IS BIDDING BEHAVIOR CONSISTENT WITH BIDDING THEORY FOR PRIVATE VALUE AUCTIONS?

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INTRODUCTION

We analyze data from first-price auction experiments with independent private values for consistency with equilibrium bidding models. Three models are considered: the log-concave model (Cox, Smith, and Walker 1988); the constant relative risk averse model (Cox, Roberson, and Smith 1982; Cox, Smith, and Walker 1982); and the risk neutral model (Vickrey 1961). These models are nested in that the log-concave model contains the constant relative risk averse model (hereafter, CRRAM) as a special case, and CRRAM contains the risk neutral model as a special case. We find that: (a) almost all of the data are consistent with the log-concave model; (b) data for about one-half of the subjects are consistent with

Research in Experimental Economics, Volume 6, pages 131-148.
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ISBN: 1-55938-607-X

CRRAM; and (c) data for only 0-10% of the subjects are consistent with the risk neutral model. We also use a loss function (Friedman 1992) to calculate the foregone expected earnings from non-risk-neutral bidding and find that they are significantly greater than zero for 90% of the subjects. Our empirical analysis uses data from some of the first-price sealed-bid auction experiments reported in Cox, Smith, and Walker (1988).

IS BIDDING BEHAVIOR CONSISTENT WITH THE LOG-CONCAVE MODEL?

The log-concave model permits bidders to differ from each other in any way that can be represented by finite-dimensional characteristic vectors that are realizations of a (finite-dimensional) random variable with integrable cumulative distribution function defined on a convex set. In the log-concave model, bidders can be risk averse, risk neutral, or risk-loving; furthermore, an individual bidder can be risk averse for some gambles and risk neutral or risk-loving for others. The only restriction that the model places on risk attitudes is that utility functions must be strictly log-concave. The class of "log-concave functions" is defined as follows. Let ϕ map $S \subset R^n$ into $T \subset R^1_+$. The function ϕ is (respectively strictly) log-concave if ln ϕ is a (respectively strictly) concave function. Note that if ϕ is a differentiable function of a single variable then it is (respectively strictly) log-concave if and only if $\phi'(x)/\phi(x)$ is (respectively strictly) decreasing on the domain of ϕ.

Log-Concave Bid Function

Let $u(y,\theta_j)$ be the utility of monetary income y to bidder j with characteristic vector, θ_j. If bidder j submits the highest bid in amount b_j, and values the auctioned item in amount v_j, then his monetary payoff is $v_j - b_j$. If bidder j does not submit the highest bid then his monetary payoff is 0. Assume that u is strictly increasing and strictly log-concave in y and normalized such that $u(0,\theta_j) = 0$ for all θ_j. Let $G_j(b_j)$ be bidder j's subjective probability that he can win the auction by submitting a bid in the amount b_j. Assume that $G_j(0) = 0$ and that G_j is strictly log-concave on $[0,\bar{b}_j]$.

Then bidder j's's expected utility of a bid in the amount b_j is

$$U_j(b_j) = G_j(b_j)\, u(v_j - b_j,\theta_j). \tag{1}$$

Assuming that U_j is differentiable, one can take logs in equation (1) and differentiate to show that

$$U_j'(b_j)/U_j(b_j) = \rho_j(b_j) - \mu(v_j - b_j, \theta_j) \tag{2}$$

where $\rho_j(b_j) = G_j'(b_j)/G_j(b_j)$, $\mu(v_j - b_j, \theta_j) = u_1(v_j - b_j, \theta_j)/u(v_j - b_j, \theta_j)$, and $u_1(v_j - b_j, \theta_j)$ is the derivative of $u(v_j - b_j, \theta_j)$ with respect to its first argument, $v_j - b_j$. Equation (2) reveals that U_j is strictly log-concave in b_j because G_j is strictly log-concave in b_j and u is strictly log-concave in $v_j - b_j$. Note that $U_j(b_j) > 0$ for all $b_j \in (0, v_j)$ because $G_j(b_j) > 0$ for all $b_j > 0$, $u(0, \theta_j) = 0$, and u is strictly increasing in $v_j - b_j$. Therefore, the strict log-concavity of U_j implies that for each v_j there exists a unique $b_j \in (0, v_j)$ that maximizes U_j.

The maximizing bid for any value is such that equation (2) equals zero; hence for the optimal bid function, $b(v_j, \theta_j)$, one has

$$\rho_j(b(v_j, \theta_j)) = \mu(v_j - b(v_j, \theta_j), \theta_j). \tag{3}$$

Differentiation of equation (3) yields

$$b_1(v_j, \theta_j) = \frac{\mu_1(v_j - b(v_j, \theta_j), \theta_j)}{\rho_j'(b(v_j, \theta_j)) + \mu_1(v_j - b(v_j, \theta_j), \theta_j)} \tag{4}$$

where $b_1(v_j, \theta_j)$ is the derivative of the bid function, $b(v_j, \theta_j)$, with respect to its first argument, v_j. Since μ_1 and ρ_j' are both negative by strict log-concavity, equation (4) implies

$$0 < b_1(v_j, \theta_j) < 1 \tag{5}$$

Now assume that the θ_j, $j = 1, \ldots, n$, are independently drawn from the probability distribution with integrable cdf Φ on the convex set Θ. Also assume that the v_j, $j = 1, \ldots, n$, are independently drawn from the uniform distribution on $[v_\ell, v_h]$. Let $\pi(b, \theta_j)$ be the v - inverse of the bid function, $b(v, \theta_j)$. Assume that the bidders all have the same rational expectations such that

$$G_j(b) = \left[\int_\Theta \frac{\pi(b, \theta) - v_\ell}{v_h - v_\ell} d\Phi(\theta)\right]^{n-}, \quad j = \;, \ldots, n \tag{6}$$

Then the argument in Cox, Smith, and Walker (1988, pp. 62-65) shows that $b(v,\theta)$ is a Bayesian-Nash equilibrium bid function.

Empirical Tests

We tested the data for properties that correspond to containment of individual bid function slopes in the (0,1) interval, as in statement (5). We conducted two types of tests for positive monotonicity of individual bids with respect to values, one based on rank correlation and the other based on fitted cubic equations. First consider the Spearman rank correlation between bids and values, $C_{b,v}$. In order to test the data for consistency with positive monotonicity, we tested the null hypothesis

$$H_0^1\colon\ C_{b,v} > 0. \tag{7}$$

We could not reject the null hypothesis in statement (7) for any subject at conventional levels of significance (p values ≤ 0.1). In addition, we can always reject the hypothesis that excludes positive correlation; that is, the hypothesis

$$H_*^1\colon\ C_{b,v} \leq 0 \tag{8}$$

can be rejected for every subject at conventional levels of significance.

Our second type of test for positive monotonicity involves fitting a cubic equation to the data and then evaluating its slope at each observation. We used OLS to estimate the parameters of

$$b_{it} = \beta_{i1}\, v_{it} + \beta_{i2}\, v_{it}^2 + \beta_{i3}\, v_{it}^3 + \varepsilon_{it} \tag{9}$$

using the observed bids, b_{it}, and values, v_{it}, for each of the 40 subjects.[1] The estimated cubics are good fits to the data: 29 out of the 40 $\bar{R}^2$ exceed 0.99; 36 out of the 40 $\bar{R}^2$ exceed 0.95; and 40 out of the 40 $\bar{R}^2$ exceed 0.90. The estimated parameter variance/covariance matrices obtained from OLS estimation of the cubic bid functions were used to calculate the standard errors for the estimated slopes of these bid functions. From these standard errors, a standard normal approximation was used to test various restrictions on the slopes of the bid functions.

In order to test the data for consistency with positive monotonicity, we tested the null hypothesis

$$H_0^2\text{: } \hat{\beta}_{i1} + 2\hat{\beta}_{i2}v_{it} + 3\hat{\beta}_{i3}v_{it}^2 > 0, \textit{ for each } v_{it}.$$

We could not reject the null hypothesis in statement (10) at 10% significance in 999 of the 1,000 tests. (The number of tests, 1,000, equals the number of subjects, 40, times the number of observations per subject, 25.) We also tested the data for consistency with nonpositive monotonicity; that is, we tested the null hypothesis

$$H_*^2 \quad \hat{\beta}_{i1} + 2\hat{\beta}_{i2}v_{it} + 3\hat{\beta}_{i3}v_{it}^2 \le 0, \textit{ for each } v_{it}$$

The null hypothesis in statement (11) could be rejected, at 10% significance, in 957 of the 1,000 tests. It could be rejected for every v_{it} for 27 out of the 40 subjects.

Our test that bid function slopes are less than one is also based on the estimated parameters of equation (9). To test the data for consistency with the less-than-one slope restriction, we tested the null hypothesis

$$H_0^3 \quad \hat{\beta}_{i1} + 2\hat{\beta}_{i2}v_{it} + 3\hat{\beta}_{i3}v_{it}^2 < 1, \textit{ for each } v_{it}.$$

At the 10% level, the null hypothesis in equation (12) could be rejected in only 26 of the 1,000 tests. It could not be rejected for any v_{it} for 33 out of the 40 subjects. We also tested the data for consistency with the no-less-than-one slope hypothesis; that is, we tested the null hypothesis

$$H_*^3 : \hat{\beta}_{i1} + 2\hat{\beta}_{i2}v_{it} + 3\hat{\beta}_{i3}v_{it}^2 \ge 1, \text{ for each } v_{it}.$$

The null hypothesis in equation (13) could be rejected, at 10% significance, in 649 of the 1,000 tests.

We have tested the data for consistency with three properties of the log-concave model with our tests of the hypotheses, H_0^i, $i = 1, 2, 3$. We have also tested the data for consistency with the class of models that exclude these three properties of the log-concave model with our tests of the hypotheses, H_*^i, $i = 1, 2, 3$. Results from the H_0^i tests reveal that the data are highly, although not perfectly, consistent with the log-concave model. Results from the H_*^i tests reveal that the data are highly inconsistent with the class of models that excludes the three properties of the log-concave model.

IS BIDDING BEHAVIOR CONSISTENT WITH THE CONSTANT RELATIVE RISK AVERSE MODEL?

The constant relative risk averse model (CRRAM) is the special case of the log-concave model in which bidders have log-linear utility of income functions. Thus, for CRRAM the individual characteristic vectors, θ_i, are the scalars, r_i, and the convex set, Θ, is the interval, $(o, r_h]$. Further, the utility of income functions, $u(y, \theta_i)$, are the power functions, y^{r_i}, for which the $1 - r_i$ are the individuals' coefficients of constant relative risk aversion.

CRRAM Bid Function

The CRRAM equilibrium bid function has a linear segment and a nonlinear segment. The linear segment is

$$b_i = v_\ell + \frac{n-1}{n-1+r_i}(v_i - v_\ell), \; for\; v_i \in [v_\ell, v_i^*],$$

where: b_i is the amount of bidder i's bid; n is the number of bidders; $1 - r_i$ is bidder i's coefficient of constant relative risk aversion; v_i is the private value of the auctioned item to bidder i; v_ℓ is the lower bound on the support, $[v_\ell, v_h]$, of the uniform distribution from which the v_i are independently drawn; and v_i^* is the "knot" which joins the linear and nonlinear parts of bidder i's bid function. If $1 - r_h$ is the coefficient of constant relative risk aversion (or risk preference) for the least risk averse (or most risk-preferring) bidder in the population from which bidder i's rivals are drawn, then the knot occurs at the value

$$v_i^* = v_\ell + \frac{n-1+r_i}{n-1+r_h}(v_h - v_\ell).$$

v_i^* is the private value for which a CRRAM bidder with risk attitude parameter, r_i, would bid an amount equal to the highest possible bid by the least risk averse bidder in the population of bidders, $b(v_h, r_h)$.

Spline Function Estimation

The stochastic bid function for the *i*th bidder in an *n*-person auction can be represented by the spline function

$$b_{it} = v_\ell + \frac{n-1}{n-1+r_i}(v_{it} - v_\ell) + D^*_{it}\, g_i(v_{it}) + u_{it},\, t = 1, \ldots T$$

where

$$D^*_{it} = 1 \text{ if } v_{it} \geq v_i^{'}$$
$$= 0 \text{ otherwise,}$$

and $g_i(\cdot)$ is concave. An alternative expression for the bid function is obtained by rearranging terms in equation (16) to obtain:

$$b_{it} = \beta_{0i} + \beta_{1i}\, v_{it} + D^*_{it}\, g_i\,(v_{it}) + u_{it}\,,$$

where

$$\beta_{0i} = \frac{v_\ell r_i}{n-1+r_i}$$

and

$$\beta_{1i} = \frac{n-1}{n-1+r_i}.$$

Next, we represent the function $g(\cdot)$ by its second-order Taylor series approximation:[2]

$$g_i(v_{it}) = \beta_{2i}(v_{it} - v_i^*) + \beta_{3i}(v_{it} - v_i^*)^2. \qquad (21)$$

Substitution of equation (21) into equation (18) yields

$$b_{it} = \beta_{0i} + \beta_{1i}\, v_{it} + \beta_{2i}D^*_{it}\ (v_{it} - v_i^*) + \beta_{3i}D^*_{it}\ (v_{it} - v_i^*)^2 + u_{it},$$

where $\beta_{1i} + \beta_{2i} > 0$ and $\beta_{3i} < 0$. Since the restrictions on β_{0i} and β_{1i} implied by the theory are not imposed at this point, we refer to equation (22) as the unrestricted (or naive) bid function. We estimated equation (22) with nonlinear least squares by searching for the value of v_i^* that minimizes the sum of squared residuals, $\sum_{t=1}^{T} \hat{u}^2_{it}$ *for* $v_\ell < v_i^* < v_h$.

If $r_h = 1$ then the theoretical relationship between the risk aversion parameter r_i and the threshold value v_i^* is given by

$$r_i = n\left[\frac{v_i^* - v_h}{v_h - v_\ell}\right] + 1. \tag{23}$$

Upon substituting equation (23) for r_i in equations (19) and (20), we obtain the following parameter restrictions:

$$\beta_{0i} = \frac{v_\ell[n(v_i^* - v_h) + v_h - v_\ell]}{\text{[illegible]}}$$

and

$$\beta_{1i} = \frac{(n-1)(v_h - v_\ell)}{n(v_i^* - v_\ell)}.$$

Our restricted bid function is obtained by substituting equations (24) and (25) into equation (22), to obtain

$$b_{it}^* = \beta_{2i} D_{it}^* (v_{it} - v_i^*) + \beta_{3i} D_{it}^* (v_{it} - v_i^*)^2 + u_{it},$$

where

$$\begin{aligned} b_{it}^* &= b_{it} - \beta_{0i} - \beta_{1i} v_{it} \\ &= b_{it} - \frac{v_\ell[n(v_i^* - v_h) + v_h - v_\ell]}{n(v_i^* - v_\ell)} - \frac{(n-1)(v_h - v_\ell)}{n(v_i^* - v_\ell)} v_{it} \end{aligned} \tag{27}$$

In the case where $v_\ell = 0$, we have

$$b_{it}^* = b_{it} - \frac{(n-1)v_h}{n v_i^*} v_{it}.$$

We estimated the restricted bid function (26) by searching for the value of the knot, v_i^*, that minimizes the sum of squared residuals, $\sum_{t=1}^{T} \hat{u}_{it}^2$ for v_i^* contained between the maximum risk neutral equilibrium bid and the maximum value, that is $v_\ell + \left(\frac{n-1}{n}\right)(v_h - v_\ell) \leq v_i^* < v_h$. The restricted bid function admits only risk averse or risk neutral bidding behavior. Using equation (23), we find that our estimator of r_i is given by

$$\hat{r}_i = n\left(\frac{\hat{v}_i^* - v_h}{v_h - v_\ell}\right) + 1,$$

where $\hat{v}_i^*$ is the value of v_i^* that minimizes the sum of squared residuals. When $v_\ell = 0$, our estimator of r_i is simply

$$\hat{r}_i = n\left(\frac{\hat{v}_i^* - v_h}{v_h}\right)$$

If we approximate the distribution of the bid function disturbance terms by the mean zero normal distribution, then minimizing the sum of squared residuals of the bid function is equivalent to maximizing the likelihood function. The experimental data used in our bid function estimation were generated from an experimental design in which the lower bound and upper bound of the support on the distribution of values were $v_\ell = 0$ and $v_h = 10$. Also, the number of bidders was $n = 4$. Therefore, the log likelihood function corresponding to the restricted two-part bid function specified in equations (26) and (27) is given by

$$\ell n L = -\frac{T}{2}\ell n\left(2\pi\sigma_{ui}^2\right) - \frac{\sum_t \left[b_t - \alpha\left(\frac{v_{it}}{\hat{v}_i^*}\right) - \beta_{2i} D_{it}^* \left(v_{it} - v_i^*\right) + \beta_{3i} D_{it}^* \left(v_{it} - v_i^*\right)^2\right]^2}{2\sigma_{ui}^2} \quad (31)$$

where $\alpha = \frac{n-1}{n} v_h = 7.5$. In order to obtain estimated standard errors for the parameter estimates, $\hat{v}_i^*$ and $\hat{r}_i^*$, we employ the computationally simple method introduced in Berndt et al. (1974) that relies only on first derivatives of the likelihood function. Let $\hat{h}_{it}$ denote the first derivative of the likelihood function for observation t for subject i:

$$\hat{h}_{it} = \partial \ell n f(v_{it}, D_{it}^*, \hat{\Upsilon}_i)/\partial \hat{\Upsilon}_i,$$

where $f(\cdot)$ is the density function, and $\hat{\Upsilon}_i$ is the estimated bid function parameter vector whose elements are $\hat{\beta}_{2i}$, $\hat{\beta}_{3i}$, and $\hat{v}_i^*$. The asymptotic variance-covariance matrix for the parameters of the bid function is estimated according to

$$est.\ Asy.\ Var(\hat{\Upsilon}_i) = \left[\sum_t \hat{h}_{it}\hat{h}'_{it}\right]^{-1} \tag{33}$$

The estimated standard error for $\hat{r}_i$ is easily obtained from the estimated standard error of $\hat{v}_i^*$ by noting the linear relationship between these estimators given by equations (29) or (30).

Few observations were greater than or equal to the estimated threshold values because there were only 25 periods in each experiment. Out of the combined samples there were only 148 out of 1,000 observations that were greater than or equal to the estimated threshold values. On an individual subject basis, the number of these observations ranged from 0 to 6. The implication of this is that the parameters of the quadratic segment of the spline function could not be estimated in some cases, and in others the parameters were estimated with little precision.

Empirical Results

Table 1 presents the estimated values of $\hat{v}_i^*$ and $\hat{r}_i$ and their associated standard errors for each subject in each of the 10 experiments. The threshold value, $\hat{v}_i^*$, ranges from 7.56 to 10 with a corresponding range of r_i from 0.024 to 1. All of the estimated threshold values were statistically significant. With four exceptions, the risk aversion parameter was statistically significant. These four exceptions are for subject 3 in experiment 7, subject 4 in experiment 9, and subjects 1 and 2 in experiment 10. The lack of statistical significance is equivalent to the failure to reject the hypothesis that the threshold value is equal to 7.5.

Table 2 reports the p values for (asymptotic) F tests of selected pairwise model comparisons. The second column of Table 2 reports the p values for the test of the restricted two-part bid function (equation [26]) against the naive two-part bid function (equation [22]). The restricted two-part bid function would not be rejected in favor of the naive two-part bid function for the data for 19 out of the 40 subjects. A couple of these cases are marginal, however. Thus, the bidding behavior of about 50% of the subjects is consistent with CRRAM on the basis of this stringent test.

Table 1. Restricted Two-Part Bid Function Estimates and Standard Errors

Experiment/Subject	$\hat{v}_i^*$	$\hat{\sigma}_{\hat{v}_i^*}$	$\hat{r}_i$	$\sigma_{\hat{r}i}$
1.1	8.17	.085	.268	.034
1.2	8.79	.339	.516	.136
1.3	8.42	.171	.368	.068
1.4	8.77	.176	.508	.070
2.1	8.77	.248	.508	.099
2.2	7.87	.033	.148	.013
2.3	7.97	.230	.188	.092
2.4	7.89	.058	.156	.023
3.1	8.23	.132	.292	.053
3.2	8.82	.244	.528	.098
3.3	8.42	.090	.368	.036
3.4	8.90	.114	.560	.046
4.1	7.75	.032	.100	.013
4.2	7.83	.003	.132	.001
4.3	7.93	.038	.172	.015
4.4	9.09	.081	.636	.032
5.1	7.96	.195	.184	.078
5.2	7.89	.059	.156	.024
5.3	7.66	.018	.064	.007
5.4	8.25	.061	.300	.024
6.1	8.38	.095	.352	.038
6.2	8.45	.009	.380	.039
6.3	9.23	.436	.692	.174
6.4	8.60	.214	.440	.085
7.1	7.71	.062	.084	.025
7.2	8.58	.087	.432	.035
7.3	7.74	.298	.096	.119
7.4	7.74	.038	.096	.015
8.1	7.91	.046	.164	.018
8.2	9.58	.313	.832	.125
8.3	8.20	.056	.280	.023
8.4	10.00	a	1.000	a
9.1	8.67	.178	.468	.071
9.2	8.68	.050	.472	.020
9.3	8.48	.134	.392	.054
9.4	7.56	.072	.024	.029
10.1	8.10	.804	.240	.322
10.2	7.67	.154	.068	.061
10.3	7.85	.075	.140	.030
10.4	8.15	.308	.260	0.123

Note: [a]Standard errors were not defined since there were no observations for $v \geq 10$.

Table 2. F Tests of Bid Function Restrictions—p Values

Experiment/ Subject	*Restr. 2 part/ Naive 2 part*	*Risk neutral/ Restr. 2 part*	*Risk neutral/ Naive 2 part*	*Risk neutral/ Naive linear*
1.1	.010	.000	.000	.000
1.2	.029	.003	.000	.002
1.3	.677	.000	.000	.000
1.4	.002	.000	.000	.000
2.1	.020	.000	.000	.098
2.2	.000	.000	.000	.000
2.3	.140	.000	.000	.000
2.4	.140	.000	.000	.000
3.1	.142	.000	.000	.000
3.2	.000	.000	.000	.000
3.3	.103	.000	.000	.000
3.4	.276	.000	.000	.105
4.1	.693	.000	.000	.000
4.2	.000	.000	.000	.000
4.3	.049	.000	.000	.000
4.4	.035	.107	.005	.115
5.1	.660	.000	.000	.000
5.2	.015	.000	.000	.000
5.3	.003	.000	.000	.000
5.4	.002	.000	.000	.000
6.1	.010	.000	.000	.000
6.2	.195	.000	.000	.000
6.3	.102	.436	.048	.423
6.4	.025	.000	.000	.046
7.1	.037	.000	.000	.000
7.2	.000	.000	.000	.000
7.3	.392	.000	.000	.000
7.4	.531	.000	.000	.000
8.1	.165	.000	.000	.000
8.2	.111	.421	.050	.160
8.3	.126	.000	.000	.000
8.4	.004	1.000	.004	.095
9.1	.016	.000	.000	.000
9.2	.021	.000	.000	.000
9.3	.756	.000	.000	.000
9.4	.088	.000	.000	.000
10.1	.000	.065	.000	.000
10.2	.860	.000	.000	.000
10.3	.167	.000	.000	.000
10.4	.367	.001	.001	.016

IS BIDDING BEHAVIOR CONSISTENT WITH THE RISK NEUTRAL MODEL?

The risk neutral model is the special case of CRRAM in which the utility function power, r_i, equals 1 for all bidders. The equilibrium bid function for the risk neutral model was first derived in the classic paper by Vickrey (1961).

Risk Neutral Bid Function

With n bidders and auctioned item values independently drawn from the uniform distribution on $[v_\ell, v_h]$, the risk neutral equilibrium bid function is

$$b_i = v_\ell + \frac{n-1}{n}(v_i - v_\ell), \; for \, v_i \in [v_\ell, v_h$$

Bid Function Estimation

The data can be analyzed for consistency with the risk neutral model in various ways. We begin with the spline function approach. In the case of the risk neutral model, $r_h = r_i = 1$, which by equation (15) implies $v_i^* = v_h$. Making the appropriate substitutions in equations (19) and (20), we obtain the following parameter restrictions:

$$\beta_{0i} = \frac{v_\ell}{n};$$

and

$$\beta_{1i} = \left(\frac{n-1}{n}\right)$$

Our risk neutral restricted bid function is obtained by substituting equations (35) and (36) into equation (22) and setting $D_{it}^* = 0$ to obtain

$$b_{it}^* = u_{it},$$

where

$$b_{it}^{*} = b_{it} - \frac{v_\ell}{n} - \left(\frac{n-1}{n}\right)v_{it}$$

The sum of squared residuals from equation (37) are simply calculated as $\sum_{t=1}^{T} (b_{it}^{*})^2$. For $v_\ell = 0$, equation (35) implies that $\beta_{0i} = 0$; therefore, the risk neutral bid function restriction is given by

$$b_{it}^{*} = b_{it} - \left(\frac{n-1}{n}\right)v_{it}$$

A naive linear model is one with no restrictions on β_{0i} and β_{1i}. Such a model would simply be specified as

$$b_{it} = \beta_{0i} + \beta_{1i}v_{it} + u_{it}. \tag{40}$$

Empirical Results

Consider once again the estimated values of $\hat{v}_i^*$ and $\hat{r}_i$ and their standard errors in Table 1. In only two cases would one fail to reject the hypothesis that the threshold value, $\hat{v}_i^*$, was equal to 10 (or, equivalently, that the risk aversion parameter, $\hat{r}_i$, was equal to 1) at conventional levels of significance. In other words, in only two cases would one fail to reject risk neutrality on the basis of this test. These cases occurred in experiment 8 and involved subjects 2 and 4.

Table 2 presents other tests of the risk-neutral model. The third column of Table 2 reports the p values for the test of the risk-neutral model against the restricted two-part bid function (equations [26] and [27]). There are only four cases in which we fail to reject the risk neutral bid function in favor of the restricted two-part bid function, that is, there are only four out of 40 subjects for whom we fail to reject the risk neutral model in favor of CRRAM. One of these cases is marginal. A tougher test for the risk neutral model involves testing it against the naive two-part bid function (equation [22]). This is the stringent test that we applied to CRRAM. The p values for this test are reported in the fourth column of Table 2. The risk neutral model is rejected in favor of the naive two-part bid function for all of the 40 subjects. The fifth column of Table 2 reports the p values for tests of the risk neutral model (equation

[39]) against the naive linear model (equation [40]). The risk neutral model is rejected in favor of the naive linear model in all but four cases, and one of these is marginal. Thus, depending on which test is used, the behavior of only 0-10% of the subjects is consistent with the risk neutral model.

Loss Function Evaluation

Harrison (1989) argued that one cannot reject the hypothesis of risk-neutral bidding behavior if one uses his metrics to measure the importance of expected foregone earnings from non-risk-neutral bidding. Cox, Smith, and Walker (1992) explained the theoretical problems in Harrison's reasoning. Friedman (1992) explained that Harrison's metrics ignore most of the data on subjects' bidding behavior and proposed an alternative loss function. We apply Friedman's loss function to the data as follows. Substitute $r_i = 1$ in equation (14) to get the risk neutral theoretical bid function; then use that bid function to calculate the risk neutral bid associated with the item value for each bidder i and period t:

$$B_{it} = v_\ell + \frac{n-1}{n}(v_{it} - v_\ell). \tag{41}$$

Next, calculate the sign variable

$$Z_{it} = \begin{cases} +1, & \text{if } b_{it} > B_{it} \\ 0, & \text{if } b_{it} = B_{it} \\ -1, & \text{if } b_{it} < B_{it} \end{cases} \tag{42}$$

The loss variable is calculated as

$$L_{it} = \left\{\frac{n}{(n-1)(v_h - v_\ell)}\right\}^{n-1} \left\{[B_{it} - v_\ell]^{n-1}[v_{it} - B_{it}] - [b_{it} - v_\ell]^{n-1}[v_{it} - b_{it}]\right\},$$

where b_{it} and v_{it} are the observed bids and values. Finally, the test statistic is defined as

$$x_{it} = Z_{it} L_{it}.$$

Results from evaluating Friedman's loss function are presented in Table 3. We report a (one-tailed) test of the null hypothesis that expected foregone earnings from non-risk-neutral bidding are zero, against the alternative that they are positive (as in risk-averse bidding). At 10% significance, expected foregone earnings are significantly greater than zero for 36 out of the 40 subjects. The four subjects for whom the null hypothesis is *not* rejected have expected foregone earnings that are: (a) positive but insignificant for two subjects; and (b) negative for two subjects. The bid function tests reported in Table 2 rejected the risk neutral model in favor of CRRAM for 36 out of the 40 subjects. We observe that two out of the four test-separated subjects are the same in the two tests and two are distinct in each test.

SUMMARY AND CONCLUSIONS

We have tested three nested bidding models using data from some of the first-price sealed-bid auctions reported in Cox, Smith, and Walker (1988). The data are highly consistent with properties of the log-con-

Table 3. *t*-Tests of Friedman's Loss Metric—*p* Values

Experiment/Subject	*p Value*	*Experiment/Subject*	*p Value*
1.1		6.1	
1.2		6.2	
1.3		6.3	
1.4		6.4	
2.1		7.1	
2.2		7.2	
2.3		7.3	
2.4		7.4	
3.1		8.1	
3.2		8.2	
3.3		8.3	
3.4		8.4	
4.1		9.1	
4.2		9.2	
4.3		9.3	
4.4		9.4	
5.1		10.1	
5.2		10.2	
5.3		10.3	
5.4		10.4	

cave model. Furthermore, the data are highly inconsistent with the class of models that excludes these properties of the log-concave model.

We next asked whether the data are consistent with equilibrium bid functions with simple parametric forms. We tested CRRAM and its risk neutral special case using a spline function estimation procedure. CRRAM was rejected in favor of an unrestricted two-part bid function for 21 out of 40 subjects. Thus we found that data for about 50% of the subjects are consistent with CRRAM. The risk neutral model was rejected in favor of an unrestricted two-part bid function for every subject. Using other, less stringent tests we found that data for 5-10% of the subjects were consistent with the risk neutral model.

We next used Friedman's (1992) loss function to calculate the foregone expected earnings from non-risk-neutral bidding for each subject. We found that foregone expected earnings were significantly greater than zero for 36 out of the 40 subjects. Therefore, most of the subjects bid as if they were risk averse, not risk neutral, and they gave up significant amounts of expected earnings in order to do so.

ACKNOWLEDGMENTS

We are grateful for financial support from the National Science Foundation (grant numbers SBR-9108888 and SBR-9311351). Capable research assistance was provided by Lucy Atkinson Eldridge and Yu Mei.

NOTES

1. As an alternative to equation (9), we also estimated the parameters of cubic equations in which the intercepts were *not* forced to be zero. Slope tests for these equations yield essentially the same results as those reported in the paper. We report the test results for the equations with zero intercepts because the theoretical model yields bid functions with zero intercepts.
2. Most of the data are in the domains of the linear parts of the CRRAM bid functions. There are not sufficient observations in the nonlinear parts of the bid functions of many subjects to estimate the parameters of the second-order approximation. Thus, use of a higher order polynomial approximation is ruled out by the data.

REFERENCES

Berndt, E., B. Hall, R. Hall, and J. Hausman. 1974. "Estimation and Inference in Nonlinear Structural Models." *Annals of Economics and Social Measurement* 3/4: 653-66.

Cox, J.C., B. Roberson, and V.L. Smith. 1982. "Theory and Behavior of Single Object Auctions." Pp. 1-43 in *Research in Experimental Economics*, Vol. 2, edited by V.L. Smith. Greenwich, CT: JAI Press.

Cox, J.C., V.L. Smith, and J.M. Walker. 1982. "Auction Market Theory of Heterogeneous Bidders." *Economics Letters* 9: 319-325.

Cox, J.C., V.L. Smith, and J.M. Walker. 1988. "Theory and Individual Behavior of First-Price Auctions." *Journal of Risk and Uncertainty* 1(March): 61-99.

Cox, J.C., V.L. Smith, and J.M. Walker. 1992. "Theory and Misbehavior of First-Price Auctions: Comment." *American Economic Review* 82(December): 1392-1412.

Friedman, D. 1992. "Theory and Misbehavior of First-Price Auctions: Comment." *American Economic Review* 82(December): 1374-1378.

Harrison, G.W. 1989. "Theory and Misbehavior of First-Price Auctions." *American Economic Review* 79(September): 749-762.

Vickrey, W. 1961. "Counterspeculation, Auctions, and Competitive Sealed Tenders." *Journal of Finance* 16(May): 8-37.