \ldots > \gamma_T$. Monotonicity additionally requires all the $\gamma$ being positive. Hence we have the sufficiency of the Theorem. It is straightforward to verify the necessity, so we ignore it here.

At last, there are few things we would like to note here. First, our simple characterization here is incomplete as we have mentioned before, since we do not take into considerations all the regions but only the quadratic region. Second is that the domain we are working on in this section is not the usual space of all contingent consumption streams, defined in the usual recursive way. For example, given a compact set $C$, define $Z_1$ by $\Delta(C)$ and $Z_t$ recursively by $\Delta(C \times Z_{t-1})$. The final domain could be identified as $Z_\infty \sim \Delta(C \times Z_\infty)$. We do not work on this domain due to simplicity considerations. However, it will certainly be interesting for us to explore preferences for diversifications in this standard domain in the future.
Appendix B: A Numerical Example

The following is the numerical optimal solution with the parameters specified as \( \delta = 0.99, \ \phi = 0.01 \). X-axis corresponds to interest rate while Y-axis corresponds to optimal allocation in sooner payments.
$x \frac{(1 - 0.99x)}{0.01 + 100x} / (x^2 + 1)$ optimal solution for treatment Uncorrelated
Appendix C: Experimental Instructions

Welcome to our study on decision making. The instructions are simple and if you follow them carefully and make good decisions, you could earn a considerable amount of money which will be paid to you in cheques before you leave today. Different subjects may earn different amounts of money. What you earn today depends partly on your decisions, and partly on chance. All information provided will be kept confidential. Information in the study will be used for research purposes only. If you have any questions, please raise your hand to ask our experimenters at any time. Cell phones and other electronic devices are not allowed, and please do not communicate with others during the experiment.

Earn Money:

To begin, you will be given $12 as show up fee. You will receive this payment in two payments of $6 each. The two $6 minimum payments will come to you at two different times. These times will be determined in the way described below. Whatever you earn from the study today will be added to these minimum payments.

In this study, you will make 56 choices over how to allocate money between two points in time, one time is sooner and one is later. Both the sooner and later times will vary across decisions. This means you could be receiving payments as soon as one week from today, and as late as 20 weeks from today.

It is important to note that the payments in this study involve chance. There could be a chance that your sooner payment, your later payment or both will not be sent at all. For each decision, you will be fully informed of the chance involved for the sooner and later payments. Whether or not your payments will be sent will be determined at the END of the experiment today. If, by chance, one of your payments is not sent, you will receive only the $6 minimum payment.

Once all 56 decisions have been made, we will randomly select one of the 56 decisions as the decision-that-counts. This will be done in three stages. First, we will pick a number from 1 to 56 at random to determine which one is the decision-that-counts and the corresponding sooner and later payment dates. We will then determine whether the payments will be sent based on chances, which we will describe in details later. Last, we will use the resolved chances to determine your actual earnings.
Note, since all decisions are equally likely to be chosen, you should make each decision as if it will be the decision-that-counts. When calculating your earnings from the decision-that-counts, we will add to your earnings the two $6 minimum payments. Thus, you will always get paid at least $6 at the chosen earlier time, and at least $6 at the chosen later time.

IMPORTANT: We will sign you cheques with the specified date at the end of today’s experiment. Under Singapore banking practices, a cheque can be cashed only on or within 6 months of the date of the cheque. It is very IMPORTANT that you do not try to cash before the date of the cheque, since you will not be able to get the money, and it will also incur a $40 lose for the experimenter.

How it Works:
In each decision you are asked to divide 100 tokens between two payments at two different dates: Payment A (which is sooner) and Payment B (which is later). Tokens will be exchanged for money. The tokens you allocate to Payment B (later) will always be worth at least as much as the tokens you allocate to Payment A (sooner). The process is best described by example.

In the table below, in row 3, each token you allocate to one week later is worth $0.18, while each token you allocate to five weeks later is worth $0.20. So, if you allocate all 100 tokens to one week later, you may earn 100x$0.18 = $18 (+ $6 minimum payment) on this date and nothing on five weeks later (+ $6 minimum payment). If you allocate all 100 tokens to five weeks later, you may earn 100x$0.20 = $20 (+ $6 minimum payment) on this date and nothing on five week later (+ $6 minimum payment). You may also choose to allocate some tokens to the earlier date and some to the later date. For instance, if you allocate 60 tokens to one week later and 40 tokens to five weeks later, then one week later you may earn 60x$0.18 = $10.04 (+ $6 minimum payment) and five weeks later you would earn 40x$0.20 = $8 (+ $6 minimum payment). The Payoff Table shows some of the token-dollar exchanges at all relevant token exchange rates, which applies to all decisions in this experiment.
**Sample Decision Making Sheet**

<table>
<thead>
<tr>
<th>Feb</th>
<th>In Each Row ALLOCATE 100 TOKENS BETWEEN</th>
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<tbody>
<tr>
<td></td>
<td>PAYMENT A (ONE WEEK from today) AND PAYMENT B (FIVE WEEKS from today)</td>
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<table>
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<tr>
<th>No A Tokens</th>
<th>Rate A $ per token</th>
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<th>Rate B $ per token</th>
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In Each Row ALLOCATE 100 TOKENS BETWEEN

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Chance of Receiving Payments:
Each decision sheet lists the chances that each payment will be sent. Each decision in that sheet share the same chances that the payments will be sent. There are four cases.

Case A
If this decision were chosen as the decision-that-counts, both PAYMENT A and PAYMENT B will be sent for sure.

Case B
There are some chance that PAYMENT A and PAYMENT B will not be sent. If this decision were chosen as the decision-that-counts, we would determine the actual payments by throwing TWO ten-sided dices. There is 50% chance that PAYMENT A will be sent by throwing the first dice; there is 50% chance that PAYMENT B will be sent by throwing the second dice. Specifically, if the first dice tossed is odd, PAYMENT A will be sent; otherwise PAYMENT A will not be sent. If the second dice tossed is odd, PAYMENT B will be sent; otherwise PAYMENT B will not be sent.

Case C
There are some chance that PAYMENT A and PAYMENT B will not be sent. If this decision were chosen as the decision-that-counts, we would determine the actual payments by throwing ONE ten-sided dice. There is a 50% chance that both PAYMENT A and PAYMENT B will be sent, determined by the dice. Specifically, if the dice tossed is odd, both PAYMENT A and PAYMENT B will be sent; and there will be no payments if the dice tossed is even.

Case D
There are some chance that either PAYMENT A or PAYMENT B will not be sent. If this decision were chosen as the decision-that-counts, we would determine the actual payments by throwing ONE ten-sided dice. There is a 50% chance that either PAYMENT A or PAYMENT B will actually be sent, determined by the dice. Specifically, if the dice tossed is odd,
PAYMENT A will be sent while PAYMENT B will not be sent; and PAYMENT B will be sent if the dice tossed is even while PAYMENT A will not be sent.

**Things to Remember:**

- You will always be allocating exactly 100 tokens.
- Tokens you allocate to Payment A (sooner) and Payment B (later) will be exchanged for money at different rates. The tokens you allocate to Payment B will always be worth at least as much as those you allocate to Payment A.
- Payment A and Payment B will have different types of chance. You will be fully informed of the chances.
- On each decision sheet you will be asked 7 questions. For each decision you will allocate 100 tokens. Allocate exactly 100 tokens for each decision row, no more, no less.
- At the end of the study a random number will be drawn to determine which is the decision-that-counts. Because each question is equally likely, you should treat each decision as if it were the one that determines your payments. The payments you chose will actually be sent or not will be determined by chance, which is put down on the decision-that-counts.
- Your payment, by cheque, will be given to you today.
**Decision Making Sheet** [Note that other decision making sheets are presented in similar manner.]

In this decision sheet, there is a 50% chance that either PAYMENT A or PAYMENT B will be sent. Specifically, if the dice tossed is odd, PAYMENT A will be sent; and PAYMENT B will be sent if the dice tossed is even.

Please indicate your allocation of 100 tokens in each of the following 7 scenarios.

<table>
<thead>
<tr>
<th>Feb</th>
<th>In Each Row ALLOCATE 100 TOKENS BETWEEN</th>
<th>PAYMENT A (ONE WEEK from today)</th>
<th>AND</th>
<th>PAYMENT B (FIVE WEEKS from today)</th>
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<td>No A Tokens</td>
<td>Rate A $ per token</td>
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*Note: Tokens at $0.20 each one week later & tokens at $0.20 each five weeks later.*