Learning about Learning in Games through Experimental Control of Strategic Interdependence

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June, 2002

Abstract

We conduct experiments in which humans repeatedly play one of two games against a computer decision maker that follows either a reinforcement learning or an Experience Weighted Attraction algorithm. Our experiments show these learning algorithms more sensitively detect exploitable opportunities than humans. Also, learning algorithms respond to detected payoff increasing opportunities systematically; however, the responses are too weak to improve the algorithms' payoffs. Human play against various decision maker types doesn't significantly vary. These factors lead to a strong linear relationship between the humans' and algorithms' action choice proportions that is suggestive of the algorithm's best response correspondence.

1 Introduction

Identifying how people adapt their behavior in repeated strategic decision making tasks has emerged as a central question, and a difficult question, in the social sciences. Most research has addressed this question by formulating hypotheses about the learning process, then embedding the hypotheses in a parametric model, and then estimating the model's parameters from experiments using human subjects. The model is then evaluated either by goodness-of-fit statistical criteria or by using it to generate simulations that are compared to human play. Unfortunately, using this approach makes it difficult to determine how well a model reflects actual human learning, because current econometric techniques generate exceedingly high rates of Type I and Type II errors in this setting (Salmon [21] (2001)).

These econometric difficulties stem from the data generating process implied by the models. A *learning in games* model is generally comprised of two components: a rule that assigns a value to each of a player's actions conditional upon the history of play, and another rule that converts a player's action values to a probability distribution over the player's action set – in effect, a mixed strategy. Notice the conversion of the observable history of play into a current action choice involves a player's action values and mixed strategy, both of which are unobservable – or latent – variables. Identifying the underlying adjustment rules of these latent variables is a challenging econometric problem. Further, players in these models simultaneously use a common history of play to update their respective latent variables, and the resulting interdependencies only make matters worse.

In this study we control strategic interdependence through hybrid experiments in which a human repeatedly plays a simple game against a computer-implemented learning algorithm. Initially we conduct experiments in which humans play against humans, and use this data to estimate parameters of alternative learning models. Then we conduct the hybrid experiments with new human subjects and generate parallel simulations in which computer plays against computer. Our experiments thus allow us to observe how the play of a learning algorithm adjusts against actual human behavior, in addition to how it adjusts against the hypothesized model of human play.

This suggests a benchmark for evaluating learning models: a model can be considered the true model if human versus computer play is indistinguishable from human versus human play. When differences in play *can* be identified, the differences should suggest how we can refine the model. Our hybrid procedure can also help in developing new learning models: by systematically varying the kind of strategies against which people are required to play, we can obtain descriptions of how humans play against a variety of strategies. These descriptions can then serve as the empirical results that new models of learning must take into account.

We implement two prominent learning models in our hybrid experiments: Erev and Roth's [6] (1998) reinforcement learning model and Camerer and Ho's [2] (1999) experienced weighted attraction model.[1] The structure of the two models is similar to nearly all models in this genre: starting values are assigned to each of the players' actions; after each stage game, the values are updated according to an adaptive rule; and each player selects his next stage game action according to a probabilistic choice rule, where the probabilistic choice rule assigns higher probabilities to actions with greater values. Both of the models include unobservable parameters whose values are estimated from experimental data. Uniformly across studies, estimated parameter values imply that the adaptive rules have significant memory. As a consequence, the impact of recent outcomes on action values (and thus on mixed strategies) is very small. This leads to inert adjustment rules: mixed strategies, and the resultant pattern of action choices, are sluggish and exhibit little change from period to period.

Our technique reveals properties about both human learning and learning models that could not be discovered through pure human experimentation or through pure simulation. We find that human play does not significantly vary depending on whether the opponent is a human or is one of the learning algorithms we consider. In contrast, the joint algorithm-human play differs markedly from joint algorithm play in a simulation. For example, when humans' action frequencies deviate from the Nash equilibrium proportions, the algorithms' mixed strategies systematically adjust towards their pure strategy best responses. These adjustments result in a strikingly linear relationship between the learning models' and humans' action choice frequencies. Moreover, the linear relationship is consistent with the computer players' best response correspondence. While adjustments by the algorithms are remarkably

[1] There are many other similarly structured models worth studying with our technique, but models in this class tend to generate similar play (Salmon [21]) and consequently most of the potential insights can be gained through evaluation of just one or two models of this type.

systematic, the magnitude of the adjustments is quite weak – indeed, too weak to result in statistically significant gains in payoffs.

Our experimental study is just one of several that exploit laboratory control to better measure some of the latent variables underlying human play in games.[2] The findings, coupled with those reported in this paper, paint a picture of human learning different from the one upon which current learning models are based. Specifically, experiments with unique mixed strategy Nash equilibrium games have shown that humans' beliefs about opponent play are highly volatile from period to period (Nyarko and Schotter [16](2002)), and correspondingly players' mixed strategies exhibit significant variability with substantial switching between pure and mixed strategy play (Shachat [22] (2002)). Furthermore, humans are also successful at significantly increasing their payoffs when computerized opponents play either stationary non-equilibrium fixed mixed strategies (Lieberman [12](1961) and Fox [7] (1972)) or highly serially correlated action sequences (Messick [14] (1967) and Coricelli [4](2001)). In summary, human play is characterized by volatile beliefs, variable mixed strategy choices, and successful exploitation of some strategies. In contrast, the learning models we evaluate generate beliefs that are inert, make only minor mixed strategy adjustments from period to period, and don't take advantage of calculated payoff-increasing opportunities.

We proceed with a more detailed discussion of several past studies that incorporate human versus computer game play. Then we present the two learning models adopted in our study. In the fourth section we discuss the games used in our experiments and our experimental procedures. Section 5 covers our experiment results, findings and interpretations. In conclusion, we integrate our results with other experimental results to provide a summary of human play in games and contrast this with current learning models.

2 Literature Review

In a number of studies, human players and computerized decision makers researchers have interacted in strategic environments. This technique has been used to identify social pref-

[2] For example Camerer, Johnson, Rymon, & Sen [3](1993) and Crawford, Costa-Gomes, & Broseta [5](2000) studied information look-up patterns of subjects. Also, Nyarko & Schotter [16] (2002) elicited subjects' beliefs of opponents' future actions.

erences in strategic settings (Houser and Kurzban [10] (2000), and McCabe et al. [13] (2001)), to establish experimental control over player expectations in games (Roth and Shoumaker [20] (1983) and Winter and Zamir [24] (1997)), and to identify how humans play against particular strategies in games (as in Walker, Smith, and Cox [23] (1987)). In this section, we discuss the last type of study and summarize results on how humans play against unique minimax solutions, non-optimal stationary mixed strategies, and variants of the fictitious play dynamic (with deterministic choice rules) in repeated constant-sum games with unique minimax solutions in mixed strategies.

All of the studies we discuss incorporated fixed human-computer pairs playing repetitions of one of the zero-sum games presented in Table 4. [3] Studies by Lieberman [12] (1961), Messick [14] (1967), and Fox [7] (1972) all contain treatments where humans played against an experimenter-implemented minimax strategy. In these studies, the human participants were not informed of the explicit mixed strategy adopted by their computerized counterparts. [4] All three studies reach the same conclusion: human play does not correspond to the minimax prediction, and only in the Fox study does the human play adjust – albeit weakly – towards the minimax prediction. These results are not surprising: when a "computer" adopts its minimax strategy the expected payoffs of a human player's actions are all equal.

This indifference is not present when the computer adopts non-minimax mixed strategies. Lieberman [12] and Fox [7] both studied human play against non-optimal stationary mixed strategies and discovered that human players do significantly adjust their play (although not to the extent of exclusively playing the pure strategy best response) and also significantly increase – in a statistical sense – their payoffs above minimax value levels. In the relevant Lieberman treatment, subjects played against the experimenter for a total of 200 periods. In the first 100 periods, the experimenter played his minimax strategy of (.25, .75) and then in the final 100 periods the experimenter played a non-minimax strategy of (.5, .5). Human players were not informed that their opponent had adjusted his strategy. Human play adjusted from best responding approximately 20 percent of the time right after the

[3] In some of these studies the experimenters implemented stationary mixed strategies by using pre-selected computer generated random sequences in their non-computerized experiments.

[4] When reported, human participants were instructed something similar to, "The computer has been programmed to play so as to make as much money as possible. Its goal in the game is to minimize the amount of money you win and to maximize its own winnings." (Messick [14], page 35)

experimenter began non-minimax play, to best responding approximately 70 percent of the time by the end of the session. This shift toward the best response was also a shift towards the human's minimax strategy, making it difficult to differentiate between the attractiveness of the minimax strategy and the best response.

In one of Fox's treatment, each human participant played 200 periods against a computer which played the non-minimax mixed strategy (.6, .4) for the entire session. This design placed the human's best response, (1, 0), on the opposite side of (.5, .5) from the human's minimax strategy, (.214, .786). Human play started slightly above (.5, .5) and then slowly adjusted towards the pure strategy best response over the course of the experiment. Specifically, human players were best responding approximately 75 percent of the time by the latter stages of the experiment. These experiments demonstrated that human participants will adjust their behavior to take advantage of (but not as much as possible) exploitable stationary mixed strategies. Furthermore, the human subjects in both studies statistically improved their payoffs.

Messick [14] and Coricelli [4] (2001) conducted experiments to evaluate how human players respond when playing against variations of fictitious play.[5] These experiments are notable in that the computer's strategy was responsive to the actions selected by its opponent. Messick studied human subjects matched against two fictitious play algorithms: one with unlimited memory and the other with only a five period memory. Against unlimited memory fictitious play, human players earned substantially more than their minimax payoff level. Human players s earned an even greater average payoff against limited memory fictitious play. In the study by Coricelli, there are two treatments (both utilizing the game form introduced by O'Neill [17] (1985)) in which human participants play against unlimited memory fictitious play with and without a belief bias. This bias holds that human subjects tend not to repeat their "P" action. In both treatments human participants win significantly more often against the algorithms than they do against human opponents.[6] Establishing that humans can "outgame" these algorithms is significant, though not surprising. It is well known

[5] In the original formulations of fictitious play (Brown [1](1951) and Robinson [18](1951)) a player uses the empirical distribution of the entire history of his opponent's action choices as his belief of the opponent's current mixed strategy and then chooses a best response to this belief.

[6] Human versus human data for this conclusion are taken from O'Neill [17] (1985) and Shachat [22](2002).

that in games with a unique mixed strategy equilibrium, the fictitious play algorithm can generate strong positively serially correlated action choices that are easily exploited.[7] It was this speculated vulnerability that partially motivated game theorists to propose and study adaptive learning models which incorporated probabilistic choice as a key component.[8]

To summarize, experiments pairing human subjects against algorithms in constant sum games with strictly mixed strategy solutions have taught us: (1) that human players do not tend to play their minimax strategy in response to opponents playing their minimax strategy, (2) human players exploit (but not fully) opponents who play mixed strategies significantly different from their minimax strategy, and (3) human players exploit adaptive algorithms which generate highly serially correlated action choices.

3 Response Algorithms

In this section we describe Erev and Roth's [6](1998) Reinforcement learning model and Camerer and Ho's [2](1999) Experience Weighted Attraction model. Our descriptions of the model formulation and estimation techniques follow the original presentations as close as possible. Nonetheless, we only consider 2×2 games and in some instances we simplify notation without changing the models.

3.1 Reinforcement Learning

Erev and Roth's model (hereafter ER) is motivated by the reinforcement hypothesis from psychology: an action's score is incremented by a greater amount when it results in a "positive" outcome rather than a "negative" outcome. More formally, let $R_{ij}(t)$ denote player i's score for his jth action prior to the game at iteration t; let $\sigma_{ij}(t)$ denote the probability that i chooses j at iteration t; and let X_i denote the set of player i's possible stage-game payoffs. The two initial conditions for the dynamical system are (1) at the initial iteration, each of a player's actions has the same probability of being selected (in the 2×2 case each action is

[7] See Jordan [11](1993) and Gjerstad [9](1996).

[8] For example, see cautious fictitious play proposed by Fudenberg and Levine [8] (1995), and the two learning models we utilize in this study.

chosen with probability one-half), and (2) the score of actions are updated according to

$$R_{ij}(1) = \sigma_{ij}(1)S(1)\overline{X_i},$$

where $S(1)$ is an unobservable strength parameter, which influences the player's sensitivity to subsequent experience, and $\overline{X_i}$ is the absolute value of player i's payoff averaged across all action profiles.

After each iteration, each action's score is updated as follows

$$R_{ij}(t+1) = (1-\phi)R_{ij}(t) + \left((1-\varepsilon)I_{(a_i(t)=j)} + \frac{\epsilon}{2}\right)(\pi_i(j, a_{-i}(t)) - \min\{X_i\}),$$

where ϕ is an unobservable parameter that discounts past scores, $I_{(a_i(t)=j)}$ is an indicator function for the event that player i selected action j in period t, ε is an unobservable parameter determining the relative impacts on the scores of the selected versus the unselected action, and $\pi_i(j, a_{-i}(t))$ is i's payoff when he plays action j against the deleted action profile $a_{-i}(t)$. Also player i's minimum possible payoff for any action profile, $\min\{X_i\}$, is subtracted from $\pi_i(j, a_{-i}(t))$ as a normalization to avoid negative scores. The second component of the model, a probabilistic choice rule, is specified as

$$\sigma_{ij}(t) = \frac{R_{ij}(t)}{\sum_k R_{ik}(t)}.$$

For each game we consider, parameters of the model are estimated along the lines suggested by Erev and Roth. We estimate the values of $S(1)$, ϕ, and ε by minimizing the mean square error of the predicted proportions of Left play in 20-period trial blocks for the human versus human treatments. More specifically, for each fixed triple of parameter values from a discrete grid we proceed as follows: we simulate the play of 500 fixed pairs engaging in 200 iterations, and then we calculate separately the frequency of Left play by the 500 Row players and by the 500 Column players in each 20-period block. These frequencies are the model's predictions for that triple of parameter values. The grid is then searched for the optimal parameters.

3.2 Experience-Weighted Attraction

We use the version of EWA developed by Camerer & Ho [2](1999). While the structure of the EWA formulation is similar to the ER learning model, it adopts a different parametric form of probabilistic choice and it updates actions' scores according to what actions actually earned in past play, and what actions hypothetically would have earned if they had been played.

According to EWA, subjects choose stage-game actions probabilistically according to the logistic distribution

$$\sigma_{ij}(t) = \frac{e^{\lambda R_{ij}(t)}}{\sum_k e^{\lambda R_{ik}(t)}},$$

where at stage t player i chooses action j with probability $\sigma_{ij}(t)$, where λ is the inverse precision (variance) parameter, and where $R_{ij}(t)$ is a scoring function, as in the ER model, albeit defined (i.e., updated) differently. The updating of $R_{ij}(t)$ involves a "discounting" factor $N(t)$, which is updated according to $N(t+1) = \rho N(t) + 1$ for $t \geqq 1$, where ρ is an unobservable discount parameter and $N(1)$ is an unobservable parameter, interpreted as the strength of experience prior to the beginning of play. The score $R_{ij}(t)$ is updated as follows:

$$R_{ij}(t+1) = \frac{N(t)\phi R_{ij}(t) + ((1-\varepsilon)I_{(a_i(t)=j)} + \frac{\varepsilon}{2})\pi_i(j, a_{-i}(t))}{N(t+1)},$$

where $\pi_i(j, a_{-i}(t))$, ϕ, and ε are interpreted as in the Erev and Roth model. Initial scores, $R_{ij}(1)$ for each i and j, are additional unobservable parameters.

Parameters of the EWA model are estimated via maximum likelihood. It is worth noting that EWA is a flexible specification that includes several other models as special cases. For example, a simple reinforcement learning model, which has a different parametric form than ER, is generated when $N(1) = 0$, $\varepsilon = 0$, and $\rho = 0$; and probabilistic fictitious play is generated when $\varepsilon = \rho = \phi = 1$.[9]

4 Experimental Procedure

There are three basic steps in our experimental methodology. First, we collect baseline data samples consisting of fixed human versus human pairs that play 100 or 200 rounds of one of two 2×2 games. Second, we estimate parameters for the two learning models separately for each of the two games. In the third step, a new sample of humans play one of the two games

[9] We refer the reader to Camerer and Ho [2](1999) for more discussion of how EWA can emulate various models and for a more complete interpretation of the parameters.

against an estimated learning algorithm. We proceed by describing the two games we used and then present more details on the outlined steps.

4.1 The Two Games

The first game we consider is a zero-sum asymmetric game called Pursue-Evade. This game was introduced by Rosenthal, Shachat, and Walker [19](2002) (hereafter RSW). The normal form representation of the game is given in Table 1. The minimax solution (and Nash equilibrium) of this game is symmetric with each player choosing Left with probability of two-thirds.

There are several reasons why this game is a strong candidate to use in our study. (1) Zero-sum games eliminate social utility concerns often found in experimental studies of games, thereby mitigating some behavioral effects that might arise if a human suspects he is playing against a computer rather than another human. (2) With some standard behavioral assumptions, the repeated game has a unique Nash equilibrium path which calls for repeated play of the stage game Nash equilibrium. This eliminates potential repeated game effects that the algorithms are not designed to address. (3) Pursue-Evade is a simple game in which the Nash equilibrium predictions differ from equiprobable choice. This generates a powerful test against the alternative hypothesis of equiprobable play.

We selected our second game to pose a more difficult challenge to the learning algorithms. We refer to our second game, presented in Table 2, as Gamble-Safe. Each player has a Gamble action (Left for each player) from which he receives a payoff of either two or zero and a Safe action (Right for each player) which guarantees a payoff of one. This game has a unique mixed strategy in which each player chooses his Left action with probability one-half, and his expected Nash equilibrium payoff is one. Notice that this game is not constant-sum; therefore the minimax solution need not coincide with the Nash equilibrium. In this game, Right is a pure minimax strategy for both players that guarantees a payoff of one. A game for which minimax and Nash equilibrium solutions differ but generate the same expected payoff is called a non-profitable game.[10] The potential attraction of the minimax strategy can (and does) prove to be difficult for the learning algorithms which, loosely speaking, have

[10] Morgan and Sefton [15](2002) present an excellent study of human play in non-profitable games.

best response flavors.

4.2 Protocols

4.2.1 Human versus Human Baselines

For the human versus human baseline play in the Pursue-Evade game we use the data generated by RSW. In their hand-run experiments, a pair of subjects were seated on the same side of a table with an opaque screen dividing them. The Evader was given an endowment of currency. Each player was given two index cards: one labelled Left and the other labelled Right. At each iteration the players slid their chosen cards face down to the experimenter seated across the table. Then the experimenter simultaneously turned over the cards, executed the payoffs, and recorded the actions. Twenty pairs of human subjects played this treatment: fourteen for 100 periods and six for 200 periods.

The human versus human baseline experiments for the Gamble-Safe game were executed via computerized interaction. Each subject was seated at a separate computer terminal such that no subject could observe the screen of any other subject. Within a pair, each subject either played the Row or Column role for the entire experiment. Fifteen pairs of subjects participated in this treatment; five pairs who played 100 periods and ten pairs who played 200 periods. At the beginning of each repetition, a subject saw a graphical representation of the game on the screen. A Column player's display of the game was transformed so that he appeared to be a Row player. Thus, each subject selected an action by clicking on a row, and then confirmed his selection. Each subject was free to change his row selection before confirmation. Once an action was confirmed, a subject waited until his opponent also confirmed an action. Then a subject saw the outcome highlighted on his game display, as well as a text message stating both players' actions and his own earnings for that repetition. Finally, at all times a history of past play was displayed to the subject. This history consisted of an ordered list with each row displaying the number of the iteration, the actions selected by both players, and the subject's earnings.

4.2.2 Human versus Algorithm Treatments

We conducted our hybrid treatments using both the experimental software and protocol used for the Gamble-Safe game baseline.[11] In these treatments, two human subjects played against each other for the first 23 repetitions of the game. Then, unbeknownst to the human pair, they stopped playing against each other and for the remainder of the experiment they each played against a computer that implemented either the EWA or ER learning algorithm.

We used an initial phase of human versus human play to minimize the impact of estimated initial score values of actions and focus our evaluation on the dynamics of the algorithm. During the first 23 repetitions, we allowed the action value scores to "prime" themselves with the play generated by the subjects. (Although updating of scores was determined by the parameter estimates obtained from the baseline treatments). That is, even though the response algorithms were not selecting actions during the first 23 repetitions, the scores were still being updated according to the specifications of the previous section. For example, consider the 24th repetition of a game. The human Row player now faces a computer that plays the Column role. Moreover, during the first 23 repetitions, the computer Column player updated the scores associated with Column's actions based on the observed actions of both humans.

We adopted a simple technique to make the "split" seamless from the subjects' perspectives. From period twenty-four on, the two human/computer pairs had no interaction except for the timing of how action choices were revealed. Specifically, although the computers generated their action choices instantly, the computers didn't reveal their choices until both humans had selected their actions. This protocol preserved the natural timing rhythm established by the humans in the first twenty-three stage games.

In summary, we have two treatment variables; the stage game and the type of opponent. The data samples we have for each treatment cell are given in Table 3.[12]

[11] For the Pursue-Evade game, the Evader was given a currency endowment.

[12] We explain in the next section why we have no observations for the EWA Gamble-Safe treatment.

5 Baseline Results and Model Estimation

Our experimental baselines are the human versus human play in each of the two games. Inspection of the aggregate data reveals that play in the two games departs from the Nash equilibrium and the dynamic features of the data suggest non-stationarity of play. After estimating the unobserved parameters of the learning models, we simulated large numbers of experiments based upon these estimated versions of the models. Simulations reveal that the learning models generate aggregate choice frequencies similar to the experimental data, but only weakly mimic the experimental data time series. Furthermore, the simulations do not reveal striking differences between the two learning models.

We use the data from RSW as the Pursue-Evade game baseline data set. Figure 1 shows contingency tables for the data aggregated across subject pairs and stage games. A graph of the time series of the average proportion of Left play for the Row and Column players is shown below each table. Each observation in a series is the average across a twenty period time block. As noted by RSW, the contingency table is distinctly different from the Nash equilibrium predictions (the numbers in parentheses) and Column subjects play Left significantly more often than the Row subjects.[13] In the block average time series, we see that the Column series almost always lies above the Row series and that both series exhibit an increasing trend.

Using this data, RSW estimated the parameters of both the ER and EWA models. As noted by RSW, both models have some success in explaining the deviation. Using the estimated models we simulated 10,000 experiments of twenty pairs playing the Pursue-Evade game for 200 iterations. Averages from the 10,000 simulated experiments were used to construct contingency tables and time series in the same format as those presented for the baseline data. These results are presented alongside the baseline results in Figure 1. Unsurprisingly, given the respective objective functions used to select model parameters, casual observation suggests that the EWA model generates an expected contingency very close to the human baseline and the ER model more accurately mimics dynamics in the times series.

We provide a corresponding analysis for the Gamble-Safe game in Figure 2. In the con-

[13] Moreover, the Column subject plays Left more frequently than his Row counterpart in almost all pairs.

tingency table for the baseline data we observe that the Row subjects play Right significantly more than Left, while Column subjects played Left more often. This result partly comes from two pairs in which the Row and Column subjects' action profile sequence eventually converged to the profile (Safe, Gamble). This is evident around the midpoint of the times series for the baseline treatment, where we see the Column and Row subjects' series diverge.

This convergence to minimax play by the Row subjects in these two pairs is problematic for the maximum likelihood estimation used in the EWA model. Specifically, the long strings of Left by Column leads the EWA model to assign a near zero probability to Right (Safe) by Row for any possible parameter values. However, since Row is repeatedly choosing Right in these instances there is a zero likelihood problem in estimating the EWA parameters. Rather than violate the maximum likelihood criterion for parameter selection specified by Camerer and Ho we chose not to conduct a Human versus EWA treatment for this game.

Since the ER model parameter selection does not rely upon maximum likelihood estimation we obtain estimates which generate the best fit for the baseline data. Interestingly we see that the ER contingency table is remarkably similar to the baseline table. However, the predicted ER dynamics are excessively smooth and do not resemble the baseline time series. We believe this failure results from the inability of the model to incorporate the heterogenous behavior that occurs when some players adopt the minimax strategy and other players use adaptive strategies.

Comparison of the experimental data to simulations based upon estimated versions of the learning models suggests that the learning models successfully capture some features of the humans' disequilibrium behavior. However, time series views of the simulation data exhibit much smoother and less extreme dynamics than the experiment data, which suggests that learning models are not as responsive as humans and tend to simply "fit" aggregate human choice frequencies.

6 Analysis of Human/Algorithm Interaction

In the previous section we used a common technique of comparing experimental data to simulation results to evaluate the appropriateness of alternative learning models. Now we proceed to present analysis of human/algorithm interaction which reveals a significantly different story. Action choice frequencies by the algorithms are more responsive to opponents' play than the humans' action choice frequencies. Moreover, the action frequencies by each algorithm adjust linearly toward the best response to its opponent's non-equilibrium action frequencies. However, the magnitude of these adjustments is too small to generate payoff gains for the learning algorithms. Finally, we see that human play does not vary significantly whether the opponent is another human or a learning algorithm. Examination of the human/human experiments and the model simulations don't reveal these results.

6.1 Learning Algorithm Response to Opponents' Play

We now introduce pair-level data to better highlight differences in play across treatments. Inspection of the Row and Column players' proportions of Left play in each pair reveals surprising differences from purely human play and the simulations reported in the prior section. The learning algorithms are quite responsive to human deviations from Nash equilibrium play. Specifically, the algorithms' frequencies of Left play have a striking linear correlation to their human opponents' Left play proportions. Moreover, these linear relationships are consistent with a linear approximation of the algorithms' best response correspondences.

These results are most easily seen in Figures 3 - 5. Each of these figures is a 2×2 array of scatterplot panels. The rows of each panel array correspond to the decision maker type for the Row player: the top row indicates human decision maker and the bottom row indicates computer decision maker. Similarly the columns of each panel array correspond to the decision maker type for the Column player: the left column for human and the right column for computer. Hence the upper left panel is from the human/human baselines, the lower right panel is from the algorithm/algorithm simulations, and the off-diagonal panels are from the human/algorithm and algorithm/human experiments.

The scatterplots show the proportions of Left play by the Row and Column players in each pair after the first 23 iterations. In the simulation panel we only use the data from a single simulated experiment with twenty pairs playing 200 iterations. Also, Each off-diagonal scatterplot displays a trend line, which is obtained by regressing the Computers' proportions of Left on the Humans proportions of Left. We discuss these regressions shortly.

Examination of these figures reveals important common results across the two games and learning models. Comparisons between the two main diagonal panels reveal consistent differences and similarities between human/human play and pure simulations of model interaction. Both types of interactions generate uncorrelated "clouds" with the simulations' clouds exhibiting much smaller dispersion.[14] This raises the issue of whether the learning models are quite aggressive in adaptation and quickly converge to an equilibrium or instead the models are quite insensitive to opponents' play and just stubbornly mimic human aggregate frequencies. We can ask a similar question regarding human play. Do the humans' dispersed clouds result from high variance in the humans' propensities to play Left coupled with little response to the opponents' play or is it the result of differential skill in human play in which some humans more successfully exploit other humans' play?

Inspection of the human and learning algorithm interactions answers these questions. In contrast to the model simulations and human/human play, the scatter plots of human and learning algorithm interactions (found in the off-diagonal panels of Figures 3 - 5) exhibit strongly correlated interactions. This is evident by the tight clustering of the data along the plotted regression lines. Also, in each case the regression line is in the direction of the computer players' best response correspondence (the dashed correspondence given on each scatterplot). In other words, the computer "better" responds instead of best responds. This is best illustrated by an observation in the upper right scatter plot of Figure 3. In this scatterplot, Column ER players play against human Row players in the Gamble-Safe game. One of the human players chose his Minimax strategy, Right, exclusively and his computer ER opponent best responded to this only about 70 percent of the time. Hence, we see that (1) the frequency of Left by the learning algorithms move toward (but not all the way to) the best response to their opponents' frequencies, and (2) the magnitude of these responses by the algorithms is described by a surprisingly predictable linear relationship.

Table 5 gives some quantitative support for these observations by presenting the OLS results of regressing the learning algorithms' Left frequencies on their human counterparts' Left frequencies.[15] A learning algorithm that is highly sensitive and adjusts systematically to

[14] F-tests reject the significance of the presented regression lines; this gives statistical support for claims of no correlation.

[15] Note that the regression lines displayed in the upper-right panel of Figures 3 - 5 differ from the regression

opponents' play should generate regressions that explain a high percentage of the variance of the algorithm's Left frequencies, and the estimated slope coefficient should be consistent with the best response correspondence. These features are found in the Table 5 regressions: the slope of each regression has the correct sign, three of the regressions have exceedingly large adjusted R^2 statistics, and a fourth is still quite large considering the data is cross sectional. These adjusted R^2 results reflect the tight clustering to the fitted regression line observed in the scatterplots and correspondingly the detection and systematic reaction by the learning algorithms to calculated payoff-increasing opportunities. Correspondingly, F-tests for these four regressions do not reject the significance of the regressions at the 5 percent level of significance. Interestingly, the two cases where F-tests reject the regressions are when the EWA and ER algorithms assume the Column role in the Pursue- Evade game. We do not see a reason for the differential performance, but do note that the mean of the computers' data is close to their minimax strategy in this case.

6.2 Learning Algorithms' Lack of Effective Exploitation

Previous arguments established that the learning algorithms sensitively detect opponents' exploitable action choice frequencies and then the algorithms respond with a systematic but tempered reaction in the direction of their best response. However, we will now see that these statistically significant responses are too weak in magnitude to generate statistically significant payoff gains. Table 6 presents the average stage game winnings for all decision maker types when pitted against a human for each role and game. If the learning algorithms successfully exploit human decision makers we would expect the algorithms in each game and role to have greater winnings than a human when playing against a human in the competing role. The average stage game winnings in Table 6 do not exhibit this trait.

The reported average stage game payoff statistics are calculated by first taking the total session payoffs for each decision maker who plays against a human, and dividing by the number of stage games played.[16] Then we partition these decision makers according to the

results in Table 5. This is because the figures show the plot of Row proportion Left regressed on Column proportion Left, while the table reports Computer proportion Left regressed on Human proportion Left.

[16] We normalize this way because in the baseline data for Pursue-Evade and Gamble-Safe some pairs played 100 stage games and others 200.

game played, role played, and decision maker type. Finally, we report the average stage game payoffs across decision makers in each partition. For each game and player role we conduct t-tests with the null hypothesis that on average a non-human decision maker earns the same as a human when the opponent is a human. At a 5 percent level of significance we fail to reject the null hypothesis in four out of the six tests. In the two rejections, the human average exceeds the algorithm average.

Why don't the learning algorithms, which are sensitive and responsive to opponent play, generate higher payoffs than humans? The answer is twofold. First, the two games we consider have fairly flat payoff spaces in the mixed strategy domains presented in Figures 3 - 5. Thus a pair must be far removed from the Nash equilibrium to generate large payoff deviations from Nash equilibrium payoffs. Second, whenever the algorithm calculates a difference between its two action scores, it adjusts choice probabilities without assessing whether this difference is statistically significant. If this difference is not statistically significant, then there is no adjustment that can generate a real increase in payoff. Alternatively, an adjustment to a statistically significant score difference may also fail to generate a real increase in payoffs. Why? We have already seen that algorithms adjust in statistically significant ways, but these adjustments are relatively small in magnitude. These weak adjustments are the product of probabilistic choice rules, which were adopted to avoid generating transparent serially correlated choice patterns.

6.3 Human Play Conditional On Opponent Decision Maker Type

Past studies have demonstrated that humans play differently against Nash equilibrium strategies than they do against other humans. However, we also have presented arguments that play by learning algorithms is more responsive to opponents' decisions than human play is. A natural question to ask is, do humans play differently against learning algorithms than they do against other humans? To answer this question we compare the empirical distributions of the proportions of Left play by humans when facing the different decision-making types as presented in the scatter plots of Figures 3 - 5. We report a series of Kolmogorov-Smirnov two-sample goodness-of-fit tests (hereafter denoted KS) comparing the distributions of Left play proportions against human opponents to Left play proportions against the alternative algorithms. The main result is that we can't find differences in human play except in the case when the human is the Row player in the Pursue-Evade game.

Figure 6 shows the empirical CDFs of proportion of Left play by human Row players as they face human, ER, and EWA Column decision maker types in the Pursue-Evade game. Additionally, the figure reports the results of Kolmogorov-Smirnov tests of whether the Human's distribution of Left play frequencies differs when facing an algorithm opponent as opposed to a human opponent. Previously we have observed that the learning algorithms performed differently in the Column role of the Pursue-Evade game than in any other situation. This trend continues as the proportions of Left by humans in the Row role are significantly different when facing each learning algorithm than when facing another human.

Next we consider the CDFs generated by human Column players when playing against Human, ER, and EWA Row decision maker types in the Pursue-Evade game. We see in Figure 7 that play against human opponents is statistically indistinguishable from play against both EWA and ER opponents.

Next, we turn our attention to human play in the Gamble-Safe game. Figure 8 shows that human Row players' CDFs of proportion of Left play are not statistically different as they face Human and ER Column decision maker types. Finally, the CDFs and associated KS tests generated by human Column players in the Gamble-Safe game are shown in Figure 9. We see that play against human opponents differs from play against ER opponents at the six-percent level of significance.

7 Discussion

Through experiments in which humans play games against computer-implemented learning algorithms, we have established that humans do not detect nor exploit the non-stationary but rather inert mixed strategy processes of the ER and EWA algorithms. Our experiments also establish that the learning models are more sensitive than humans in detecting exploitable opponent play. Furthermore, our experiments reveal that the learning algorithms' action choice frequencies respond uniformly and linearly to opponents' non-equilibrium action choice frequencies. However, the corresponding mixed strategy adjustments of the learning models to detected exploitable play are too weak to increase their payoffs.

Our results, in conjunction with those of other studies, reveal a different depiction of human learning in games than those suggested by currently proposed models of adaptive behavior. First, through the technique of pitting humans against algorithms we know that humans successfully increase their payoffs (but not as much as possible) against non-optimal but stationary mixed strategy play and against adaptive play that generates highly serially correlated action sequences. On the other hand humans do not exploit the subtle dynamic mixed strategy processes of the learning models examined in this paper.

Some sources of behavioral departure between learning models and humans are identified in experiments that elicit subjects' beliefs (Nyarko and Schotter [16]) or subjects' mixed strategies (Shachat [22]). Elicited beliefs are highly volatile and often times correspond to a belief that one action will be chosen with certainty. Similarly elicited mixed strategies show erratic adjustments and a significant amount of pure strategy play.

This set of stylized facts establishes benchmarks which new learning models should explain. Furthermore, the use of human/algorithm interactions can play an important role in future efforts to identify how humans adapt in strategic environments. First, the technique brings increased power in evaluating proposed models and overcomes some current econometric and numerical limitations. Second, this technique can be used to identify human learning behavior through the adoption of carefully selected algorithms and the subsequent measurement of human responses to these algortihms. For example, one could determine the extent to which humans can exploit serially correlated strategies by adjusting the level of variance incorporated in the probabilistic choice rule of a cautious fictitious play algorithm; or one could determine human ability to detect and exploit non-minimax mixed strategies by systematically varying the computer's mixed strategy across opponents in a matching pennies game. In these instances, the algorithms are not being evaluated but rather used as carefully chosen stimuli to generate informative measurements of human behavior.

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		Column player	
		L	R
Row player	L	1, -1	0, 0
	R	0, 0	2, -2

Table 1: Pursue-Evade

		Column player	
		L	R
Row player	L	2, 0	0, 1
	R	1, 2	1, 1

Table 2: Gamble-Safe

	Opponent treatment		
Game treatment	Human	EWA	ER
Pursue-evade	40	30	30
Gamble-safe	30	0	24

Table 3: Number of subjects that participated in each treatment.

Zero-Sum Games Used In Previous Studies

(Humans are row player, Payoffs are for row player, minimax strategy proportions are next to action names)

Lieberman

	E1 (.25)	E2 (.75)
S1 (.75)	3	-1
S2 (.25)	-9	3

Messick

	A (.556)	B (.244)	C (.2)
a (.400)	0	2	-1
b (.111)	-3	3	5
c (.489)	1	-2	0

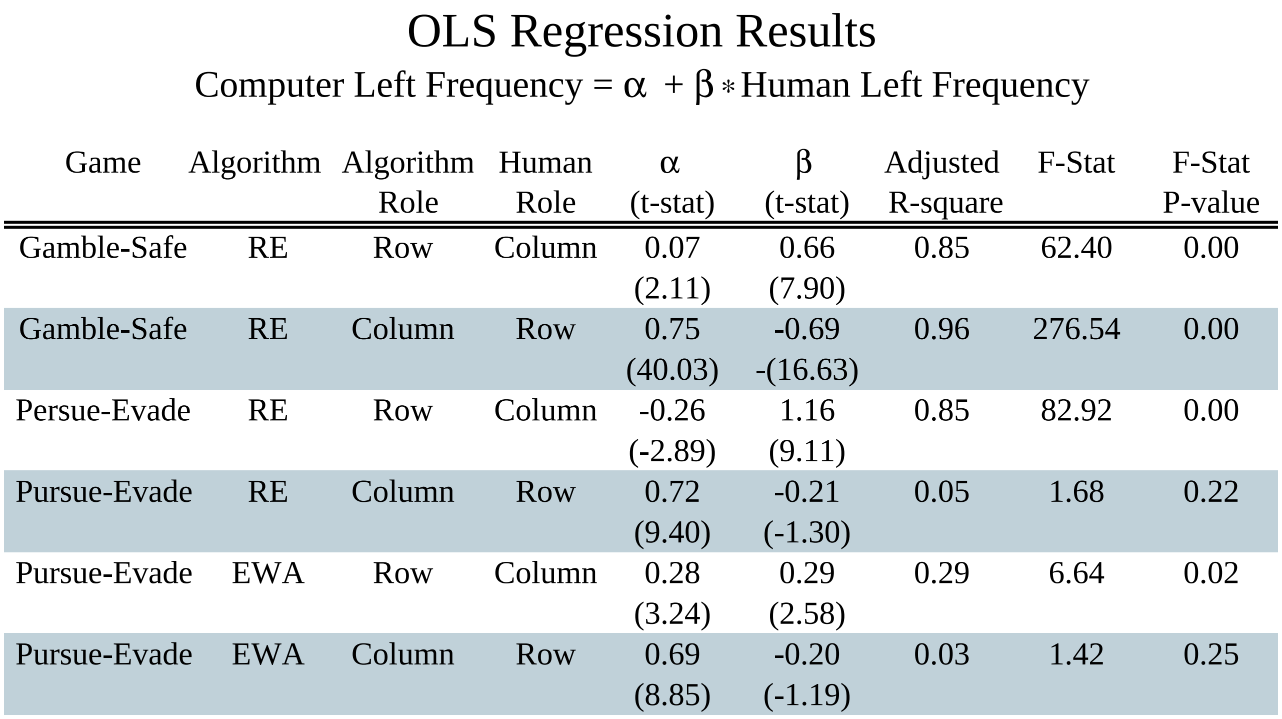
Fox

	a1 (.426)	a2 (.574)
b1 (.214)	6	-5
b2 (.786)	-2	1

Coricelli (Introduced by O’Neill)

	G (.2)	R (.2)	B (.2)	P (.4)
G (.2)	-5	5	5	-5
R (.2)	5	-5	5	-5
B (.2)	5	5	-5	-5
P (.4)	-5	-5	-5	5

Table 4:



OLS Regression Results

Computer Left Frequency = $\alpha + \beta * $Human Left Frequency

Game	Algorithm	Algorithm Role	Human Role	α (t-stat)	β (t-stat)	Adjusted R-square	F-Stat	F-Stat P-value
Gamble-Safe	RE	Row	Column	0.07 (2.11)	0.66 (7.90)	0.85	62.40	0.00
Gamble-Safe	RE	Column	Row	0.75 (40.03)	-0.69 -(16.63)	0.96	276.54	0.00
Persue-Evade	RE	Row	Column	-0.26 (-2.89)	1.16 (9.11)	0.85	82.92	0.00
Pursue-Evade	RE	Column	Row	0.72 (9.40)	-0.21 (-1.30)	0.05	1.68	0.22
Pursue-Evade	EWA	Row	Column	0.28 (3.24)	0.29 (2.58)	0.29	6.64	0.02
Pursue-Evade	EWA	Column	Row	0.69 (8.85)	-0.20 (-1.19)	0.03	1.42	0.25

Table 5:

Average Stage Game Payoffs For Decision Makers When Facing A Human Opponent

Game	Human Role	Human's Opponent	Decision Maker Avg. Payoff	T-test Statistic	Approx. d.o.f.	P-value
Gamble-Safe	Row	Human Column	1.0776	***	***	***
Gamble-Safe	Row	RE Column	1.0786	-0.012	23	0.990
Gamble-Safe	Column	Human Row	0.9888	***	***	***
Gamble-Safe	Column	RE Row	0.8983	2.187	25	0.038
Pursue-Evade	Row	Human Column	-0.6709	***	***	***
Pursue-Evade	Row	RE Column	-0.6829	0.498	32	0.622
Pursue-Evade	Row	EWA Column	-0.7205	2.312	33	0.027
Pursue-Evade	Column	Human Row	0.6709	***	***	***
Pursue-Evade	Column	RE Row	0.6395	1.285	31	0.208
Pursue-Evade	Column	EWA Row	0.6395	1.557	32	0.129

Table 6:

All Human Baseline

		Column Action	
		Left	Right
Row Action	Left	**.441** (.444)	**.169** (.222)
	Right	**.276** (.222)	**.114** (.111)

Estimated RE Model

	Column Action	
	Left	Right
Left	**.502** (.444)	**.161** (.222)
Right	**.208** (.222)	**.129** (.111)

Estimated EWA Model

	Column Action	
	Left	Right
Left	**.442** (.444)	**.204** (.222)
Right	**.239** (.222)	**.110** (.111)

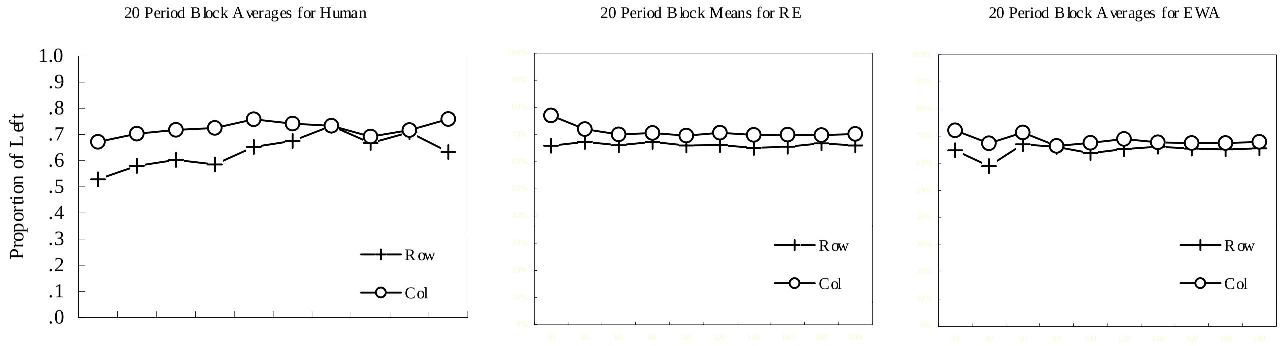


Figure 1: Baseline Data and Estimated Model Summary for Pursue-Evade Game.

All Human Baseline

Column Action

		Left	Right
Row Action	Left	**.212** (.250)	**.220** (.250)
	Right	**.309** (.250)	**.259** (.250)

Estimated RE Model

Column Action

	Left	Right
Left	**.226** (.250)	**.215** (.250)
Right	**.297** (.250)	**.262** (.250)

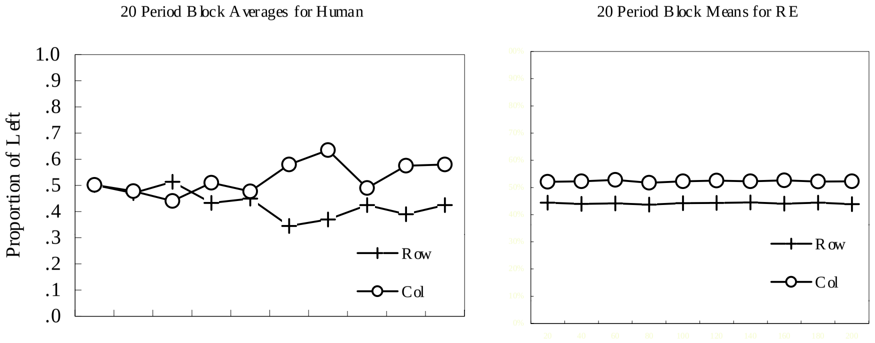


Figure 2: Baseline Data and Estimated Model Summary for Gamble-Safe Game.

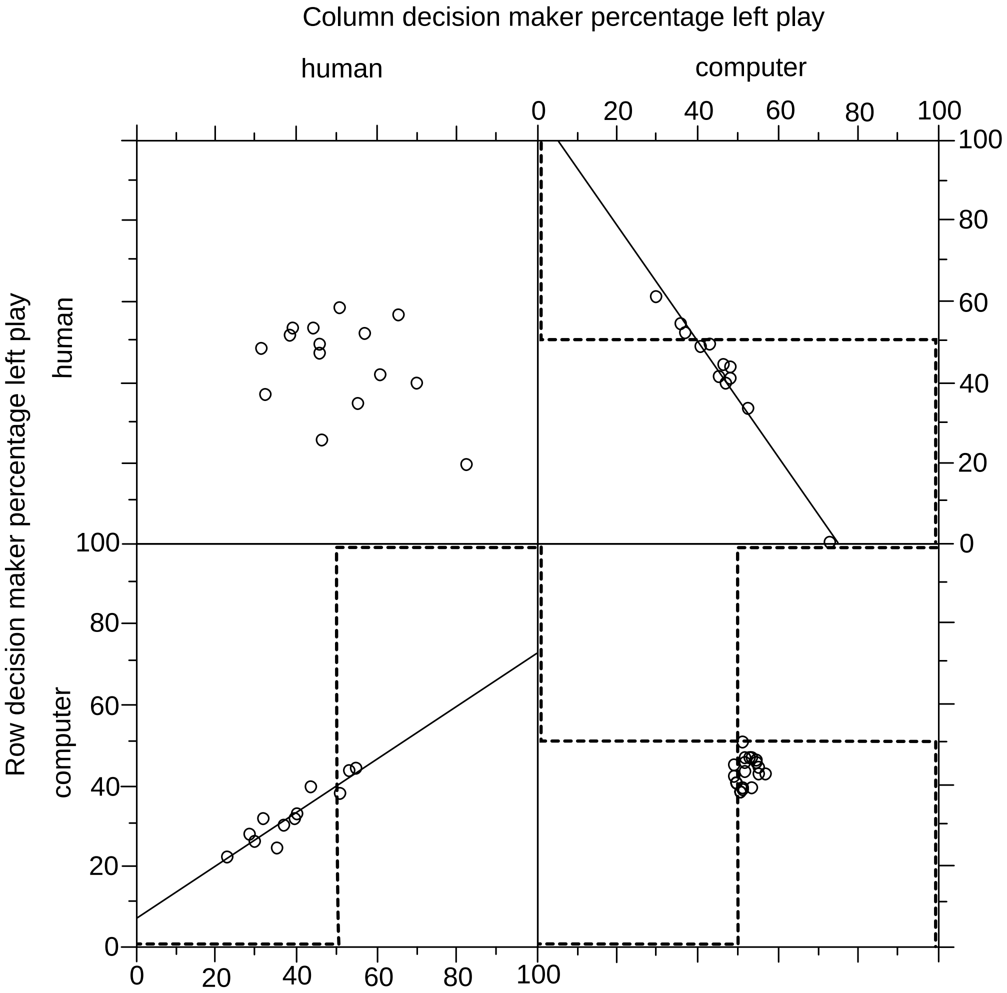


Figure 3: Gamble-Safe joint densities of proportion Left; ER interactions.

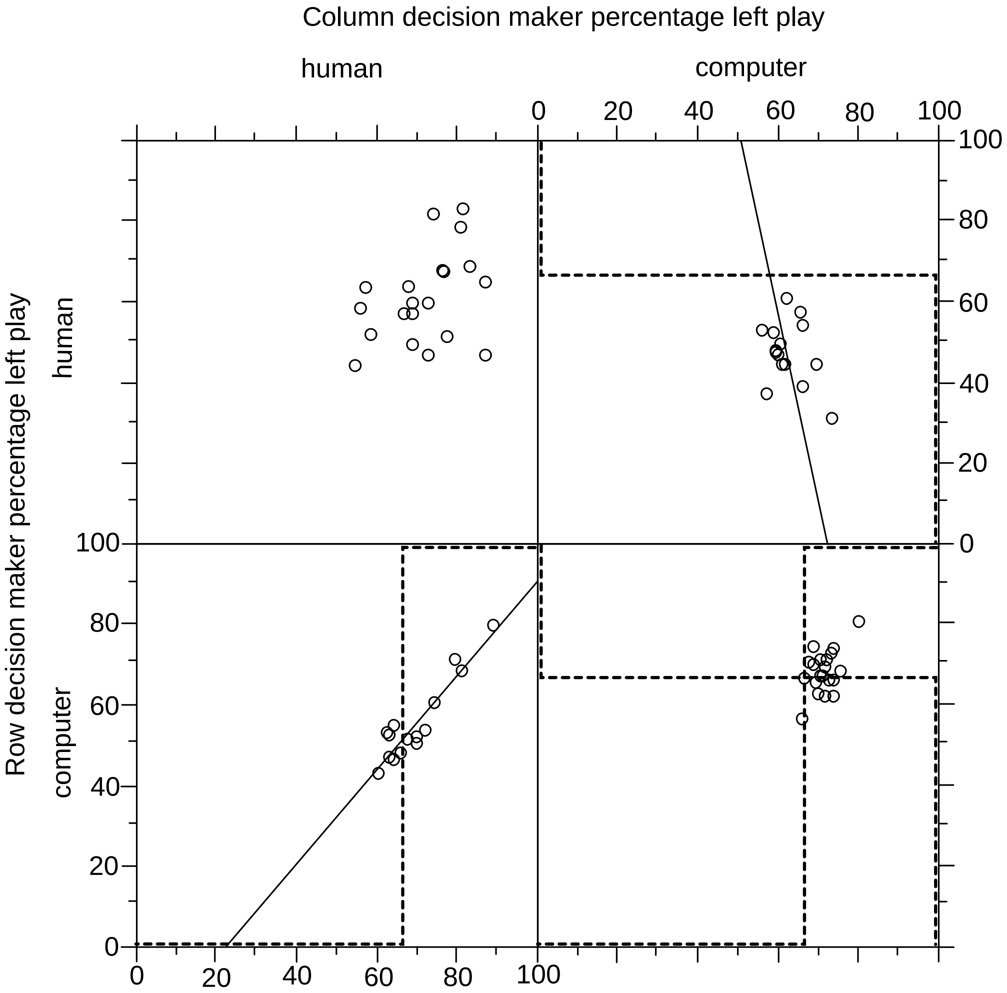


Figure 4: Pursue-Evade joint densities of proportion Left; ER interactions.

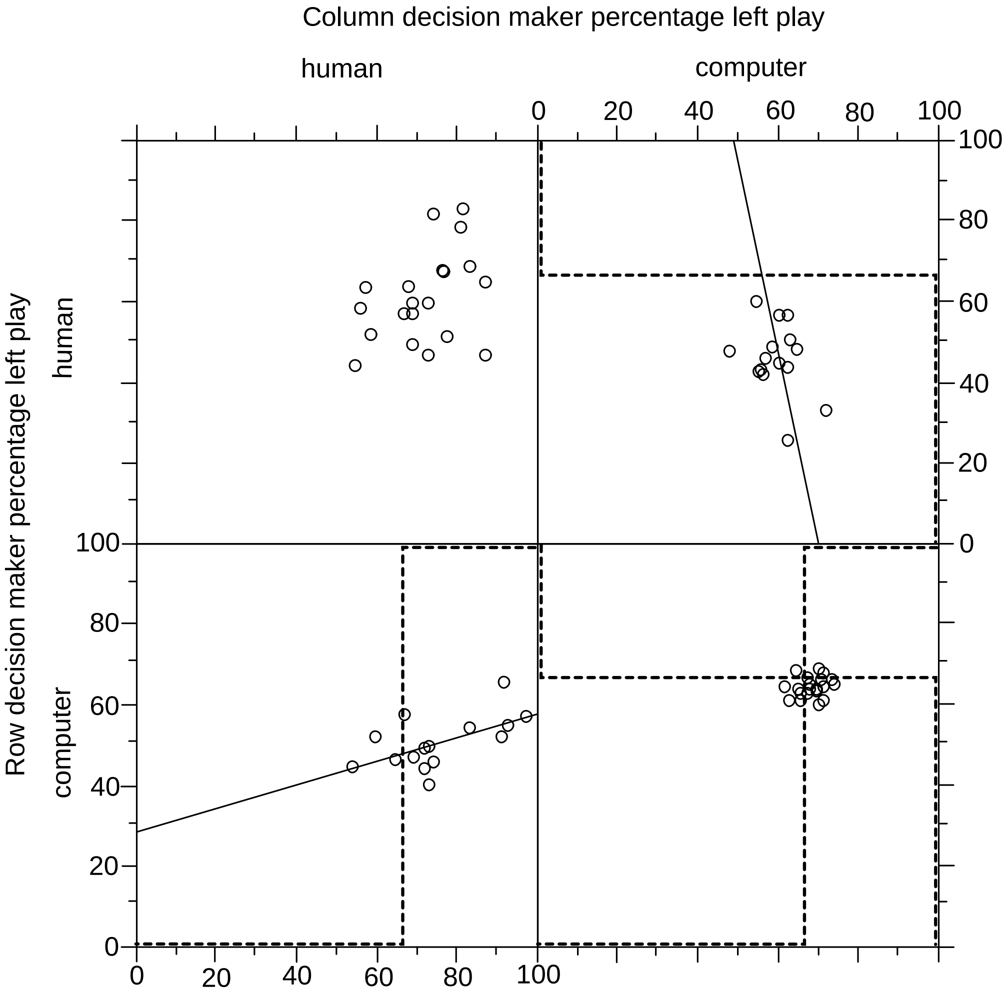
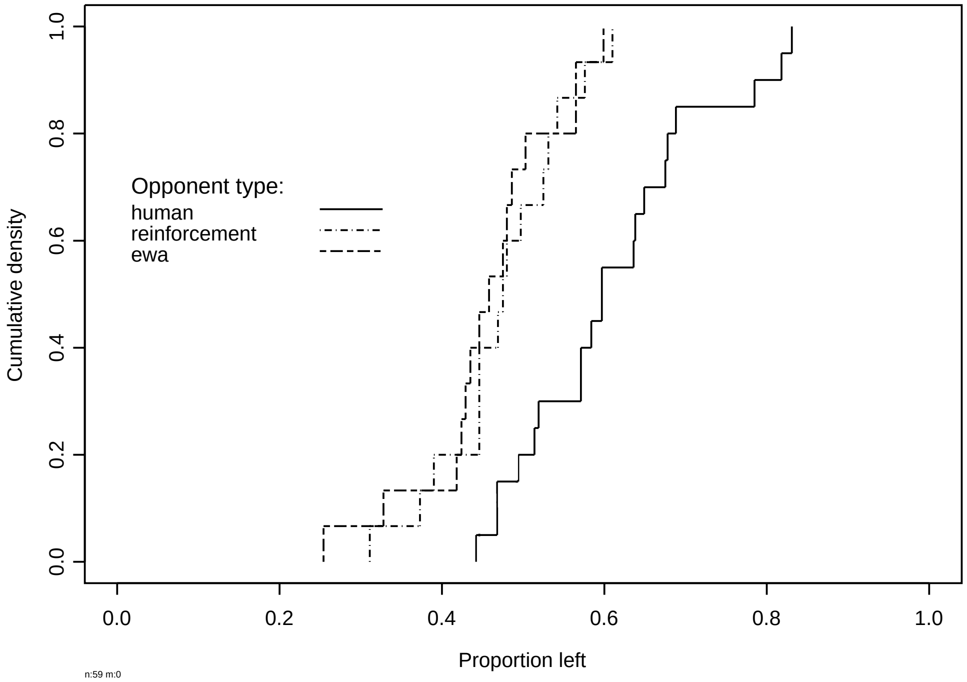
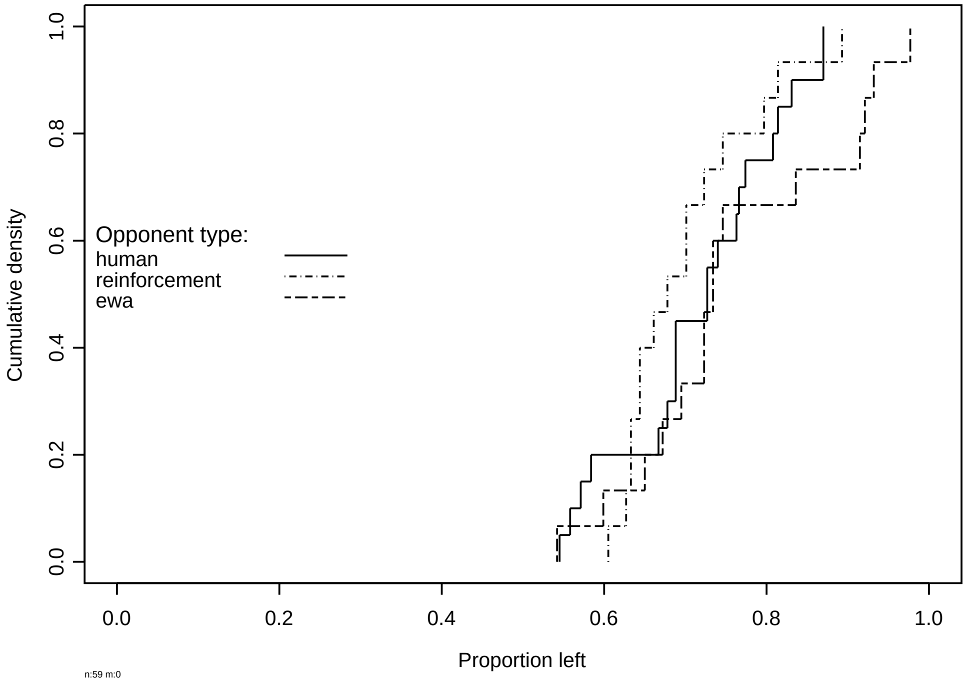


Figure 5: Pursue-Evade joint densities of proportion Left; EWA interactions.



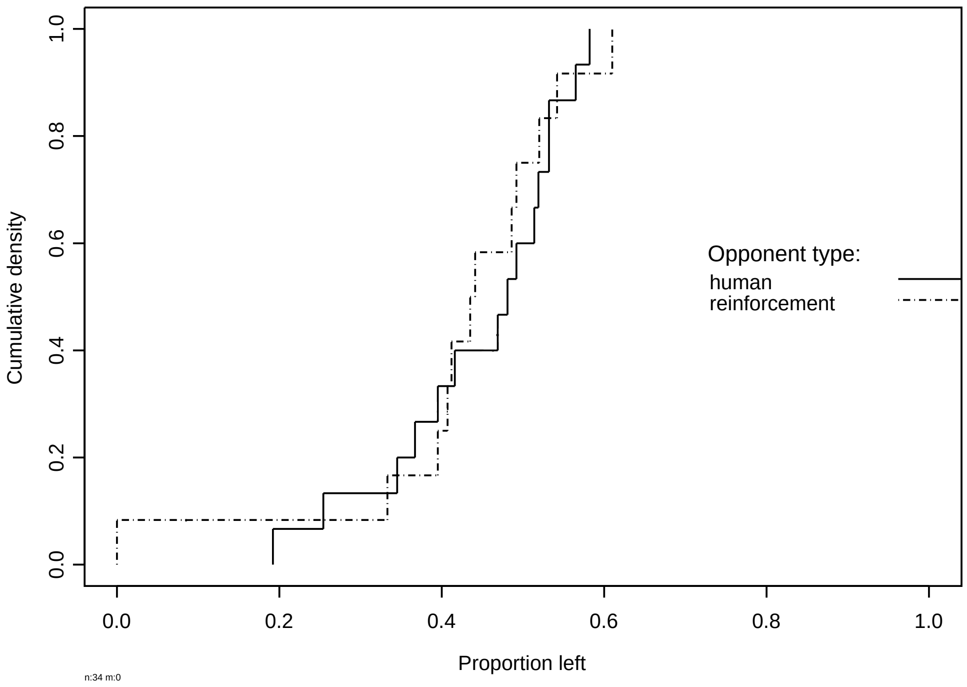
Dist. Left when facing Human tested against dist. Left when facing:	KS statistic	P-value
ER	0.567	0.005
EWA	0.633	0.001

Figure 6: Distributions of Left by Human Row players in Pursue-Evade.



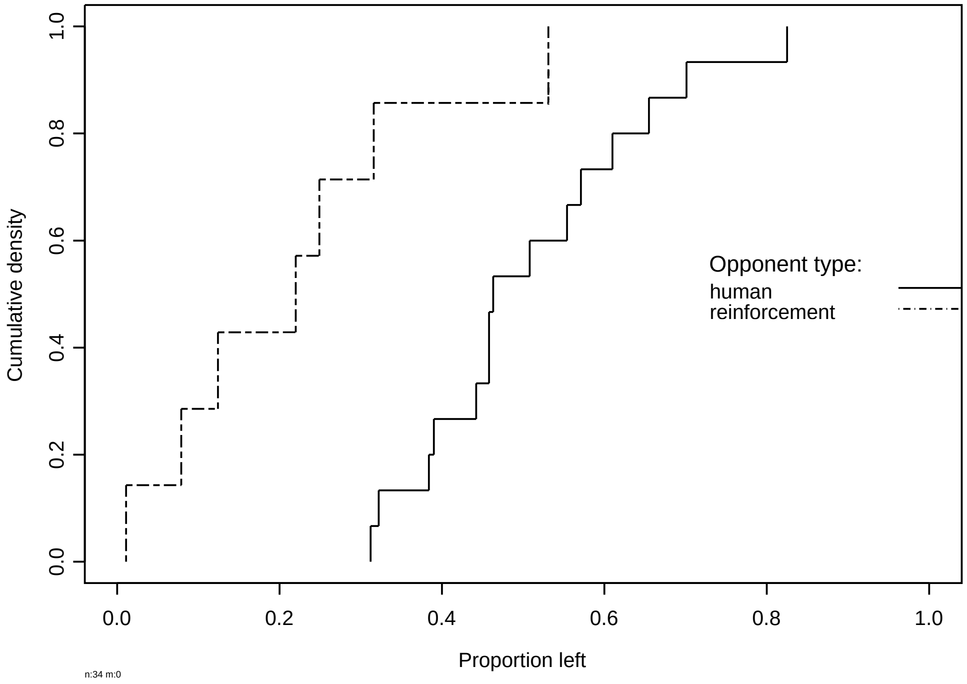
Dist. Left when facing Human tested against dist. Left when facing:	KS statistic	P-value
ER	0.283	0.435
EWA	0.267	0.507

Figure 7: Distributions of Left by Human Column players in Pursue-Evade.



Dist. Left when facing		
Human tested against		
dist. Left when facing:	KS statistic	P-value
ER	0.183	0.952

Figure 8: Distributions of Left by Human Row players in Gamble-Safe.



Dist. Left when facing Human tested against dist. Left when facing:	KS statistic	P-value
ER	0.483	0.061

Figure 9: Distributions of Left by Human Column players in Gamble-Safe.