A "Risky" Perspective on Uncertainty: Using Lower Envelope Lotteries To Examine Ambiguity Attitudes

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Abstract

We study decision-making under Knightian uncertainty using lower envelope lotteries. Lower envelope lotteries specify lower bounds on probabilities for monetary outcomes and the aggregate amount of unassigned probability mass, an objective quantity we call ambiguity. To examine ambiguity preferences we adapt α-MaxMin Expected Utility (αMEU) to this setting. The αMEU model has one more parameter than expected utility, α, that measures an everywhere-constant ambiguity attitude. Using non-parametric tests we find that 120 of 203 participants (59%) in our experiment made choices consistent with the postulates of an αMEU representation. Parameter estimates from a finite mixture model reveal that, for these 120 participants, 48% exhibited ambiguity aversion, 30% ambiguity neutrality, and 22% ambiguity seeking.

For the participants whose choices could not be rationalized with αMEU we introduce a generalization called $\alpha\beta$-MaxMin Expected Utility ($\alpha\beta$MEU). The $\alpha\beta$MEU model has one more parameter than αMEU. It retains the linear indifference curves of αMEU but allows for a non-constant ambiguity attitude. For the 41% of participants who exhibited non-constant ambiguity attitudes, finite mixture estimates of $\alpha\beta$MEU reveal that aversion to ambiguity increases as the expectation for receiving a good outcome increases. At an intuitive level these estimates typify behavior in which ambiguity is thought of as 'good' when the expectation of getting a good outcome is low, but 'bad' when the expectation of getting a good outcome is high.

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Economists make a distinction between risk and uncertainty. Risk involves fully-known probabilities while uncertainty involves unknown probabilities (Knight, 1921). Mathematically, risk is described with probabilities assigned to monetary outcomes. And in studying decision-making under risk, these probability distributions, or lotteries, are the objects of choice. Uncertainty, in contrast, is mathematically described with states of the world, monetary outcomes, and bets, which serve as a mapping rule between states and monetary outcomes. In studying decision-making under uncertainty the objects of choice are bets (also called acts by Anscombe and Aumann (1963) and others).

This paper examines decision-making under uncertainty using objects of choice that are simpler than bets/acts and are quite similar to probability distributions. We call these objects *lower envelope lotteries*.[1] Mathematically, lower envelope lotteries specify lower bounds on probabilities, $\{\underline{p}_1, \ldots, \underline{p}_K\}$, for a set of monetary outcomes, $\{z_1, \ldots, z_K\}$, and the amount of unassigned probability mass $y = 1 - \sum_{k=1}^{K} \underline{p}_k$, an objective quantity we call *ambiguity*. Formally, we denote a lower envelope lottery as $L = \left(\underline{p}_1, \ldots, \underline{p}_K, y\right)$.

Lower envelope lotteries are simpler than the bets/acts which serve as traditional objects of choice under uncertainty. Moreover, lower envelope lotteries are a simplification of the 'sets of lotteries' setting investigated theoretically by Olszewski (2007), Ahn (2008) and Dumav and Stinchcombe (2013). This simplicity is advantageous, however, because it permits a tractable and intuitive way to describe the objects of choice under varying ambiguity, especially in experimental settings. For example, Section 1 documents how the well-known, one-urn Ellsberg experiment can be described using lower envelope lotteries and plotted in a figure we call the *uncertainty triangle*.[2]

To examine ambiguity preferences we collected experimental data in which the objects of choice were two-outcome lower envelope lotteries. In each experimental choice-situation, six alternatives were available: one fully-specified lottery and five lower envelope lotteries. The design of our experiment is based on traditional demand elicitation. We systematically varied (i) the relative tradeoff between ambiguity and the minimum probabilities assigned to each outcome, a value that can be interpreted as a *price* along a linear budget, and (ii) the probability assigned to

[1] This notion is motivated by the literature on imprecise probabilities which describes partial knowledge about a probability distribution by the lower (and upper) envelopes of the set of candidate distributions. See Dempster (1967) and Shafer (1976).

[2] A figure similar to what we call the uncertainty triangle has also been used by Dumav and Stinchcombe (2013) to illustrate, for example, decreasing ambiguity aversion. See specifically Dumav and Stinchcombe (2013) Figure 2.

the good outcome, for the fully-specified lottery available in the choice situation, a value that can be interpreted as *wealth*.

To analyze participant's choices we start by assessing the generalized axiom of revealed preference (GARP). Choices that adhere to GARP can be modeled as if they resulted from maximizing a preference relation that is complete, transitive, continuous, and monotone (Varian, 1982). These assumptions serve as a foundation for most theories of choice under uncertainty.[3] In our experimental data, we find very high rates of GARP compliance – more than 95% of participants made choices consistent with the generalized axiom, a result consistent with Ahn et al. (2014). We interpret this high rate of GARP-compliance as positive news for theoretical efforts directed at choice under uncertainty.

And there has been extensive theoretical effort directed at choice under uncertainty. One widely discussed example is the α-MaxMin Expected Utility (αMEU) representation (Marinacci, 2002; Ghirardato et al., 2004). We use the αMEU representation as a parsimonious description of behavior in our setting.[4] The αMEU model has one more parameter than expected utility, α, that captures an everywhere-constant ambiguity attitude. The parsimony of αMEU, however, requires a great deal of structure for choices made in our experimental setting. And we assess this structure with a non-parametric test that determines whether a participant's choices can be rationalized with an αMEU representation.[5] The majority of participants in our experiment (59%; 120 out of 203) made choices that can be modeled with an αMEU representation. For these 120 participants, we estimate a finite mixture model. What emerges from finite mixture estimates are three distinct ambiguity preference 'types.' Two of these types correspond to ambiguity aversion and ambiguity seeking (48% and 21% of the 120 participants, respectively). The third behavioral type, can be broadly characterized as ambiguity neutral (30%).[6]

About 40% of participants (83 out of 203) made choices inconsistent with an αMEU representation. This indicates that a more flexible model is required for rationalizing their behavior.

[3] The most prominent exception being Bewley (2002).

[4] Under a mild monotonicity assumption our adaptation of αMEU is identical to the $H_{u,\alpha}$ representations developed by Olszewski (2007). We utilize the αMEU nomenclature, however, for expository convenience.

[5] Note that we do not assess the procedural validity of the multiple-priors assumption embedded in the αMEU theory. We instead use the αMEU representation as a parsimonious model for describing behavior and focus purely on whether choices can be modeled with an indifference map generated under this representation.

[6] Our two-step approach combining non-parametric assessments of individual representations and parametric finite mixture estimates is similar to the analysis in Ahn et al. (2014) but distinct from the purely parametric analyses in Hey et al. (2010), Hey and Pace (2014), and Stahl (2014).

But what type of flexibility is necessary to rationalize these participant's choices? Was it that the everywhere-constant ambiguity attitude embedded in the αMEU representation was violated, while other properties of αMEU were retained (specifically, linearity along a single indifference curve). Or, were there broader inconsistencies with αMEU (e.g. non-linearities in the indifference curves defined on the set of lower envelope lotteries)?

To explore these issues we closely examine choices made by these 83 experimental participants. This examination reveals a pattern of choices consistent with an increasing aversion toward ambiguity as the likelihood for a good outcome increases. More specifically, we find that relaxing the everywhere-constant ambiguity attitude embedded in the αMEU model can serve as a parsimonious way to rationalize these 83 participant's choices. And to parametrically characterize these varying ambiguity attitudes we introduce a model that, in an empirically expedient fashion, generalizes αMEU. We call this model $\alpha\beta$-MaxMin Expected Utility ($\alpha\beta$MEU). The $\alpha\beta$MEU model has one more parameter than αMEU and retains the linear indifference curves of αMEU but permits indifference curves to 'fan-in' or 'fan-out' across the lower envelope lottery space.[7] Again using finite mixture methods we show that under an $\alpha\beta$MEU representation these 83 participants exhibit increasing rates of aversion toward ambiguity as the expectation for a good outcome improves.

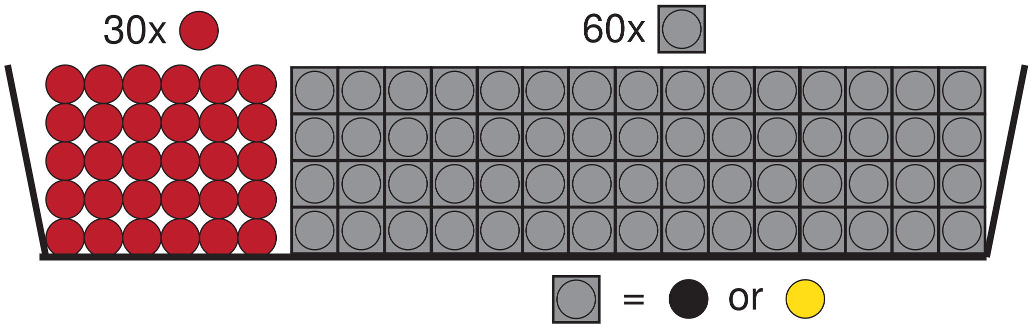
We believe that the pattern of increasing aversion toward ambiguity has a strong intuitive appeal. Consider, by way of illustration, a man who has just been diagnosed with cancer that has a 90% chance of recovery and a 10% chance of death (if left untreated). Further suppose that he faces two treatment options. The first option is conventional and conditions the chance of recovery by 5 percentage points. The second treatment option is novel and it is unknown what, if any, effect it has on the probability of recovery – it could increase the probability of death. It is easy to imagine that the man chooses the conventional treatment as the ambiguity associated with the novel treatment has the potential for a large downside. Now, suppose instead that the man's prognosis is much more grim: a 95% chance of death if the cancer is untreated. Facing the same two treatment options (increasing the chance of recovery by 5 percentage points vs. a new treatment which has an unknown effect on the likelihood of recovery) it seems quite plausible

[7] In terms of the indifference curves generated in lower envelope lottery space the $\alpha\beta$MEU model shares some similarities with the weighted utility model developed for choice under risk (Chew and MacCrimmon, 1979; Chew, 1983; Fishburn, 1983; Chew, 1989). Specifically, $\alpha\beta$MEU can produce fanning-in or fanning-out of indifference curves in the uncertainty triangle while weighted utility can produce fanning-in or fanning-out in the probability triangle.

that the man elects for the novel treatment. Indeed, in the case where death is nearly certain, ambiguity can be considered something quite good as it provides a glimmer of hope that the conventional treatment does not. While our experiment does not involve such life and death considerations, the pattern of increasing aversion to ambiguity that we capture with the $\alpha\beta$MEU model is consistent with this type of behavior. And as we discuss more thoroughly in Section 4, this pattern of behavior is also consistent with the results documented by Abdellaoui et al. (2011).

The remainder of our paper is divided into four sections. The next section discusses how the Ellsberg one-urn experiment can be described using lower envelope lotteries. Using a figure similar to a probability triangle (Marschak, 1950; Machina, 1982), we illustrate how canonical behavior in the Ellsberg experiment can be rationalized with an αMEU representation, adapted to the setting of lower envelope lotteries. Section 2 lays out our experiment design and methods. Section 3 presents our analysis of choices made in this experiment. The final section discusses our results, establishes connections between our work and the empirical and theoretical ambiguity literature, and concludes.

Figure 1: The Ellsberg Urn



1 The Ellsberg One-Urn Experiment

The Ellsberg one-urn thought experiment involves an urn that contains 90 balls (see Figure 1). Exactly 30 of the balls are **red**. Each of the remaining 60 balls are either **black** or **yellow** so that the proportion of **black** to **yellow** balls are unknown. Exactly one ball is drawn from the urn to determine the decision maker's payment, which could be $100 or $0.

In Ellsberg's first choice-situation, the decision maker has to select between a bet on **red** and a bet on **black**. If the color of the ball drawn from the urn matches the color of a decision maker's bet then he receives $100. If the ball drawn does not match his bet then he receives $0. The first two rows in Table 1 summarize these bets and the payoffs associated with each of the possible colors of balls that can be drawn from the urn. When surveyed, most people preferred to bet on **red**.

In Ellsberg's second choice-situation, the decision maker has to select between a bet on **red** or **yellow** and a bet on **black** or **yellow**. If the color of the ball drawn from the urn matches either of the colors of a decision maker's bet then he receives $100. If the ball drawn does not match his bet then he receives $0. The last two rows in Table 1 summarize these bets and the payoffs associated with each of the possible colors of balls that could be drawn from the urn. When surveyed, most people preferred to bet on **black** or **yellow**.

Table 1: The Ellsberg One-Urn Experiment Described as Lower Envelope Lotteries

Bet on ...	Drawn-Ball Red	Black	Yellow	Lower Envelope Lotteries $(\underline{p}_1, \underline{p}_0, y)$
Red $(*)$	100	0	0	$\left(\frac{1}{3}, \frac{2}{3}, 0\right) = R$
Black	0	100	0	$\left(0, \frac{1}{3}, \frac{2}{3}\right) = B$
Red or **Yellow**	100	0	100	$\left(\frac{1}{3}, 0, \frac{2}{3}\right) = RY$
Black or **Yellow** $(*)$	0	100	100	$\left(\frac{2}{3}, \frac{1}{3}, 0\right) = BY$

$*$ indicates the modal choice observed in this type of experiments.

1.1 The Ellsberg Experiment as Lower Envelope Lotteries

All of the bets in the Ellsberg experiment can be described with lower envelope lotteries. Adopt the convention $\left(\underline{p}_1, \underline{p}_0, y\right)$, where $\underline{p}_1$ is the minimum probability of receiving \$100, $\underline{p}_0$ is the minimum probability of receiving \$0, and y is the amount of ambiguity or unassigned probability.

Consider first the bet on **red**. Exactly 30 of the 90 balls in the urn are **red** so the probability of receiving \$100 is exactly $\frac{1}{3}$. Exactly 60 of the 90 balls are not **red** so the probability of receiving \$0 is exactly $\frac{2}{3}$. Denoting this as a lower envelope lottery: $\left(\frac{1}{3}, \frac{2}{3}, 0\right) = R$.

Next, consider the bet on **black**. Of the 90 balls in the urn there could be any number between 0 and 60 that are **black**. So the probability of receiving \$100 could be anywhere between 0 and $\frac{2}{3}$. This means that the minimum probability of receiving \$100 is 0. That the maximum probability of receiving \$100 is $\frac{2}{3}$ means that the minimum probability of receiving \$0 is $\frac{1}{3}$. Denoting this as a lower envelope lottery: $\left(0, \frac{1}{3}, \frac{2}{3}\right) = B$.

The third bet in the Ellsberg experiment is for **red** or **yellow**. Because there are exactly 30 **red** balls in the urn and any number between 0 and 60 **yellow** balls in the urn, the minimum probability of receiving \$100 is $\frac{1}{3}$ while the maximum is 1. Because the maximum probability of receiving \$100 is 1, the minimum probability of receiving \$0 is 0. Denoting this as a lower envelope lottery: $\left(\frac{1}{3}, 0, \frac{2}{3}\right) = RY$.

The fourth bet in Ellsberg's experiment is for **black** or **yellow**. There are exactly 60 **black** or **yellow** balls in the urn so the probability of receiving \$100 is exactly $\frac{2}{3}$. And exactly 30 of the 90 balls are **red** so the probability of receiving \$0 is exactly $\frac{1}{3}$. Denoting this as a lower envelope lottery: $\left(\frac{2}{3}, \frac{1}{3}, 0\right) = BY$. The right-most column of Table 1 shows the lower envelope lotteries for each of the four bets in Ellsberg's experiment.

1.2 A Graphical Illustration of the Ellsberg Experiment In The Uncertainty Triangle

The lower envelope lotteries that describe the Ellsberg experiment have three, non-negative entries that sum to one. This allows them to be graphically depicted with a figure similar to a probability triangle. An example of such a triangle is shown in Figure 2(a) and we refer to this as the *uncertainty triangle*. The lower-left vertex of the uncertainty triangle represents the lower envelope lottery $(0, 1, 0)$ – certainty of receiving \$0. The upper-right vertex represents $(1, 0, 0)$ – certainty of receiving \$100. The lower-right vertex represents $(0, 0, 1)$ – probabilities are completely unknown. Movements from this vertex to the left involve a decrease of ambiguity and an increase in the minimum probability assigned to the \$0 outcome, $\underline{p}_0$ – Along the horizontal leg of the uncertainty triangle no minimum probability for the \$100 outcome is specified so that $\underline{p}_0 + y = 1$. Movements upward from the lower-right vertex involve a decrease of ambiguity and an increase in the minimum probability assigned to the \$100 outcome, $\underline{p}_1$ – Along the vertical leg of the triangle no minimum probability for the \$0 outcome is specified so that $\underline{p}_1 + y = 1$.

Two of the lower envelope lotteries implied by bets in Ellsberg's experiment are fully-specified lotteries: R and BY. As can be seen in Figure 2(b), both of these lotteries lie on the hypotenuse of the uncertainty triangle. The other two lower envelope lotteries implied by the bets in Ellsberg's experiment, B and RY, lie on the horizontal-leg and vertical-leg of the triangle, respectively. The parallel, dashed lines connecting R to B, and BY to RY, indicate the pairs of lower envelope lotteries that are implied by the two choice-situations in Ellsberg's experiment.

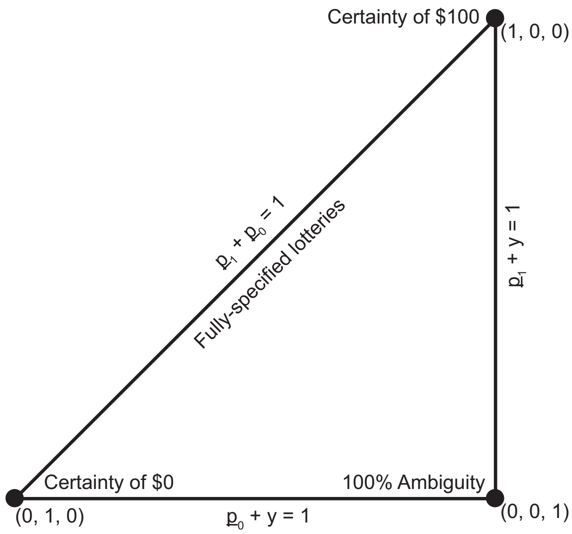
1.3 Rationalizing Choices in the Ellsberg Experiment with αMEU

The canonical pattern of choices in Ellsberg's experiment have participants betting on **red** in the first choice-situation and betting on **black** or **yellow** in the second choice-situation. Adopting the lower envelope lottery notation: $R \succ B$ and $BY \succ RY$. Figure 2(b) indicates these choices with blue circles.

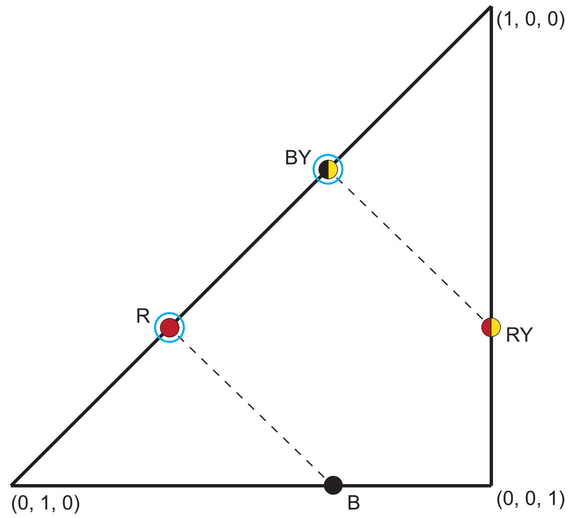
Many models have been proposed that can rationalize the canonical pattern of choices in Ells-

Figure 2: Uncertainty Triangles Illustrating Ellsberg's Experiment and how α-MaxMin Expected Utility (αMEU) can Rationalize Canonical Choices Therein

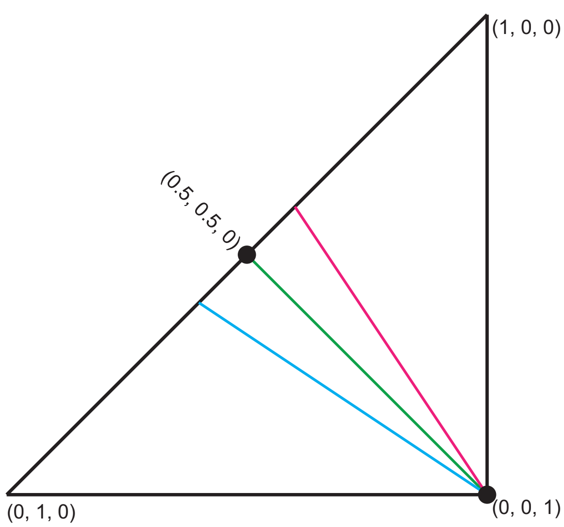
(a) The uncertainty triangle can depict two-outcome lower envelope lotteries



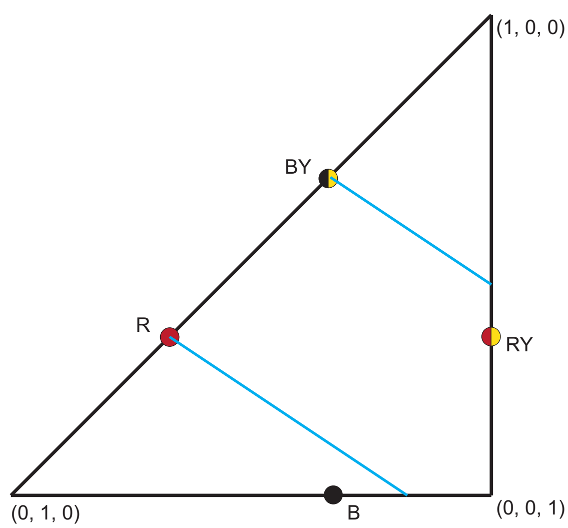
(b) Ellsberg's bets depicted in the uncertainty triangle. The blue circles show the canonical Ellsberg choices ($R \succ B$ and $BY \succ RY$)



(c) Three *distinct* ambiguity preferences: ambiguity **aversion** $(\alpha = \frac{3}{5})$, **neutrality** $(\alpha = \frac{1}{2})$, and **seeking** $(\alpha = \frac{2}{5})$



(d) Ambiguity **averse** αMEU preferences can rationalize canonical Ellsberg choices



berg's experiment.[8] One widely discussed proposal is the αMaxMin Expected Utility (αMEU) model introduced by Marinacci (2002). While the αMEU model is specified for the bets/acts commonly used to examine decision making under uncertainty, it can be adapted to the setting of lower envelope lotteries. Adopting the convention that outcomes are ordered from best to worst, so that utilities for outcomes are such that $u_1 > u_2 > \cdots > u_K$, the αMEU can be written as

$$\alpha\text{MEU}\left(\underline{p}_1, \ldots, \underline{p}_K, y\right) = \alpha\left[\left(\underline{p}_K + y\right)u_K + \sum_{k=1}^{K-1}\underline{p}_k u_k\right] + (1-\alpha)\left[\left(\underline{p}_1 + y\right)u_1 + \sum_{k=2}^{K}\underline{p}_k u_k\right]. \quad (1)$$

The expression in the left-hand brackets captures total pessimism: all ambiguity (y) is added to the minimum probability of receiving the worst outcome $(\underline{p}_K)$. The expression in the right-hand brackets captures total optimism: all ambiguity (y) is added to the minimum probability of receiving the best outcome $(\underline{p}_1)$. The ambiguity preference parameter, $\alpha \in [0,1]$, thus represents a weighting between total pessimism and total optimism. Obviously, if $\alpha = 0$ a decision maker is perfectly optimistic, such that they are said to be ambiguity seeking, while if $\alpha = 1$ a decision maker is perfectly pessimistic, which is considered ambiguity averse.[9] Also note that when a lower envelope lottery is a fully specified lottery (i.e. $y = 0$), αMEU reduces to expected utility.

The αMEU for the Ellsberg experiment can be written as:

$$\alpha\text{MEU}\left(\underline{p}_1, \underline{p}_0, y\right) = \alpha\left[\underline{p}_1 u_1 + \left(\underline{p}_0 + y\right)u_0\right] + (1-\alpha)\left[(\underline{p}_1 + y)u_1 + \underline{p}_0 u_0\right]. \quad (2)$$

In the uncertainty triangle, this two-outcome αMEU model gives rise to indifference curves that are parallel, straight-lines. And the slope of these indifference curves is determined by the ambiguity preference parameter $\alpha \in [0,1]$. The following definition elucidates the relationship between α and commonly used descriptions of behavior.

Definition 1. *An α-MaxMin Expected Utility (αMEU) decision maker is ambiguity **averse**, **neutral**, or **seeking** whenever:*

$$\alpha > \frac{1}{2}, \quad \alpha = \frac{1}{2}, \quad \alpha < \frac{1}{2} \quad (3)$$

[8]The first theories that could rationalize canonical Ellsberg behavior were David Schmeidler's Choquet Expected Utility theory (Schmeidler, 1989) and Itzhak Gilboa and David Schmeidler's MaxMin Expected Utility Theory (Gilboa and Schmeidler, 1989).

[9] The former is also known as maxmax expected utility, and the latter as maxmin expected utility (Gilboa and Schmeidler, 1989).

To clarify the geometric interpretation of ambiguity attitudes in the uncertainty triangle, consider a choice between 100% ambiguity $(0, 0, 1)$ and a 50/50 lottery $\left(\frac{1}{2}, \frac{1}{2}, 0\right)$. Figure 2(c) plots these two alternatives in the uncertainty triangle. The perfectly ambiguous alternative is at the lower-right vertex of the triangle and the 50/50 lottery is halfway between the endpoints of the hypotenuse. Because there is a one-to-one tradeoff between the ambiguity and the probabilities attached to each outcome, an ambiguity **averse** decision maker would strictly prefer the 50/50 lottery, so that

$$\frac{1}{2}u_0 + \frac{1}{2}u_1 > \alpha u_0 + (1-\alpha)u_1 \quad \leftrightarrow \quad \alpha > \frac{1}{2}, \tag{4}$$

an ambiguity **neutral** decision maker would be indifferent, such that

$$\frac{1}{2}u_0 + \frac{1}{2}u_1 = \alpha u_0 + (1-\alpha)u_1 \quad \leftrightarrow \quad \alpha = \frac{1}{2}, \tag{5}$$

and an ambiguity **seeking** decision maker would strictly prefer the ambiguous option so that

$$\frac{1}{2}u_0 + \frac{1}{2}u_1 < \alpha u_0 + (1-\alpha)u_1 \quad \leftrightarrow \quad \alpha < \frac{1}{2}. \tag{6}$$

Figure 2(c) depicts three indifference curves, one for each of these three, distinct types of ambiguity preferences. The pink indifference curve depicts an ambiguity **seeking** preference (specifically, $\alpha = \frac{2}{5}$). The green indifference curve depicts an ambiguity **neutral** preference ($\alpha = \frac{1}{2}$). And the blue indifference curve depicts an ambiguity **averse** preference ($\alpha = \frac{3}{5}$). Figure 2(d) illustrates how these ambiguity **averse** preferences can rationalize the canonical pattern of choices in the Ellsberg experiment. Similar to how expected utility produces parallel, straight-line indifference curves in the probability triangle, αMEU produces indifference curves that are parallel straight-lines in the uncertainty triangle. So, an ambiguity **averse** indifference curve that has $R \succ B$ must also have $BY \succ RY$, which is the canonical pattern of choices in Ellsberg's experiment.

2 Experimental Design

To study ambiguity preferences we expanded upon the (implied) design of Ellsberg's one-urn experiment in lower envelope lottery space. Similar to the Ellsberg experiment, we used only two monetary outcomes: 60 or 20 Swiss Francs.[10] In contrast to the Ellsberg experiment, our design had many more choice-situations, each with several alternatives. Broadly speaking, our experimental design was a traditional demand elicitation mechanism for minimum probabilities and ambiguity. We systematically varied (i) the relative tradeoff between minimum probabilities and ambiguity, a value we interpret as the price for a Walrasian budget, and (ii) the probability assigned to the 60 Swiss Franc (CHF) outcome for the fully-specified lottery available in a choice-situation, a value we interpret as wealth.

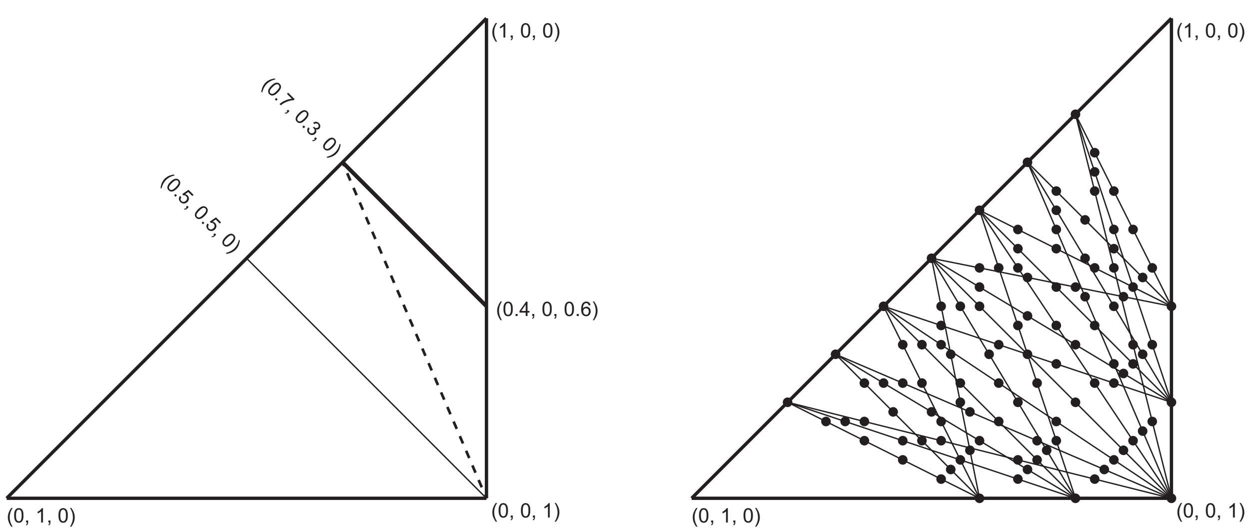
To illustrate how our design can be viewed as traditional demand elicitation, consider the three example budgets in Figure 3(a). The thin, solid-line represents a budget in which the tradeoff between ambiguity and the minimum probabilities of receiving each outcome is one-to-one. This is easy to see when moving from the end of the budget at the lower-right vertex (100% ambiguity), to the end of the budget at the midpoint on the hypotenuse (the 50/50 lottery). This movement

[10] At the time of the experiment, one Swiss Franc (CHF) was worth approximately \$1.10.

Figure 3: The Design Of Our Experiment

(a) Three budgets illustrating a wealth increase (thin line to thick line) and price change (thick line to dashed line)

(b) The 25 budgets used in our experiment, each featuring six alternatives



completely reduces the 100% ambiguity into an equal division of probabilities. Next, consider the budget represented with the thick, solid-line connecting the point $(0.4, 0, 0.6)$ on the vertical leg of the triangle to the 70/30 lottery on the hypotenuse. This budget retains the one-to-one tradeoff between ambiguity and minimum probabilities, but increases the wealth, or the probability for the CHF 60 outcome in the fully-specified lottery available from that budget. Contrast that with the dashed line, which keeps wealth constant, but changes the tradeoff between minimum probabilities and ambiguity such that a one unit reduction in ambiguity results in a more-than-one unit increase in the minimum probability of CHF 60, and a smaller-than-one-unit increase in the minimum probability of CHF 20. Put simply, the minimum probability for CHF 60 is relatively cheap.

The budgets used in our experiment are plotted in Figure 3(b). The black lines depict the budgets. The dots on each line depict the alternatives available on that budget. This set of budgets provide a wide range of price and wealth variation that permits a close examination of preferences towards ambiguity.[11]

2.1 Elicitation Software

Participant choices were elicited using software programmed with the Psychtoolbox Matlab libraries freely available at http://www.psychtoolbox.org. Figure 4 shows three screenshots from our elicitation software. The row of boxes at the bottom of each screenshot, each with a thumbnail image, provide visual depictions of all of the lower envelope lotteries available in the choice situation. When participants moved their mouse over a box, the corresponding thumbnail image was drawn as a large image in the center of the screen. The large image provides detailed outcome, minimum probability, and ambiguity information for the alternative depicted by the highlighted thumbnail. Figure 4(a) is a depiction of the thin budget line in Figure 3(a). The right-most alternative is the 100% ambiguous alternative and the left-most alternative is the 50/50 lottery. Figure 4(b) depicts the same one-to-one tradeoff but in a "high wealth" setting – The probability assigned to the CHF 60 outcome in the fully-specified lottery is 70%. The screenshot in 3(b) depicts the thick budget line that is closest to the upper-right vertex in Figure 3(a). Figure 4(c) depicts the

[11] Table 4 in Appendix B provides a detailed summary of the prices and wealths for each of these budgets. Also in Appendix B, Table 5 provides a comprehensive list of the lower envelope lotteries available on each budget.

same "wealth" as in 4(b) but the tradeoff between ambiguity and minimum probabilities is tilted heavily in favor of the CHF 60 outcome – it is the dashed budget line in Figure 3(a).

Participants were instructed to select the alternative they wanted most in each choice-situation. They indicated their choice by clicking the mouse on the relevant box, such that it was depicted as the large image in the middle of the screen. After clicking the box "Confirm" and "Cancel" buttons appeared. Clicking the "Cancel" button allowed a different alternative to be selected, while clicking the "Confirm" button moved on to the next choice-situation.

2.2 Resolution of Uncertainty

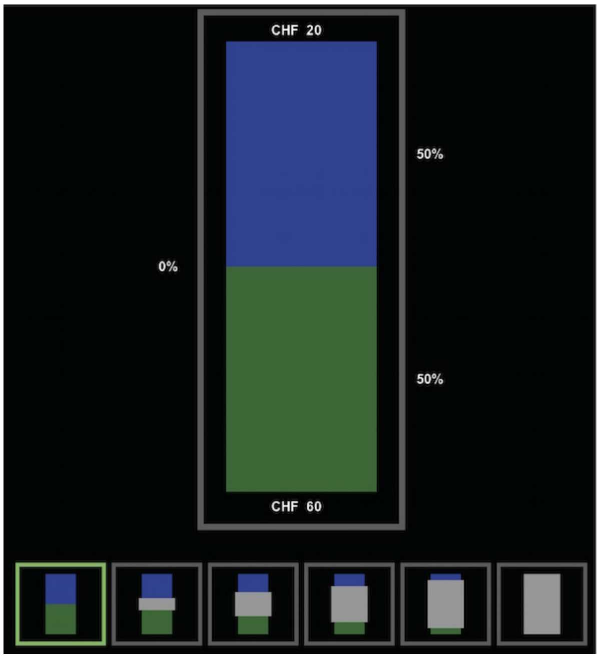
The choice-situations in our experiment made available a large number of distinct lower envelope lotteries. To be able to resolve the uncertainty for each of these lower envelope lotteries, using a single source, we constructed an urn with 100 balls in it. Each ball in our urn had an integer between 1 and 100 printed on it. Our urn was constructed in such a way that the number of balls between any odd-number and any larger, even-number was exactly the count between the numbers. Figure 5 provides an example of how this works for the interval 1 to 10. There were exactly 10 balls in our urn that could have the numbers 1 through 10 on them. What was unknown was how many of the balls numbered 1 through 10 had an even-number on them and how many had an odd-number on them. It was possible that each ball had an even-number on it or that each ball had an odd-number on it.[12] This allows us to resolve the uncertainty associated with all of the lower envelope lotteries used in our experiment.

To illustrate how the 100-ball urn could be used to resolve the uncertainty in each of the lower envelope lotteries in our experiment, consider the examples detailed in Figure 6. The first example, detailed in Figure 6(a), is the 50/50 lottery, $\left(\frac{1}{2}, \frac{1}{2}, 0\right)$. In the urn there were exactly 50 balls numbered 1 through 50, and exactly 50 balls numbered 51 through 100. This allowed us to resolve the uncertainty of this lottery. If the ball drawn form the urn had any number 1 through 50 on it,

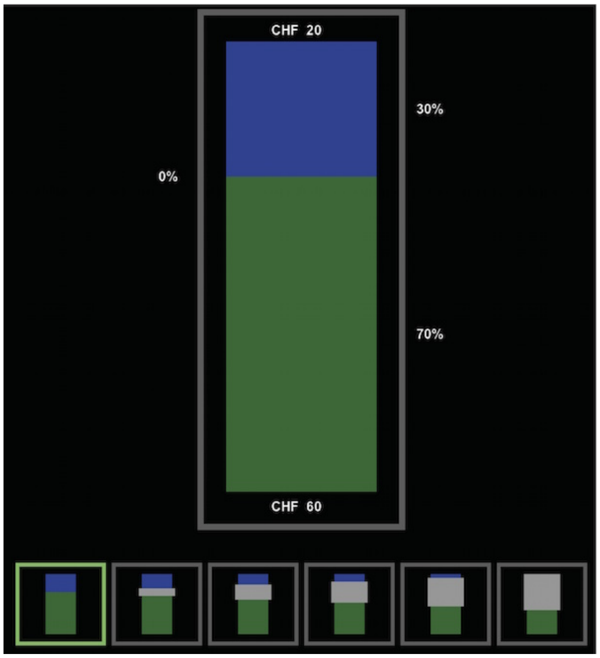
[12]Following Hey et al. (2010), we wanted to insure that participants were not concerned about experimenters 'stacking the urn' against the participants. Thus, experimenters were blind to the exact composition of the urn. This was accomplished by giving various students, faculty, and staff in the Department of Economics at the University of Zürich envelopes with two balls, a pen, and instructions telling them to, for example, write 7 on both balls, write 8 on both balls, or 7 on one ball and 8 on the other ball, and then to seal the envelope. The balls were placed in the urn, and quality control for readability and consistency with the instructions, was done by a person not involved with the experiment. Experimental participants were told how the urn was constructed during the instructions, and informed that experimenters were blinded to the exact composition of the urn.

Figure 4: Screenshots From Our Elicitation Software

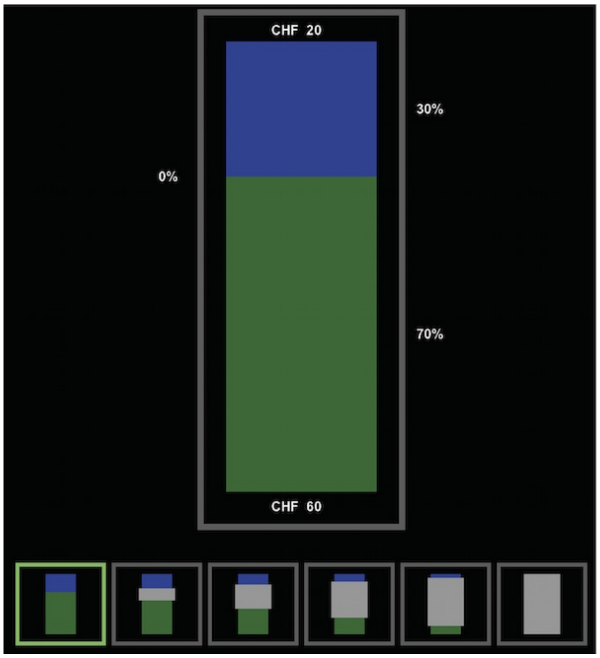
(a) The thin budget line in Figure 3(a)



(b) The thick budget line in Figure 3(a)



(c) The dashed budget line in Figure 3(a)



the 50/50 lottery would pay out CHF 20. If the ball drawn had any number 51 though 100 on it, the lottery would pay out CHF 60.

The second example, detailed in Figure 6(b), is the fully ambiguous lower envelope lottery, $(0, 0, 1)$. In the urn there were an unknown number of balls with even-numbers on them and an unknown number of balls with odd-numbers on them. This allowed us to resolve the uncertainty of this fully ambiguous lower envelope lottery. If the ball drawn from the urn had an odd-number on it, the alternative paid out CHF 20. If the ball drawn from the urn had an even-number on it, the alternative paid out CHF 60.

The final example, detailed in Figure 6(c), is a lower envelope lottery with strictly positive minimum probabilities and ambiguity, $(\frac{2}{10}, \frac{3}{10}, \frac{5}{10})$. In the urn there were exactly 30 balls numbered 1 to 30, 50 balls numbered 31 to 80, and exactly 20 balls with numbers 81 to 100. For the 50 balls numbered 31 to 80, it was unknown how many had an odd-number on them and how many had an even-number on them. This allowed us to resolve the uncertainty of this lower envelope lottery. If the ball drawn from the urn had any number 1 to 30 on it, the alternative paid out CHF 20. If the ball drawn from the urn had any number 81 to 100 on it, the alternative paid out CHF 60. If the ball was an odd-number between 31 and 80, the alternative paid out CHF 20. And if the ball was an even-number between 31 and 80 the alternative paid out CHF 60.

Figure 5: The 100-Ball Urn Used In Our Experiment

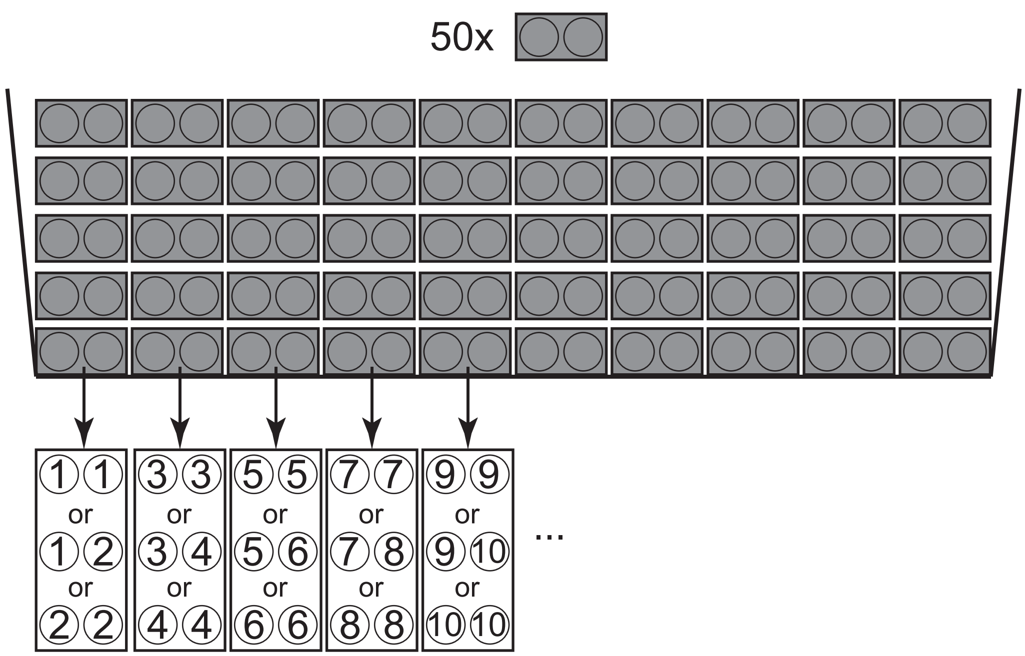
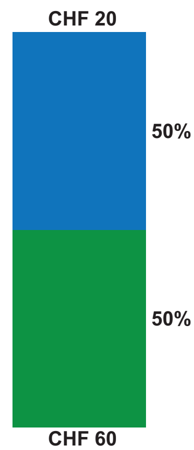


Figure 6: Three Examples Showing How The 100-Ball Urn Was Used To Resolve Uncertainty For The Two-Outcome (CHF 60/20) Lower Envelope Lotteries $\left(\underline{p}_{60}, \underline{p}_{20}, y\right)$ In Our Experiment

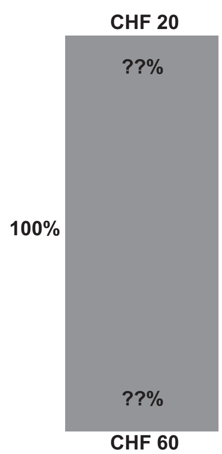
(a) The lower envelope lottery $\left(\frac{1}{2}, \frac{1}{2}, 0\right)$



Resolution of uncertainty

Number on Ball	Payment
1 to 50	CHF 20
51 to 100	CHF 60

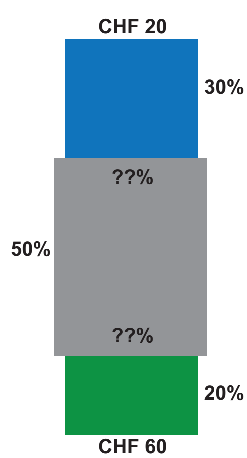
(b) The lower envelope lottery $(0, 0, 1)$



Resolution of uncertainty

Number on Ball	Odd or Even	Payment
1 to 100	Odd	CHF 20
	Even	CHF 60

(c) The lower envelope lottery $\left(\frac{2}{10}, \frac{3}{10}, \frac{5}{10}\right)$



Resolution of uncertainty

Number on Ball	Odd or Even	Payment
1 to 30	Both	CHF 20
31 to 80	Odd	CHF 20
	Even	CHF 60
81 to 100	Both	CHF 60

2.3 Making Draws From The Urn

To make draws from our urn we used a commercially available bingo blower. As can be seen in Figure 7, we occluded the windows on our bingo blower with purple masking tape so that balls could be seen mixing inside the blower but the numbers on the balls could not be seen. Before participants entered the lab they were told that a bingo blower would be used as a randomization device to determine their payment in the experiment. To familiarize participants with the bingo blower it was turned on while they entered the laboratory.

The bingo-blower we used can create a queue of balls in the drawing tube. To avoid ball-queuing in the drawing tube, we inserted a stopper. As can be seen in Figure 7, the stopper was a wooden dowel, cut to the same length as the drawing tube, with a large wooden ball attached to the top. When participants were at the cashier's desk, they removed the stopper so that one ball was "drawn" by the bingo blower. After the participant left the cashier's desk, the drawn-ball and stopper were replaced in the bingo blower.

Figure 7: The Bingo Blower And Stopper Used To Make Draws From The Urn

3 Analyzing Choices

In this section we analyze choices made by 203 participants in experimental sessions conducted at the Decision Science Laboratory, located in the ETH Zurich. All experimental sessions were conducted during one week in July, 2014. All experimental procedures were consistent with a protocol approved by the ethics committee at the University of Zürich.

Our analysis proceeds in three steps. The first step assesses whether participant's choices adhere to the generalized axiom of revealed preferences (GARP). Choices that adhere to GARP can be rationalized with a preference relation that is complete, transitive, continuous, and monotone, foundational assumptions which underlie the majority of theories of choice under uncertainty. Because our experiment was designed as a set of intersecting linear budgets, it has excellent power for detecting violations of GARP. In general, we find very high rates of GARP compliance.

The next analytical step uses non-parametric tests to assess whether individual choices can be rationalized using an αMEU representation. Using these non-parametric tests, we identify a large subset of participants (120 out of 203) whose choices can be represented with αMEU. We then take this subset of αMEU-representable participants and calibrate ambiguity preferences using finite mixture models. We start with just one class of preferences (i.e. a homogeneous preference model) and, one-by-one, increase the number of possible preference classes. This procedure lets the data speak as to the preference-types present in the αMEU-representable participants. Three preference types emerge clearly from this mixture-model analysis. Interestingly, these three types correspond exactly to ambiguity aversion, neutrality, and seeking, in the context of an αMEU representation.

The final step in our analysis examines choices made by the 83 participants which cannot be rationalized with an αMEU representation. We closely examine the choice data to understand why αMEU failed as a parsimonious model. This ad-hoc examination is not as technically rigorous as our analysis of the αMEU representation. We do, however, propose a model, $\alpha\beta$ MaxMin Expected Utility ($\alpha\beta$MEU), that provides a good fit for this sub-sample of participants. The $\alpha\beta$MEU model retains the linear indifference curves of αMEU but its additional parameter, β, allows for fanning-in or fanning-out across the uncertainty triangle.

3.1 Assessing the Generalized Axiom of Revealed Preference (GARP)

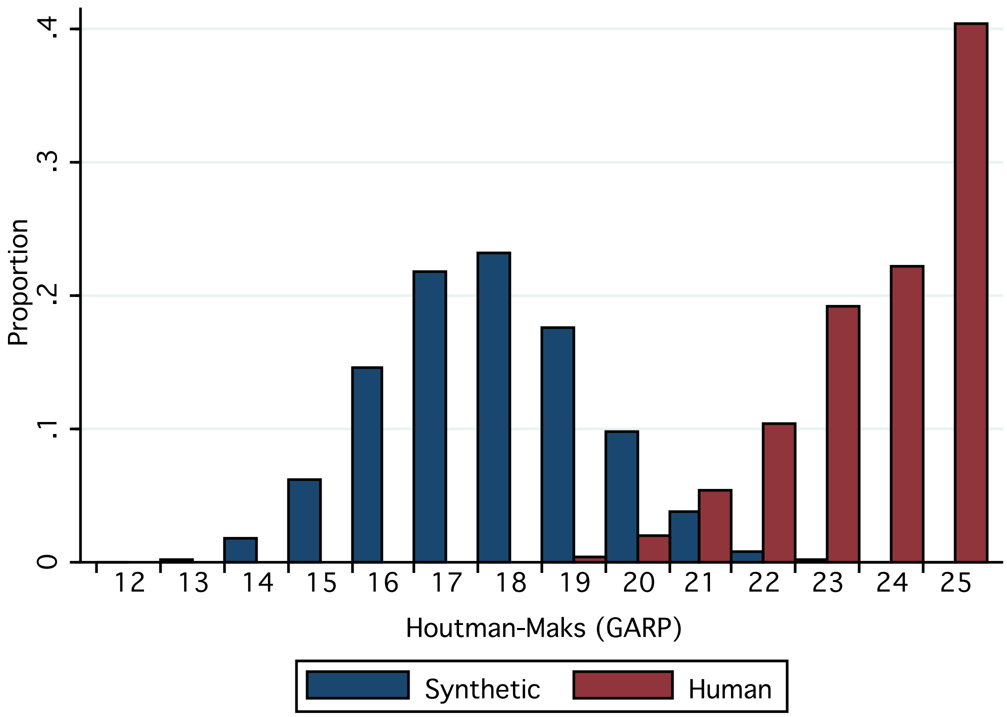
Choices from a set of Walrasian budgets (with strictly positive prices) are consistent with the maximization of a preference relation that is complete, transitive, continuous, and monotone if and only if they adhere to the generalized axiom of revealed preference (Varian, 1982; Afriat, 1967). In an experimental setting such as ours, the generalized axiom is an easy-to-test condition and Varian (1982) provides an easy-to-implement algorithm. Technical details are in Appendix A.1.

Adherence to an axiomatic criterion is binary: Either choices conform perfectly to GARP of they do not. While taking a binary perspective on GARP-compliance can be informative, a widely accepted approach is to determine, conditional on observing a violation, just "how badly" choices departed from perfect GARP-compliance. A common measure used in this regard is Houtman-Maks (HM). Houtman-Maks is the largest subset of choices which are GARP-compliant. So, for example, if we see that a full set of 25 choices violate GARP but, by removing one offending choice, the remaining choices are GARP-compliant, this would represent a Houtman-Maks of 24.

To determine whether a participant's choices are GARP-compliant, we compare their Houtman-Maks to a critical value derived from a Monte Carlo simulation. Our Monte Carlo simulation uses choices for 5,000 synthetic experimental participants, each with a uniform random choice rule. We calculated Houtman-Maks for each synthetic. We interpret the distribution of all 5,000 synthetics' Houtman-Maks as a sampling distribution and use it to determine critical values. This sampling distribution is illustrated with the blue bars in Figure 8(a), and exact values are reported in 8(b). The critical value for the 95% confidence level implied by this sampling distribution is 21. Relative to this critical value, 198 of our 203 experimental participants (98%) had a Houtman-Maks that met or exceeded this threshold (i.e. $\text{HM} \geq 21$). If the cutoff for GARP-compliance is raised to the critical value for the 99% confidence level of 22, 187 participants (92%) had a Houtman-Maks that met or exceeded this threshold (i.e. $\text{HM} \geq 22$). Generally speaking, we find very high rates of GARP-compliance.

Figure 8: Houtman-Maks For The Generalized Axiom of Revealed Preference (GARP)

(a) A High Rate of GARP-compliance Illustrated by the Distribution of Houtman-maks for Experimental Participants (red) and a Sampling Distribution Implied by 5,000 Synthetics With a Uniform Random Choice Rule (Blue)



(b) Supporting Data

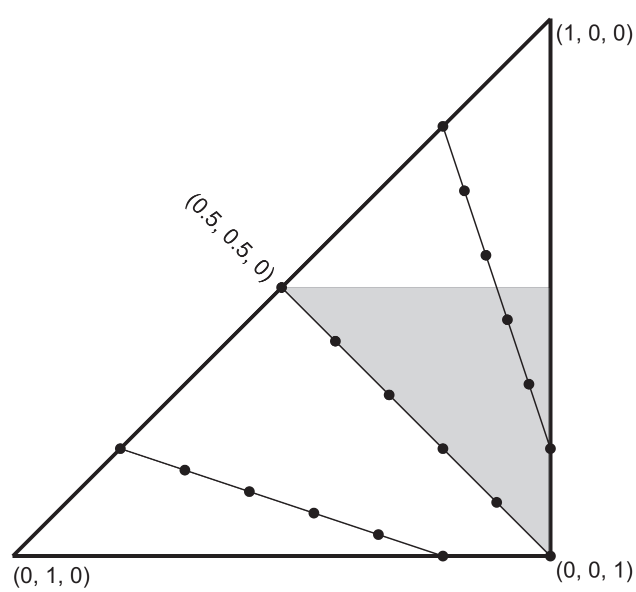
	Humans			Synthetics		
Houtman-Maks	Freq.	%	Cum.	Freq.	%	Cum.
12	0	0	0	2	0.04	0.04
13	0	0	0	10	0.2	0.24
14	0	0	0	86	1.72	1.96
15	0	0	0	309	6.18	8.14
16	0	0	0	728	14.56	22.7
17	0	0	0	1,093	21.86	44.56
18	0	0	0	1,158	23.16	67.72
19	1	0.49	0.49	885	17.7	85.42
20	4	1.97	2.46	489	9.78	95.2
21	11†	5.42	7.88	188	3.76	98.96
22	21†	10.34	18.23	43	0.86	99.82
23	39†	19.21	37.44	7	0.14	99.96
24	45†	22.17	59.61	1	0.02	99.98
25	82†	40.39	100	1	0.02	100
Total	203	100	-	5,000	100	-

† – Exceeds the critical value for the 95% confidence level established by 5,000 synthetics with a uniform random choice rule.

3.2 Assessing The α-MaxMin Expected Utility (αMEU) Representation

In the uncertainty triangle, the αMEU model exhibits indifference curves that are linear and parallel. This geometry necessitates a particular pattern of choices in an experiment like ours. Consider, for example, the three budgets, each with a distinct slope, depicted in Figure 9. Suppose that the 50/50 lottery was chosen from the middle budget (i.e. the budget connecting the 100% ambiguous alternative to the 50/50 lottery). Any linear indifference curve that rationalizes this choice must lie in the shaded area. And because αMEU indifference curves must be parallel, the choice from any steeper budget, like the upper-most budget in Figure 9, must also be the lottery on the hypothenuse. When a budget is flatter than the middle budget, however, no such prediction can be placed on choices. If the original choice from the middle budget was the 100% ambiguous alternative, then the reverse structure would be required. Flatter budgets would require the most ambiguous alternative available to be selected, while no such requirement could be placed on choices from steeper budgets. If the original choice was an alternative from the strict interior of the middle budget, any αMEU indifference curve that rationalizes this choice must lie on top of that budget line. So, for any steeper budget the lottery must be selected while for any flatter budget the most ambiguous alternative must be selected. The comprehensive technical details for testing the αMEU representation in this way, and algorithmic details, can be found in Appendix A.2.

Figure 9: Testing For The Linear And Parallel Indifference Curves Required Under αMEU

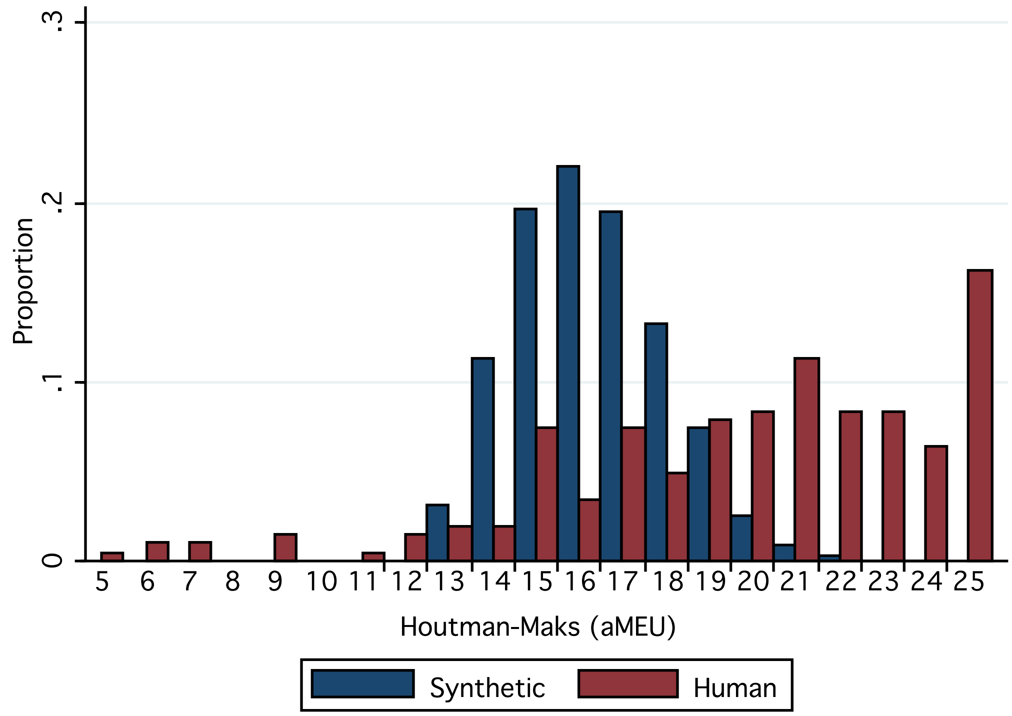


Paralleling the logic of assessing "how badly" choices can depart from the generalized axiom, we adapt the logic of Houtman-Maks (HM) to our test of the αMEU representation. We define "Houtman-Maks αMEU" as the largest subset of choices that conform to our test. Figure 10(a) is a histogram of HM αMEU for the 203 participants in our experiment (red bars). The supporting data are in 10(b). The blue bars in Figure 10(a) show the sampling distribution for HM αMEU implied by 5,000 synthetic experimental participants who had a 50/50 choice rule for just the endpoints of the budgets – these 5,000 synthetics' choices were restricted to be either the fully-specified lottery or the most ambiguous alternative. The critical value for the 95% confidence level for this sampling distribution was 20.[13] Of the 203 participants in our experiment, 120 (59.1%) had an HM αMEU that met or exceeded this critical value. Considering the critical value for the 99% confidence level of 21, 103 participants (50.7%) had an HM αMEU that met or exceeded this value. Generally speaking, αMEU is a good representational assumption for the majority of our experimental participants.

[13]The careful reader will notice that this sampling distribution is truncated at 13. This is a natural feature of the synthetics' 50/50 choice rule – the smallest subset that has choices at only one 'corner' of the budget is 13. Choices that are at only one corner are trivially representable with αMEU.

Figure 10: Houtman-Maks For The αMEU Representation

(a) Histogram



(b) Supporting Data

	Humans			Synthetics		
Houtman-Maks	Freq.	%	Cum.	Freq.	%	Cum.
5	1	0.49	0.49	0	0	0
6	2	0.99	1.48	0	0	0
7	2	0.99	2.46	0	0	0
9	3	1.48	3.94	0	0	0
10	0	0	3.94	0	0	0
11	1	0.49	4.43	0	0	0
12	3	1.48	5.91	0	0	0
13	4	1.97	7.88	156	3.12	3.12
14	4	1.97	9.85	562	11.24	14.36
15	15	7.39	17.24	983	19.66	34.02
16	7	3.45	20.69	1,098	21.96	55.98
17	15	7.39	28.08	972	19.44	75.42
18	10	4.93	33	665	13.3	88.72
19	16	7.88	40.89	371	7.42	96.14
20	17‡	8.37	49.26	126	2.52	98.66
21	23‡	11.33	60.59	45	0.9	99.56
22	17‡	8.37	68.97	18	0.36	99.92
23	17‡	8.37	77.34	3	0.06	99.98
24	13‡	6.4	83.74	1	0.02	100
25	33‡	16.26	100	0	0	100
Total	203	100	-	5,000	100	-

‡ – Exceeds the critical value for the 95% confidence level established by 5,000 synthetics with a 50/50 choice rule for only the endpoints of a budget.

3.2.1 Estimating α-MaxMin Expected Utility (αMEU) Parameters

To estimate αMEU preference parameters we assume decision makers possess a random utility: $RU(L)$. Denoting L^{jk} as the k^{th} lower envelope lottery available in the j^{th} choice-situation, we assume random utility has an additive error component (ε):

$$RU(L^{jk}) = \alpha\text{MEU}(L^{jk}) + \varepsilon^{jk} \tag{7}$$

The systematic, αMEU-component of random utility, for the CHF 60 and CHF 20 outcomes available in our experiment, can be written as:

$$\alpha\text{MEU}\left(\underline{p}_{60}, \underline{p}_{20}, y\right) = \alpha\left[\underline{p}_{60}u_{60} + \left(\underline{p}_{20} + y\right)u_{20}\right] + (1-\alpha)\left[\left(\underline{p}_{60} + y\right)u_{60} + \underline{p}_{20}u_{20}\right]. \tag{8}$$

Normalizing outcome utilities such that $u_{20} = 0$ and $u_{60} = 1$, the expression in 8 simplifies to

$$\alpha\text{MEU}\left(\underline{p}_{60}, \underline{p}_{20}, y\right) = \underline{p}_{60} + (1-\alpha)y, \tag{9}$$

an expression with just one parameter (α) that captures an individual's constant ambiguity attitude. If we can assume that the random component of utility, the additive error ε^{jk}, is independently, identically distributed type I extreme value, then choice probabilities will take on the standard logit form,

$$\text{Prob}\left[L^{j*} = L^{jk}\right] = \frac{\exp\left[\left(\frac{1}{\sigma}\right)\alpha\text{MEU}(L^{jk})\right]}{\sum_k \exp\left[\left(\frac{1}{\sigma}\right)\alpha\text{MEU}(L^{jk})\right]}, \tag{10}$$

where L^{j*} indicates the lower envelope lottery selected in the j^{th} choice-situation, and σ is the dispersion parameter of the error component.

Under the assumption of homogeneous preferences the log-likelihood function, $f(\cdot)$, is constructed in the standard way,

$$f(\alpha, \sigma) = \sum_{i=1}^{120}\sum_{j=1}^{25}\sum_{k=1}^{6} \mathbb{1}\left[L_i^{j*} = L_i^{jk}\right] \ln\left(\text{Prob}\left[L_i^{j*} = L_i^{jk}\right]\right), \tag{11}$$

where the subscript i denotes the i^{th} of the 120 individuals with an αMEU representation, the superscript j denotes the j^{th} of 25 choice situations (budgets), and the superscript k denotes the

k^{th} of the six possible lower envelope lotteries available in a choice situation. The Iverson bracket $\mathbb{1}[\cdot]$ returns one if the condition in the bracket is true and zero otherwise.

We then relax the assumption of homogenous preferences and explore the possibility that our data were generated by multiple and distinct preference types. To do so, we employ a finite mixture approach.[14] The principal idea of such models is assigning each subject to one of C different preference types. Each type is endogenously characterized by a distinct vector of parameters, (α_c, σ_c), with $c \in \{1, ..., C\}$. The finite mixture procedure also estimates the proportion of the sample that belongs to each type: π_c. Summing over all C behavioral types yields the complete log-likelihood ($\mathscr{L}^{fm}_{\alpha\text{MEU}}$) which is given by

$$\mathscr{L}^{fm}_{\alpha\text{MEU}} = \sum_{i=1}^{120} \sum_{c=1}^{C} \pi_c f(\alpha_c, \sigma_c)\,, \tag{12}$$

where $f(\cdot)$ is the density function listed in Equation 11.[15]

For finite mixture models, the number of behavioral types has to be fixed prior to estimation. There is a large literature discussing the optimal number of types but there is no common agreement on which measure is best.[16] Evaluating the 'goodness-of-discrimination' by an entropy criterion (the normalized entropy criterion introduced by Celeux and Soromenho, 1996), we find that four preference types are optimal. However, relative to a model with $C = 3$ types, increasing the number of types does not add new qualitative insights. The three types we find can be characterized as (i) ambiguity averse, (ii) ambiguity neutral, and (iii) ambiguity seeking.[17]

3.2.2 Estimation Results: Examining Ambiguity Preference Types

Estimation results are reported in Table 2. Model 1 assumes homogeneous preferences across the sub-sample of 120 participants that are αMEU representable. The parameter estimates and bootstrapped 95% confidence interval for ambiguity preference parameter (α) are plotted in **blue**

[14] Applications of finite mixture models are discussed in El Gamal and Grether (1995), Stahl and Wilson (1995), Houser et al. (2004), Bruhin et al. (2010), Fehr-Duda et al. (2010), Conte et al. (2011), and Fehr Duda and Epper (2012).

[15] Details about estimation of finite mixture models can be found in McLachlan and Peel (2004).

[16] For a summary of this and many other practical issues related to implementing finite mixture models see McLachlan and Peel (2004).

[17] See also Bruhin et al. (2010) who employ a similar 'stopping rule' when incrementing the number of types.

in the top portion of Figure 11.[18] That the confidence interval lies above 0.5 indicates that, on average, participants exhibited ambiguity aversion.

Model 2 relaxes the restriction of homogeneous preferences and allows for two preference types. Fitted ambiguity parameters, α_1 and α_2, and their standard errors (s.e.) are shown in the middle column of Table 2. The middle portion of Figure 11 shows the bootstrapped 95% confidence intervals for these two preference types. The confidence interval for Type 1 preferences, colored in **blue**, is greater than 0.5, consistent with ambiguity aversion. The confidence interval for Type 2 preferences, colored in **red**, is less than 0.5, consistent with ambiguity seeking. The finite mixture algorithm also fits a parameter, π_1, of the proportion of the sample that is associated with the Type 1 preferences. This estimate, shown in the first row for the Type 1 preferences Table 2, indicates that about 80% of participants are ambiguity averse. The remaining 20% of participants belong to the Type 2 preference type, or ambiguity seeking.

We take the estimates for Model 3, which has three preference types, as definitive for the sub-sample of 120 participants representable with αMEU.[19] The estimates are in the right-most column of Table 2 and the bootstrapped 95% confidence intervals for the three preference types are shown in the lower portion of Figure 11. As with model 2, the **blue** and **red** confidence intervals correspond to ambiguity aversion and neutrality. The **green** confidence interval, from a strictly statistical perspective, also corresponds to ambiguity aversion. We interpret, however, this group as exhibiting ambiguity neutrality because the fitted parameter is only 3% larger than 0.5. In terms of the estimated proportions for each of these three preference types, **48%** are ambiguity averse, **30%** are ambiguity neutral, and **22%** are ambiguity seeking.[20]

[18] As McLachlan and Peel (2004), p.68 recommend using bootstrapped standard errors instead of an information-based approach for typically sized data sets.

[19] When fitting finite mixture models a natural concern is whether more preference types can be introduced to capture more of the preference heterogeneity present in the data. To address this concern we also fit models that allowed four preference types, and five preference types. The fitted proportion parameter for these fourth and fifth types (π_4 and π_5) were always zero. While these results do not rule out the possibility of more than five preference types, we take it as strong evidence that three preference types exist in our sub-sample of 120 participants representable with αMEU.

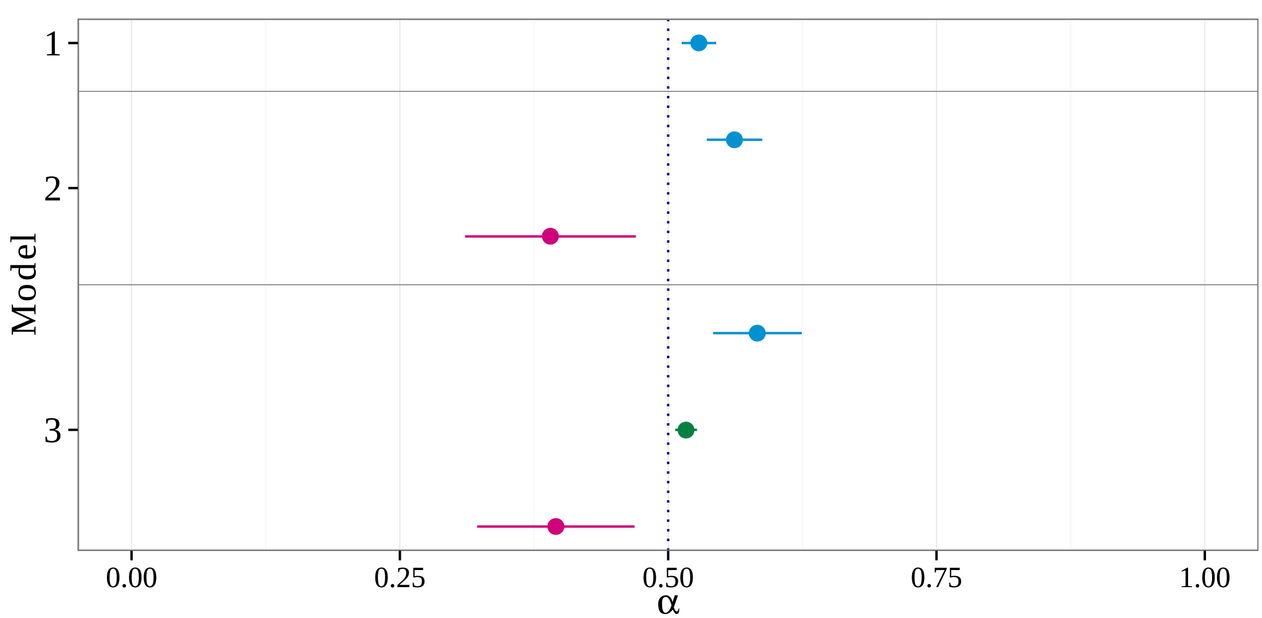
[20] We also estimated individual-level α- and σ-parameters for each of the 120 subjects. The results are comparable to the results of Model 3, when subjects are classified according to z-tests (95% significance level) against a null hypothesis of "ambiguity neutrality".

Table 2: Model Estimates For The 120 Participants Representable With α-MaxMin Expected Utility (αMEU) Preferences

		Model 1 (One Type)		Model 2 (Two Types)		Model 3 (Three Types)	
Type 1	π_1 (s.e.)	1.000	(-)	0.795*	(0.096)	0.483*	(0.084)
	α_1 (s.e.)	**0.529**$*^{1/2}$	(0.008)	**0.562**$*^{1/2}$	(0.017)	**0.583**$*^{1/2}$	(0.025)
	σ_1 (s.e.)	0.025*	(0.002)	0.020*	(0.002)	0.026*	(0.002)
Type 2	π_2 (s.e.)	-		0.205	(-)	0.302*	(0.067)
	α_2 (s.e.)	-		**0.390**$*^{1/2}$	(0.042)	**0.517**$*^{1/2}$	(0.005)
	σ_2 (s.e.)	-		0.029*	(0.005)	0.005*	(0.002)
Type 3	π_3	-		-		0.215	(-)
	α_3 (s.e.)	-		-		**0.395**$*^{1/2}$	(0.038)
	σ_3 (s.e.)	-		-		0.029*	(0.004)
Individuals		120		120		120	
Observations		3,000		3,000		3,000	
$\mathscr{L}$		-3,190		-2,882		-2,681	
BIC		6,396		5,804		5,425	

Notes: $*^{1/2}$ denotes that α_j is different from 1/2 (ambiguity neutrality) at the 95% confidence level. $*$ denotes difference from 0 at the 95% confidence level. Standard errors are calculated using 1,000 bootstrap replications.

Figure 11: Bootstrapped 95% Confidence Intervals For The Ambiguity Preference Parameters (α) In Table 2



3.3 Examining Why αMEU Was Violated

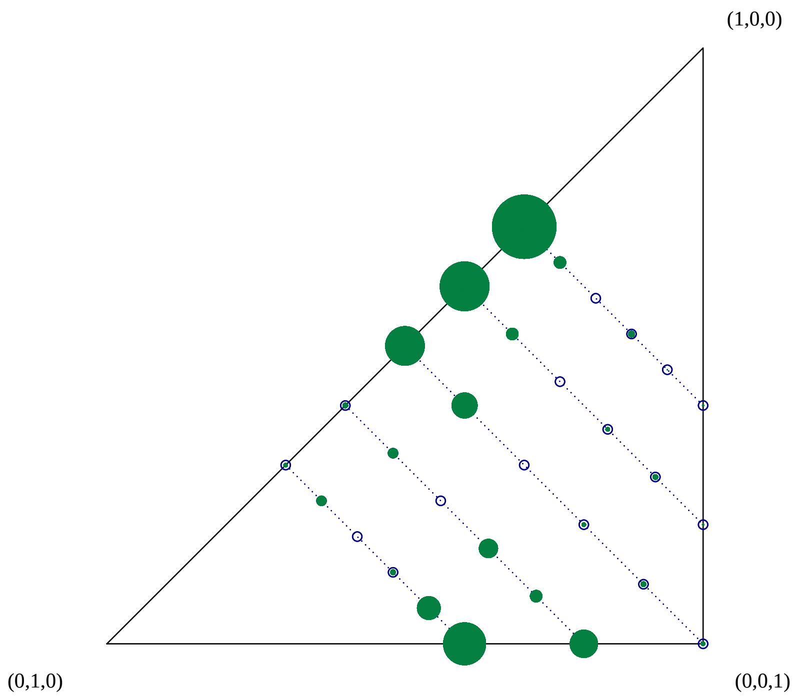
Section 3.2 reveals that a significant proportion of participants made choices that cannot be rationalized with αMEU (41%, or 83 of 203 participants; see Figure 10 for details). Two explanations for why αMEU failed as a parsimonious model seem plausible. First, it could be that the everywhere-constant ambiguity attitude embedded in the αMEU representation was violated, while the linearity of indifference curves was retained. That indifference curves would still be linear means that choices would be predominantly at the endpoints (corners) of the budgets in our experiment. In this case, the fraction of choices between the fully-specified lottery endpoint and the 'most ambiguous' endpoint should vary across the uncertainty triangle. A second explanation for why αMEU failed as a parsimonious model would be that indifference curves in the uncertainty triangle were non-linear. A pattern of choices consistent with this explanation would mean that choices were, to a large extent, on the interior of the budgets in our experiment.

For this sub-sample of 83 participants we observe very low proportions of choices on the interior of the budgets. The bubble plot in Figure 12 is illustrative, however, of the general trend observed there. It displays choices made from five budgets, each with the same price but differing wealths. As can be seen in the figure, only a small proportions of choices are on the interior of these budgets. Put another way, there are a large proportion of choices at the endpoints of the budgets. The ratio of choices between the fully-specified lottery endpoint and the most ambiguous endpoint, however, varies across the uncertainty triangle. This pattern of choices is roughly consistent with the first explanation above. We therefore consider a generalization of αMEU that relaxes the everywhere constant ambiguity attitude only.

3.3.1 The $\alpha\beta$-MaxMin Expected Utility ($\alpha\beta$MEU) Model

As an empirical expedient for accommodating a non-constant ambiguity attitude we introduce a generalization of the αMEU model. We call this generalization the $\alpha\beta$-MaxMin Expected Utility ($\alpha\beta$MEU) model. The $\alpha\beta$MEU model weakens the everywhere-constant ambiguity attitude embedded in αMEU by allowing indifference curves to fan-in or fan-out across the uncertainty triangle. The additional parameter, β, controls the extent to which indifference curves fan-in or fan-out across the uncertainty triangle. Formally, the $\alpha\beta$MEU of a lower envelope lottery is given

Figure 12: Illustrating non-constant ambiguity attitudes across the uncertainty triangle with a bubble plot of the choices made by 83 participants on five budgets with a constant price



by:

$$\alpha\beta\text{MEU}\left(\underline{p}_1,\ldots,\underline{p}_K,y\right) = (\alpha+\beta w_j)\left[\left(\underline{p}_K+y\right)u_K+\sum_{k=1}^{K-1}\underline{p}_k u_k\right] + (1-\alpha-\beta w_j)\left[\left(\underline{p}_1+y\right)u_1+\sum_{k=2}^{J}\underline{p}_k u_k\right] \quad (13)$$

where w_j is the wealth in choice-situation j (see Section 2 for more details). Adapting the $\alpha\beta$MEU model to our two-outcome experiment, and specifying wealth by the probability of receiving the better outcome (CHF 60) at the risky end of the budget j, which we denote as $(p_{60,j})$,

$$w_j = 10\left(p_{60,j}-\frac{1}{2}\right), \quad (14)$$

which gives

$$\begin{aligned}\alpha\beta\text{MEU}\left(\underline{p}_{60},\underline{p}_{20},y\right) = \quad & \left(\alpha+10\beta\left(p_{60,j}-\frac{1}{2}\right)\right)\left[\left(\underline{p}_{20}+y\right)u_{20}+\underline{p}_{60}u_{60}\right] \\ & +\left(1-\alpha-10\beta\left(p_{60,j}-\frac{1}{2}\right)\right)\left[\underline{p}_{20}u_{20}+\left(\underline{p}_{60}+y\right)u_{60}\right] .\end{aligned} \tag{15}$$

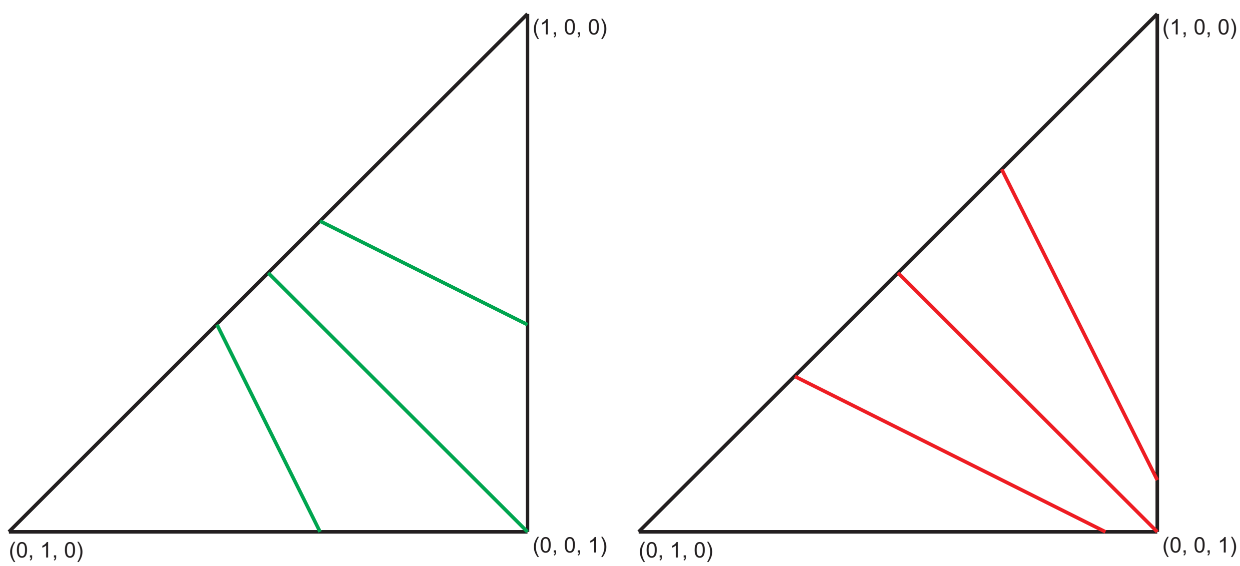
It is possible to interpret the $\alpha\beta$MEU model as introducing a systematically varying ambiguity attitude, where the parameter β captures whether ambiguity preference is increasing ($\beta > 0$) or decreasing ($\beta < 0$) as the expectation for a good outcome increases (i.e. wealth is increasing). More precisely, β measures the effect of a 0.1 increase in normalized wealth, which - in our case - is equivalent to a shift of 0.1 preassigned probability mass from the worse outcome 20 to the better outcome 60. The parameter α in our specification characterizes average ambiguity aversion, i.e. ambiguity aversion where $p_{60,j} = 1/2$.

Figure 13 shows example indifference curves for two parameterizations of the $\alpha\beta$MEU model plotted in the uncertainty triangle. Both examples have $\alpha = \frac{1}{2}$ so that the indifference curve intersecting the hypotenuse of the uncertainty triangle (the risk line) at the 50/50 lottery has a slope equivalent to ambiguity neutrality. Panel (a) shows a positive value of β which produces indifference curves that 'fan-in' across the uncertainty triangle. Put another way, positive values of β mean that aversion to ambiguity is increasing as the expectation for a good outcome increases (i.e. as w_j increases). Negative values of β, like those depicted in panel (b) of Figure 13, have the opposite pattern. Indifference curves 'fan-out' across the uncertainty triangle. Fanning-out of indifference curves is consistent with an increasing preference for ambiguity as the expectation for a good outcome increases.

Figure 13: Indifference Curves For The $\alpha\beta$-MaxMin Expected Utility ($\alpha\beta$MEU) Model

(a) Increasing aversion to ambiguity: $\alpha = \frac{1}{2}$, $\beta > 0$ ('Fanning In')

(b) Decreasing aversion to ambiguity: $\alpha = \frac{1}{2}$, $\beta < 0$ ('Fanning Out')



3.3.2 Estimating $\alpha\beta$-MaxMin Expected Utility ($\alpha\beta$MEU) Parameters

To estimate $\alpha\beta$MEU preference parameters we again assume that decision makers possess an additive error random utility (see Section 3.2.1 for details). Normalizing the outcome utilities for the systematic $\alpha\beta$MEU-component of random utility, so that $u_{20} = 0$ and $u_{60} = 1$, we have

$$\alpha\beta\text{MEU}\left(\underline{p}_{60}, \underline{p}_{20}, y\right) = \underline{p}_{60} + \left(1 - \alpha - 10\beta\left(p_{60,j} - \frac{1}{2}\right)\right) y. \tag{16}$$

Assuming that the random component of utility is independently, identically distributed type I extreme value, then choice probabilities take the standard logit form,

$$\text{Prob}\left[L^{j*} = L^{jk}\right] = \frac{\exp\left[\left(\frac{1}{\sigma}\right)\alpha\beta\text{MEU}(L^{jk})\right]}{\sum_k \exp\left[\left(\frac{1}{\sigma}\right)\alpha\beta\text{MEU}(L^{jk})\right]}, \tag{17}$$

where L^{j*} indicates the lower envelope lottery selected in the j^{th} choice-situation, and σ is the dispersion parameter of the error component. Under the assumption of homogeneous preferences the log-likelihood function, $g(\cdot)$, is constructed in the standard way,

$$g(\alpha, \beta, \sigma) = \sum_{i=1}^{83}\sum_{j=1}^{25}\sum_{k=1}^{6} \mathbb{1}\left[L_i^{j*} = L_i^{jk}\right] \ln\left(\text{Prob}\left[L_i^{j*} = L_i^{jk}\right]\right), \tag{18}$$

where the subscript i denotes the i^{th} of the 83 individuals with an $\alpha\beta$MEU representation, the superscript j denotes the j^{th} of 25 choice situations (budgets), the superscript k denotes the k^{th} of the six possible lower envelope lotteries available in a choice situation. The Iverson bracket $\mathbb{1}[\cdot]$ returns one if the condition in the bracket is true and zero otherwise. Paralleling the logic used in estimating preference parameters for αMEU (see Section 3.2.1), we relax the assumption of homogenous preferences and explore the possibility that our data were generated by multiple and distinct preference types using a finite mixture approach. Finite mixture models assign each subject to one of C different preference types and each type is endogenously characterized by a distinct vector of parameters, $(\alpha_c, \beta_c, \sigma_c)$, with $c \in \{1, ..., C\}$. The finite mixture procedure also estimates the proportion of the sample that belongs to each type: π_c. Summing over all C behavioral types yields the complete log-likelihood ($\mathscr{L}^{fm}_{\alpha\beta\text{MEU}}$) which is given by

$$\mathscr{L}^{fm}_{\alpha\beta\text{MEU}} = \sum_{i=1}^{83}\sum_{c=1}^{C} \pi_c g(\alpha_c, \beta_c, \sigma_c)\,, \tag{19}$$

where $g(\cdot)$ is the density function listed in Equation 18.

Again, the number of behavioral types (C) has to be fixed prior to estimation. We follow the same procedure as in Section 3.2.1. Namely, we increase the number of types as long as there is a meaningful interpretation of the distinct preference characteristics of each type. Here, we end up with $C = 2$ types, both showing considerable fanning across the simplex: one type revealing an average behavior which can be described as ambiguity neutral, and the other type revealing an average behavior which can be described as ambiguity averse. The normalized entropy criterion, once again, would favor one more type. However, this additional type can be broadly described as a subgroup of the more ambiguity averse type in the two-types model.

3.3.3 Estimation Results: Examining $\alpha\beta$MEU Preference Types

Estimation results are reported in Table 3. Model 1 assumes homogeneous preferences across the sub-sample of 83 participants that are $\alpha\beta$MEU representable. The parameter estimates and a bootstrapped 95% confidence region for preference parameters α and β are plotted in **blue** in panel (a) of Figure 14. The confidence intervals indicate that α is larger than 0.5 and β is positive. In general, these homogeneous parameter estimates exhibit a large degree of ambiguity aversion. This is easy to see in panel (a) of Figure 15 which plots indifference curves for Model 1 as **blue** lines in the uncertainty triangle.

Model 2 estimates two types of preferences. And we take these parameter estimates as definitive for this group of 83 participants. Fitted preference parameters, α_1, β_1, α_2, and β_2, and their standard errors (s.e.), are shown in the right column of Table 3. Bootstrapped 95% confidence regions for both preference types are shown in panel (b) of Figure 14. The confidence region for Type 1 preferences is colored **green** while the confidence region for Type 2 preferences is colored **blue**. Indifference curves for these two types are plotted in panel (b) of Figure 15. Interestingly, Type 1 preferences exhibit ambiguity seeking in the lower portion of the uncertainty triangle and ambiguity aversion in the upper portion of the triangle. Type 2 preferences are everywhere ambiguity averse with levels of ambiguity aversion approaching the MaxMin extreme (i.e. $\alpha = 1$ in

Table 3: Model Estimates For $\alpha\beta$-MaxMin Expected Utility ($\alpha\beta$MEU) Preferences For 83 Experimental Participants

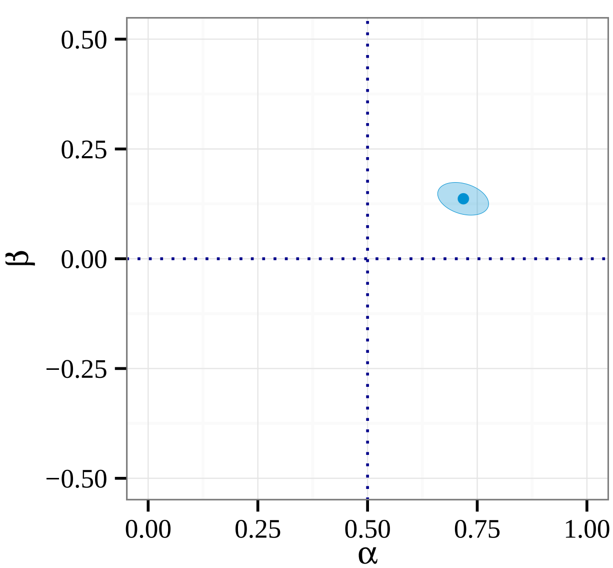
		Model 1 (One Type)		Model 2 (Two Types)	
Type 1	π_1 (s.e.)	1.000	(-)	0.428*	(0.108)
	α_1 (s.e.)	**0.719**$^{*1/2}$	(0.024)	**0.494**	(0.108)
	β_1 (s.e.)	**0.136***	(0.015)	**0.261***	(0.064)
	σ_1 (s.e.)	0.099*	(0.008)	0.137*	(0.025)
Type 2	π_2 (s.e.)	-		0.572	(-)
	α_2 (s.e.)	-		**0.785**$^{*1/2}$	(0.062)
	β_2 (s.e.)	-		**0.093***	(0.024)
	σ_2 (s.e.)	-		0.065*	(0.007)
Individuals		83		83	
Observations		2,075		2,075	
$\mathscr{L}$		-2,816		-2,720	
BIC		5,654		5,494	

Notes: $*^{1/2}$ denotes that α_j is different from $1/2$ at the 95% confidence level. $*$ denotes difference from 0 at the 95% confidence level. Standard errors are calculated using 1,000 bootstrap replications.

the αMEU representation) in the upper portion of the uncertainty triangle. Approximately 43% of this sample is estimated to be associated with the Type 1 preferences (see π_1 for Model 2 in Table 3). Broadly speaking, the parameter estimates for Model 2 are consistent with the bubble plots discussed above – aversion to ambiguity increases as the expectation for a good outcome increases.

Figure 14: Bootstrapped 95% Confidence Regions Based On 1,000 Replications For The $\alpha\beta$-MaxMin Expected Utility ($\alpha\beta$MEU) Preference Parameters Reported In Table 3

(a) Model 1 – Homogeneous Preferences



(b) Model 2 – Two Preferences Types

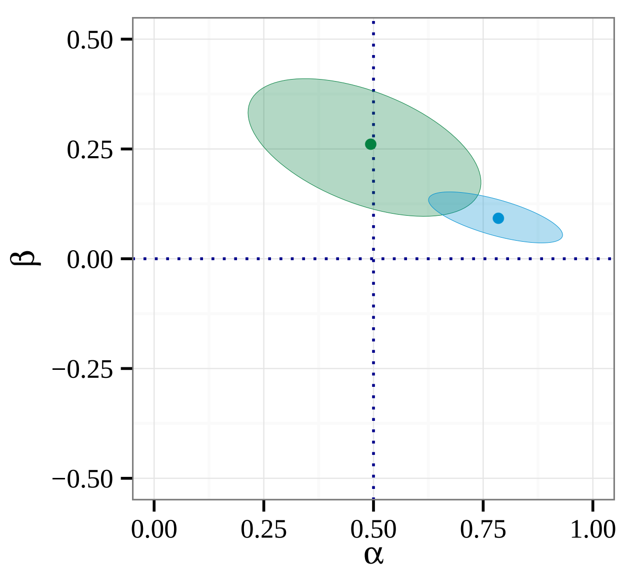
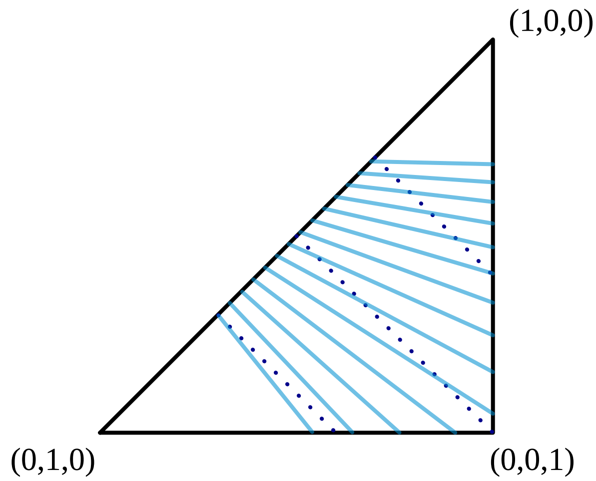
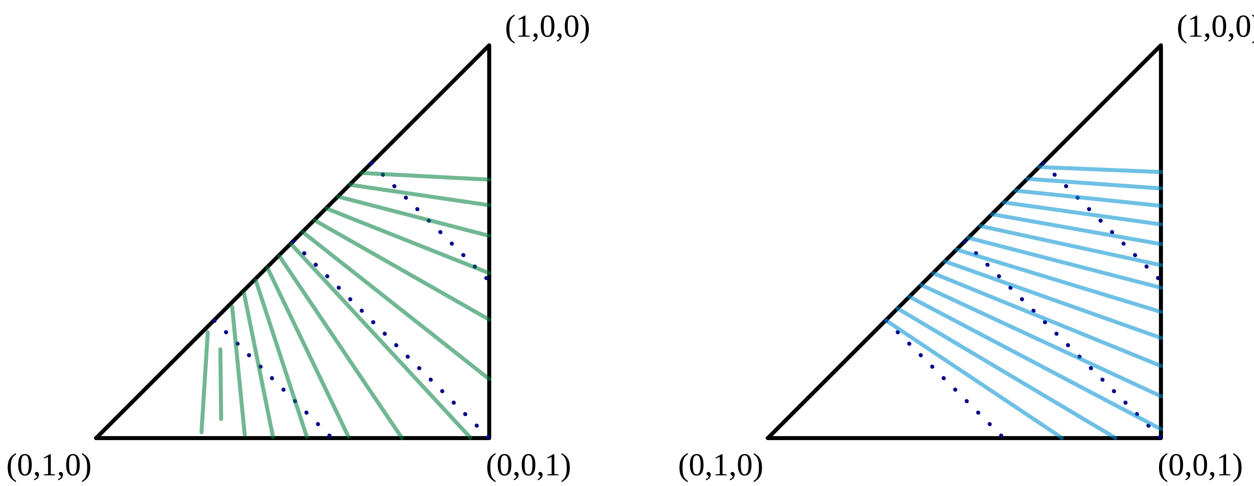


Figure 15: Indifference Curves For $\alpha\beta$-MaxMin Expected Utility ($\alpha\beta$MEU) Preference Parameters Reported In Table 3. The Dashed Lines Are For Reference And Represent Ambiguity Neutrality.

(a) Model 1 – Homogeneous Preferences



(b) Model 2 – Two Preference Types



4 Discussion and Conclusion

This paper used lower envelope lotteries as a simple framework for examining ambiguity preferences. This simplicity can be advantageous in the experimental settings frequently explored in studies of choices involving ambiguity. For example, as we documented in Section 1, the Ellsberg one-urn experiment can be described with lower envelope lotteries. And the canonical pattern of choices in the Ellsberg experiment can be rationalized using the αMEU model, adapted to the setting of lower envelope lotteries.

The setting of lower envelope lotteries, and the 'sets of probability distributions' approach more generally, is a natural complement to the Anscombe and Aumann (1963) framework typically employed in theoretical and empirical examinations of ambiguity preferences. And our analysis of choices for lower envelope lotteries acts as a complement to a number of previous reports. For example, Ahn et al. (2014) examine ambiguity preferences using an Arrow security setting similar to Choi et al. (2007). In Ahn et al. (2014)'s experiment there were three mutually exclusive states of the world. One of the states was assigned an exact probability of 1/3, while the probabilities for each of the other two states were unknown. Experimental participants allocated a budget between three Arrow securities, each of which paid off in one of the states, and zero otherwise. This experimental design allowed for an individual-level assessment of the generalized axiom of revealed preference and identification of both risk and ambiguity preferences. Using non-linear least squares, Ahn et al. (2014) found that about 60% of participants were fit best with αMEU and parameter $\alpha = 1/2$ (ambiguity neutrality). The approach employed by Ahn et al. (2014) highlights the natural complementarity between examining ambiguity preferences with Arrow securities and the set-of-probability-distributions/lower-envelope-lottery approach employed here. The approach used by Ahn et al. (2014) relies on varying monetary outcomes, under a fixed set of probability distributions, while the lower envelope lottery approach used here varies minimum probabilities and ambiguity (i.e. the set of lotteries), while holding outcomes fixed.

Hey and Pace (2014) also used experimental data in an assessment of five single-stage models of decision-making under ambiguity.[21] Their experimental design had participants betting on

[21] The five single stage models examined by Hey and Pace (2014) were (i) Subjective Expected Utility (Savage, 1954), (ii) Choquet Expected Utility (Schmeidler, 1989), (iii) α-MaxMin Expected Utility (Marinacci, 2002), (iv) Vector Expected

which color of ball (with three possible colors) would be drawn from an urn that was mixed by a bingo blower. Participants were shown the actively mixing bingo blower that contained the urn in a transparent plexiglass box.[22] Hey and Pace (2014) then used parametric estimation techniques to describe and predict choices. They too found support for α-MaxMin Expected Utility (Marinacci, 2002) as a representation for behavior in their experimental data, in addition to Vector Expected Utility (Siniscalchi, 2009).

Abdellaoui et al. (2011) examine certainty equivalents for draws from two Ellsberg-style urns that had fully-known and fully-unknown compositions. Using a flexible rank dependent utility-like setting, they introduce source functions as a method for translating events (i.e. collections of states) into a willingness to bet. By comparing source functions elicited for urns with known and unknown compositions, these authors provide a rich characterization of attitudes towards ambiguity. Interestingly, our finding of increasing aversion to ambiguity under an $\alpha\beta$MEU representation is, broadly speaking, consistent with the results documented in that paper. Referring specifically to Abdellaoui et al. (2011)'s Table 1 on pg. 706, the difference between the source functions for the unknown (U) and known (K) urns is taken as a quantitative measure of ambiguity aversion. Taking these differences shows that this measure of ambiguity aversion is increasing as the likelihood of a good outcome increases: "*For large probabilities* ($p > 0.5$), *source functions are significantly lower for urn U than for urn K [i.e. more ambiguity aversion]. For small probabilities* ($p \leq 0.5$) *there is no significant difference [i.e. no ambiguity aversion]*" (pg. 706, 2nd paragraph; text in brackets is ours). Moreover, as can be seen in Figure 3 on pg. 705 in Abdellaoui et al. (2011), the vertical distance between the known and unknown source functions (i.e. the amount of ambiguity aversion) appears to be increasing as the expectation for a good outcome improves.[23]

Moving forward, the setting of lower envelope lotteries can be used as an input toward a richer characterization of preferences toward uncertainty. One potential direction could be an exploration of whether individuals that exhibit increasing aversion toward ambiguity also commit Allais-type common-ratio/common-consequence violations of expected utility (Allais, 1953; Kah-

Utility (Siniscalchi, 2009), and (v) the Contraction Model (Gajdos et al., 2008).

[22] The transparency of the bingo blower is an important consideration when interpreting the results in Hey and Pace (2014). Transparency means that some information regarding the composition of the urn would be available to participants through visual inspection. For example, the extreme possibility of the urn being comprised of just one color of ball could be easily ruled out.

[23]Formally, Abdellaoui et al. (2011) show that aversion toward ambiguity is increasing as the number of states in an event increases, a quantity interpreted as an objective probability.

neman and Tversky, 1979; Burghart et al., 2014). For Allais-type violations it is widely taken that aversion toward risk is increasing in the direction of first order stochastic dominance (see Machina (1982), Hypothesis II and Figure 5b). Put another way, aversion toward risk is increasing as the expectation for a good outcome improves. Here, we show that many people exhibit increasing aversion toward ambiguity as the expectation for a good outcome improves. We speculate that individuals who exhibit increasing aversion toward ambiguity will also exhibit increasing aversion toward risk, (i.e. canonical common-ratio/common-consequence violations of expected utility).

Another direction that seems promising is using lower envelope lotteries to examine ambiguity preferences when a risky alternative is not available in the choice-set. Indeed, the notion of probabilistic sophistication (Savage, 1954; Machina and Schmeidler, 1992; Epstein and Zhang, 2001) hints that there should be no jumps, or discontinuities, for indifference curves between risky and ambiguous settings. Although, recent experimental work by Baillon and Bleichrodt (2015) indicates that the additivity of disjoint events required under probabilistic sophistication fails when eliciting matching probabilities for naturally occurring uncertainties.

Another prospective direction could be an exploration of whether two-stage resolution of lower envelope lotteries has an effect on the results documented in this paper. Such an extension seems natural given that Halevy (2007) documents a relationship between ambiguity aversion, as elicited with an Ellsberg urn, and violations of the reduction of compound lotteries. Lower envelope lotteries provide a simple and intuitive setting for deeper explorations of these questions, and others, in the domain of uncertainty.

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A Analytical Details

A.1 Testing For GARP

Varian (1982) provides a convenient algorithm for testing the Generalized Axiom of Revealed Preferences (GARP) and, in general, we adopt his notation and methods here. Following Burghart et al. (2014), testing GARP in this setting is computationally less intensive, and more intuitive, if we interpret lower envelope lotteries as two-dimensional demand vectors, relative to the worst outcome. To do this we transform lower envelope lotteries into two dimensions. Denoting the lower envelope lottery selected from the i^{th} budget as L^{i*}, we use the following transformation:

$$L^{i*} = \left(\underline{p}_{60}^{i*}, \underline{p}_{20}^{i*}, y^{i*}\right) \rightarrow \left(\underline{p}_{60}^{i*} + y^{i*}, \underline{p}_{60}^{i*}\right) = \left(x_1^{i*}, x_2^{i*}\right) = \mathbf{x}^{i*} \tag{20}$$

This transformation means that $\mathbf{x}^{i*}$ can be interpreted as a two-dimensional demand vector, relative to the worst outcome. Denote as $\mathbf{q}^i$ the two-dimensional vector of prices that describes the linear budget from which $\mathbf{x}^{i*}$ was selected.

The starting point for testing GARP is to construct a directly revealed preferred graph. For the 25 budgets in our experiment, and each participants choices $\left(\mathbf{x}^{1*}, \ldots, \mathbf{x}^{25*}\right)$, we construct a 25 by 25 matrix M (the directly revealed graph) whose ij^{th} entry is given by

$$m_{ij} = \begin{cases} 1 & \text{if } \mathbf{q}^i \cdot \mathbf{x}^{i*} \geq \mathbf{q}^i \cdot \mathbf{x}^{j*} \\ 0 & \text{otherwise} \end{cases} \tag{21}$$

We then construct the indirectly revealed preferred graph MT, which is the transitive closure of M. The ij^{th} entry of the closure is given by

$$mt_{ij} = \begin{cases} 1 & \text{if } m_{ij}^{25} > 0 \\ 0 & \text{otherwise} \end{cases} \tag{22}$$

where m_{ij}^{25} is the ij^{th} entry of the matrix $M^{25} = MM \cdots M$. If $mt_{ij} = 1$ and $\mathbf{q}^j \cdot \mathbf{x}^{j*} > \mathbf{q}^j \cdot \mathbf{x}^{i*}$ for some i and j (with $\mathbf{x}^{i*} \neq \mathbf{x}^{j*}$) there is a GARP violation.

To calculate Houtman-Maks, conditional on observing at least one violation of GARP, we use a brute force approach. We check whether all subsets of size 24 are consistent with GARP (using the above algorithm). If no subset of size 24 is GARP compliant we take all subsets of size 23 and check whether any of these subsets are GARP compliant. We proceed in such a manner until we find at least one subset of the data that is GARP compliant. The cardinality of that subset is the Houtman-Maks.

A.2 Testing For α-MaxMin Expected Utility (αMEU)

The αMEU model gives rise to indifference curves that linear and parallel in the uncertainty triangle. So, given a choice from one budget, the αMEU indifference curve structure lets us make predictions about choices from other budgets. To make predictions, we need to compare the steepness of the i^{th} budget, denoted by the ratio of its prices $\left(\frac{q_1}{q_2}\right)^i$, to the steepness of the j^{th} budget, denoted by the ratio of its prices $\left(\frac{q_1}{q_2}\right)^j$.[24] We say:

1. The j^{th} budget is flatter than the i^{th} budget whenever $\left(\frac{q_1}{q_2}\right)^j < \left(\frac{q_1}{q_2}\right)^i$
2. The j^{th} budget is steeper than the i^{th} budget whenever $\left(\frac{q_1}{q_2}\right)^j > \left(\frac{q_1}{q_2}\right)^i$

We can use these steepness comparisons to make an exhaustive list of predictions. Using $\mathbf{x}^{i*}$ to denote the choice from the i^{th} budget:

- **Case 1:** The choice from the i^{th} budget is the most ambiguous alternative available (i.e. the alternative was on the horizontal or vertical leg of the uncertainty triangle). Denote this "corner" alternative by $\lfloor \mathbf{x} \rfloor^i$

$$\mathbf{x}^{i*} = \lfloor \mathbf{x} \rfloor^i \;\rightarrow \begin{cases} \mathbf{x}^j = \lfloor \mathbf{x} \rfloor^j & \text{when } \left(\frac{q_1}{q_2}\right)^j < \left(\frac{q_1}{q_2}\right)^i \\ \text{no prediction} & \text{when } \left(\frac{q_1}{q_2}\right)^j > \left(\frac{q_1}{q_2}\right)^i \\ \text{no prediction} & \text{when } \left(\frac{q_1}{q_2}\right)^j = \left(\frac{q_1}{q_2}\right)^i \end{cases}$$

- **Case 2:** The choice from i^{th} budget fully-specified lottery (i.e. the alternative selected was on

[24] See Burghart et al. (2014) for a similar test in the domain of choice under risk.

the hypotenuse in the uncertainty triangle). Denote this "corner" alternative by $\lceil \mathbf{x} \rceil^i$

$$\mathbf{x}^{i*} = \lceil \mathbf{x} \rceil^i \;\rightarrow\; \begin{cases} \text{no prediction} & \text{when } \left(\frac{q_1}{q_2}\right)^j < \left(\frac{q_1}{q_2}\right)^i \\ \mathbf{x}^j = \lceil \mathbf{x} \rceil^j & \text{when } \left(\frac{q_1}{q_2}\right)^j > \left(\frac{q_1}{q_2}\right)^i \\ \text{no prediction} & \text{when } \left(\frac{q_1}{q_2}\right)^j = \left(\frac{q_1}{q_2}\right)^i \end{cases}$$

- **Case 3:** The choice from the i^{th} budget is on the "interior" (i.e. not Case 1 or Case 2). Denote this set of alternatives by $[\mathbf{x}]^i$

$$\mathbf{x}^{i*} \in [\mathbf{x}]^i \;\rightarrow\; \begin{cases} \mathbf{x}^j = \lfloor \mathbf{x} \rfloor^j & \text{when } \left(\frac{q_1}{q_2}\right)^j < \left(\frac{q_1}{q_2}\right)^i \\ \mathbf{x}^j = \lceil \mathbf{x} \rceil^j & \text{when } \left(\frac{q_1}{q_2}\right)^j > \left(\frac{q_1}{q_2}\right)^i \\ \text{no prediction} & \text{when } \left(\frac{q_1}{q_2}\right)^j = \left(\frac{q_1}{q_2}\right)^i \end{cases}$$

This provides an exhaustive list of predictions under an αMEU representation.

Given a set of predictions, it seems reasonable to assess two conditions:

- **Condition 1:** Predictions about choices are internally consistent.
- **Condition 2:** Choices are consistent with the predictions.

The first condition requires that choices from distinct budgets do not generate conflicting predictions about the choice from a third budget (also distinct). The second condition simply requires that choices are consistent with the set of (internally consistent) predictions. We say that choices are consistent with an αMEU representation if these two conditions are met.

Algorithmically, we test the Condition 1 by constructing three square matrices, $\lfloor D \rfloor$, $\lceil D \rceil$, and $[D]$, with entries defined, respectively:

$$\lfloor d \rfloor_{ij} = \begin{cases} -1 & \text{if } \mathbf{x}^{i*} = \lfloor \mathbf{x} \rfloor^i \text{ and } \left(\frac{q_1}{q_2}\right)^j < \left(\frac{q_1}{q_2}\right)^i \\ 0 & \text{otherwise} \end{cases} \tag{23}$$

$$\lceil d \rceil ij = \begin{cases} +1 & \text{if } \mathbf{x}^{i*} = \lceil \mathbf{x} \rceil^i \text{ and } \left(\frac{q_1}{q_2}\right)^j > \left(\frac{q_1}{q_2}\right)^i \\ 0 & \text{otherwise} \end{cases} \tag{24}$$

$$
[d]_{ij} = \begin{cases} -1 & \text{if } \mathbf{x}^{i*} \in [\mathbf{x}]^i \text{ and } \left(\frac{q_1}{q_2}\right)^j < \left(\frac{q_1}{q_2}\right)^i \\ +1 & \text{if } \mathbf{x}^{i*} \in [\mathbf{x}]^i \text{ and } \left(\frac{q_1}{q_2}\right)^j > \left(\frac{q_1}{q_2}\right)^i \\ 0 & \text{otherwise} \end{cases} \tag{25}
$$

Essentially, these three matrices document whether a choice from the j^{th} budget is predicted to be the most ambiguous alternative available (-1), the fully-specified lottery $(+1)$, or neither (0), based upon some original choice $(\mathbf{x}^i)$ and steepness of that original budget $\left(\frac{q_1}{q_2}\right)^i$.

Because $[D]$ is the only matrix that has both $+1$ and -1 entries (i.e. only when $\mathbf{x}^{i*} \in [\mathbf{x}]^i$ can we get predictions of both $\lfloor \mathbf{x} \rfloor^j$ and $\lceil \mathbf{x} \rceil^j$) we first check it for internal consistency of its predictions. We construct two row vectors, Max$[D]$ and Min$[D]$, where

$$
\begin{aligned} \text{Max}[D] &= [\text{Max}\{[d]_{\cdot 1}, 0\}, \text{Max}\{[d]_{\cdot 2}, 0\}, \dots, \text{Max}\{[d]_{\cdot J}, 0\}] \\ \text{Min}[D] &= [\text{Min}\{[d]_{\cdot 1}, 0\}, \text{Min}\{[d]_{\cdot 2}, 0\}, \dots, \text{Min}\{[d]_{\cdot J}, 0\}] \end{aligned} \tag{26}
$$

where $[d]_{\cdot j}$ indicates the collection of entries in the j^{th} column of $[D]$. Notice that for the element-by-element multiplication, $\text{Max}[d]_j * \text{Min}[d]_j \geq 0, \ \forall j = 1, \dots, J$ if and only if predictions are internally consistent in $[D]$.

Conditional on $[D]$ exhibiting internal prediction consistency, all that remains is to verify that all three of the following conditions hold:

$$
\begin{aligned} \text{Max}\lceil d \rceil_j \cdot \text{Min}\lfloor d \rfloor_j &\geq 0, \ \forall j = 1, \dots, J \\ \text{Max}\lceil d \rceil_j \cdot \text{Min}[d]_j &\geq 0, \ \forall j = 1, \dots, J \\ \text{Max}[d]_j \cdot \text{Min}\lfloor d \rfloor_j &\geq 0, \ \forall j = 1, \dots, J \end{aligned} \tag{27}
$$

where

$$
\begin{aligned} \text{Max}\lceil D \rceil &= [\text{Max}\{\lceil d \rceil_{\cdot 1}\}, \text{Max}\{\lceil d \rceil_{\cdot 2}\}, \dots, \text{Max}\{\lceil d \rceil_{\cdot J}\}] \\ \text{Min}\lfloor D \rfloor &= [\text{Min}\{\lfloor d \rfloor_{\cdot 1}\}, \text{Min}\{\lfloor d \rfloor_{\cdot 2}\}, \dots, \text{Min}\{\lfloor d \rfloor_{\cdot J}\}] \end{aligned} \tag{28}
$$

If predictions are internally consistent (i.e. choices satisfy condition 1) it is a straightforward exercise to verify that choices are consistent with predictions (i.e. choices satisfy condition 2). If choices satisfy both condition 1 and condition 2, they pass our test for an αMEU representation.

B Experimental Details

B.1 Choice Situation Details

Table 4 lists normalized prices q_1, q_2, their ratio $\frac{q_1}{q_2}$ and normalized wealth w for each budget line.

Table 4: Choice Situation (CS) Budget Details

CS	q_1	q_2	w	$\frac{q_1}{q_2}$
1	0.2	0.8	0.2	0.25
2	0.3	0.7	0.3	0.43
3	0.4	0.6	0.4	0.67
4	0.5	0.5	0.5	1.00
5	0.6	0.4	0.6	1.50
6	0.7	0.3	0.7	2.33
7	0.8	0.2	0.8	4.00
8	0.2	0.6	0.2	0.33
9	0.3	0.5	0.3	0.60
10	0.4	0.4	0.4	1.00
11	0.5	0.3	0.5	1.67
12	0.6	0.2	0.6	3.00
13	0.2	0.6	0.4	0.33
14	0.3	0.5	0.5	0.60
15	0.4	0.4	0.6	1.00
16	0.5	0.3	0.7	1.67
17	0.6	0.2	0.8	3.00
18	0.2	0.4	0.2	0.50
19	0.3	0.3	0.3	1.00
20	0.4	0.2	0.4	2.00
21	0.5	0.1	0.5	5.00
22	0.1	0.5	0.5	0.20
23	0.2	0.4	0.6	0.50
24	0.3	0.3	0.7	1.00
25	0.4	0.2	0.8	2.00

B.2 Exhaustive List of Each Lower Envelope Lottery

Table 5: An exhaustive list of the lower envelope lotteries available in each choice situation

	Risky End					Ambiguous End
CS	$\delta = 1.0$	$\delta = 0.8$	$\delta = 0.6$	$\delta = 0.4$	$\delta = 0.2$	$\delta = 0.0$
1	$(0.2, 0.8, 0)$	$(0.16, 0.64, 0.2)$	$(0.12, 0.48, 0.4)$	$(0.08, 0.32, 0.6)$	$(0.04, 0.16, 0.8)$	$(0, 0, 1)$
2	$(0.3, 0.7, 0)$	$(0.24, 0.56, 0.2)$	$(0.18, 0.42, 0.4)$	$(0.12, 0.28, 0.6)$	$(0.06, 0.14, 0.8)$	$(0, 0, 1)$
3	$(0.4, 0.6, 0)$	$(0.32, 0.48, 0.2)$	$(0.24, 0.36, 0.4)$	$(0.16, 0.24, 0.6)$	$(0.08, 0.12, 0.8)$	$(0, 0, 1)$
4	$(0.5, 0.5, 0)$	$(0.4, 0.4, 0.2)$	$(0.3, 0.3, 0.4)$	$(0.2, 0.2, 0.6)$	$(0.1, 0.1, 0.8)$	$(0, 0, 1)$
5	$(0.6, 0.4, 0)$	$(0.48, 0.32, 0.2)$	$(0.36, 0.24, 0.4)$	$(0.24, 0.16, 0.6)$	$(0.12, 0.08, 0.8)$	$(0, 0, 1)$
6	$(0.7, 0.3, 0)$	$(0.56, 0.24, 0.2)$	$(0.42, 0.18, 0.4)$	$(0.28, 0.12, 0.6)$	$(0.14, 0.06, 0.8)$	$(0, 0, 1)$
7	$(0.8, 0.2, 0)$	$(0.64, 0.16, 0.2)$	$(0.48, 0.12, 0.4)$	$(0.32, 0.08, 0.6)$	$(0.16, 0.04, 0.8)$	$(0, 0, 1)$
8	$(0.2, 0.8, 0)$	$(0.16, 0.68, 0.16)$	$(0.12, 0.56, 0.32)$	$(0.08, 0.44, 0.48)$	$(0.04, 0.32, 0.64)$	$(0, 0.2, 0.8)$
9	$(0.3, 0.7, 0)$	$(0.24, 0.6, 0.16)$	$(0.18, 0.5, 0.32)$	$(0.12, 0.4, 0.48)$	$(0.06, 0.3, 0.64)$	$(0, 0.2, 0.8)$
10	$(0.4, 0.6, 0)$	$(0.32, 0.52, 0.16)$	$(0.24, 0.44, 0.32)$	$(0.16, 0.36, 0.48)$	$(0.08, 0.28, 0.64)$	$(0, 0.2, 0.8)$
11	$(0.5, 0.5, 0)$	$(0.4, 0.44, 0.16)$	$(0.3, 0.38, 0.32)$	$(0.2, 0.32, 0.48)$	$(0.1, 0.26, 0.64)$	$(0, 0.2, 0.8)$
12	$(0.6, 0.4, 0)$	$(0.48, 0.36, 0.16)$	$(0.36, 0.32, 0.32)$	$(0.24, 0.28, 0.48)$	$(0.12, 0.24, 0.64)$	$(0, 0.2, 0.8)$
13	$(0.4, 0.6, 0)$	$(0.36, 0.48, 0.16)$	$(0.32, 0.36, 0.32)$	$(0.28, 0.24, 0.48)$	$(0.24, 0.12, 0.64)$	$(0.2, 0, 0.8)$
14	$(0.5, 0.5, 0)$	$(0.44, 0.4, 0.16)$	$(0.38, 0.3, 0.32)$	$(0.32, 0.2, 0.48)$	$(0.26, 0.1, 0.64)$	$(0.2, 0, 0.8)$
15	$(0.6, 0.4, 0)$	$(0.52, 0.32, 0.16)$	$(0.44, 0.24, 0.32)$	$(0.36, 0.16, 0.48)$	$(0.28, 0.08, 0.64)$	$(0.2, 0, 0.8)$
16	$(0.7, 0.3, 0)$	$(0.6, 0.24, 0.16)$	$(0.5, 0.18, 0.32)$	$(0.4, 0.12, 0.48)$	$(0.3, 0.06, 0.64)$	$(0.2, 0, 0.8)$
17	$(0.8, 0.2, 0)$	$(0.68, 0.16, 0.16)$	$(0.56, 0.12, 0.32)$	$(0.44, 0.08, 0.48)$	$(0.32, 0.04, 0.64)$	$(0.2, 0, 0.8)$
18	$(0.2, 0.8, 0)$	$(0.16, 0.72, 0.12)$	$(0.12, 0.64, 0.24)$	$(0.08, 0.56, 0.36)$	$(0.04, 0.48, 0.48)$	$(0, 0.4, 0.6)$
19	$(0.3, 0.7, 0)$	$(0.24, 0.64, 0.12)$	$(0.18, 0.58, 0.24)$	$(0.12, 0.52, 0.36)$	$(0.06, 0.46, 0.48)$	$(0, 0.4, 0.6)$
20	$(0.4, 0.6, 0)$	$(0.32, 0.56, 0.12)$	$(0.24, 0.52, 0.24)$	$(0.16, 0.48, 0.36)$	$(0.08, 0.44, 0.48)$	$(0, 0.4, 0.6)$
21	$(0.5, 0.5, 0)$	$(0.4, 0.48, 0.12)$	$(0.3, 0.46, 0.24)$	$(0.2, 0.44, 0.36)$	$(0.1, 0.42, 0.48)$	$(0, 0.4, 0.6)$
22	$(0.5, 0.5, 0)$	$(0.48, 0.4, 0.12)$	$(0.46, 0.3, 0.24)$	$(0.44, 0.2, 0.36)$	$(0.42, 0.1, 0.48)$	$(0.4, 0, 0.6)$
23	$(0.6, 0.4, 0)$	$(0.56, 0.32, 0.12)$	$(0.52, 0.24, 0.24)$	$(0.48, 0.16, 0.36)$	$(0.44, 0.08, 0.48)$	$(0.4, 0, 0.6)$
24	$(0.7, 0.3, 0)$	$(0.64, 0.24, 0.12)$	$(0.58, 0.18, 0.24)$	$(0.52, 0.12, 0.36)$	$(0.46, 0.06, 0.48)$	$(0.4, 0, 0.6)$
25	$(0.8, 0.2, 0)$	$(0.72, 0.16, 0.12)$	$(0.64, 0.12, 0.24)$	$(0.56, 0.08, 0.36)$	$(0.48, 0.04, 0.48)$	$(0.4, 0, 0.6)$

Each lower envelope lottery listed above can be constructed as the convex combination $\delta R + (1 - \delta)Y$, where R represents the lower envelope lottery at the "Risky End" of the budget, and Y represents the lower envelope lottery at the "Ambiguous End" of the budget.

B.3 Instructions

Welcome

Please wait to open this packet until you are instructed to do so.

Instructions Page 1

Introduction

This is an experiment about decision-making. It should take about 60 minutes. On your desk is a sheet of paper that has a number written on it. You will be identified only by this number not by your name. All of your payment will be given to you privately, today, in cash, here at the lab.

During this experiment you will be presented with a series of *choice-situations*. In each choice-situation there are several *alternatives* from which you can choose. These alternatives are a list of possible monetary *amounts* and information about the probability of receiving those amounts. All of the alternatives in this experiment will have only two possible amounts you can receive: CHF 60 or CHF 20. For some alternatives the probability of receiving CHF 60 or CHF 20 will be partially or fully unknown when you choose. For other alternatives the probability of receiving the CHF 60 or CHF 20 will be fully known when you choose. We will explain the alternatives and choice-situations in more detail shortly.

When this experiment is over, one choice-situation will be randomly selected to count for actual payment. You will receive the alternative you selected in that choice-situation and you will draw one ping-pong ball from the bingo blower you saw on your way into the lab to determine whether you will receive CHF 60 or CHF 20 from that alternative. I will explain payments in more detail shortly.

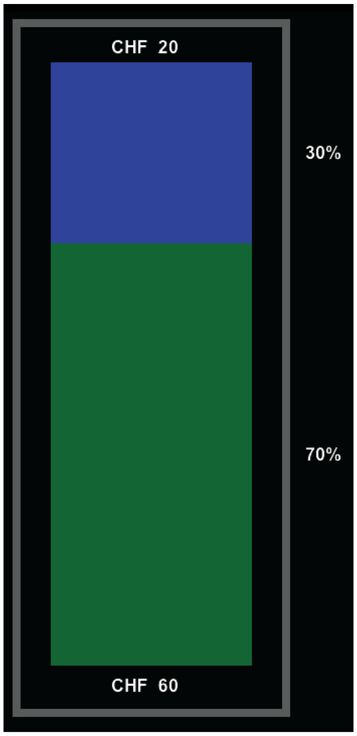
Please wait to turn the page until you are instructed to do so.

Instructions Page 2

Types of Alternatives

In this experiment you will be making choices between alternatives. All of the alternatives in today's experiment will have two possible amounts and these amounts will always be in Swiss Francs (CHF). The two amounts you can receive from any alternative are either CHF 60 or CHF 20. In this way, all alternatives will be similar to each other.

Alternatives will be different from each other in two ways. First, alternatives will differ in the probability of receiving either the CHF 60 or the CHF 20. The second way alternatives will differ from each other is in the information you know about the probability of receiving either CHF 60 or CHF 20. The remainder of this section will describe these features and explain how the alternatives will be visually laid-out.



Alternatives with fully-known probabilities

For some alternatives you will know the exact probability of receiving the CHF 60 or CHF 20 amounts. We will call these "alternatives with fully-known probabilities." The figure to the right is an example of how alternatives with fully-known probabilities will be presented.

The two possible amounts, CHF 60 and CHF 20, are written below and above the vertical bar, respectively. Alternatives will always be presented so that the CHF 60 amount is on the bottom and the CHF 20 amount is on the top.

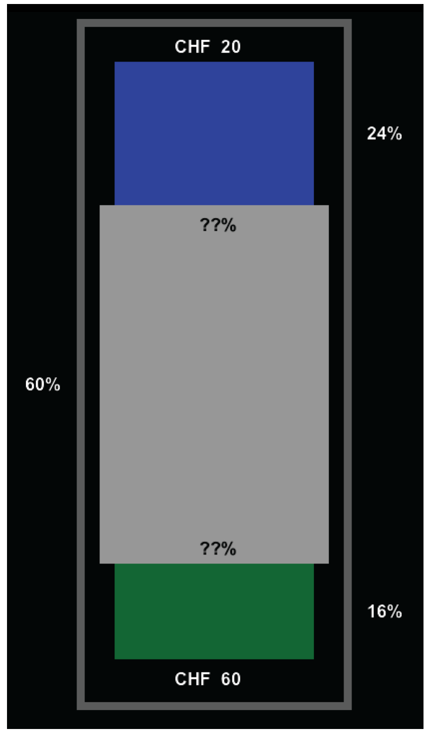
The green block on the bottom is a visual representation of the probability of receiving the CHF 60 amount. Blocks with this green color will only ever be used to represent information about the probability of receiving the CHF 60 amount.

The blue block on the top is a visual representation of the probability of receiving the CHF 20 amount. Blocks with this blue color will only ever be used to represent information about the probability of receiving the CHF 20 amount.

The exact probability of receiving each amount is shown as a percentage (out of 100%) in text, to the right the colored blocks. For the alternative shown to the right, the probability of receiving CHF 60 is 70%, or 70 out of 100. The probability of receiving CHF 20 is 30%, or 30 out of 100. Note that the probabilities add up to 100%.

The height of each of the colored blocks is proportional to the probabilities. You can see this in the alternative shown above — the block representing the probability of receiving the CHF 60 is taller than the block representing the probability of receiving CHF 20. This is because the probability of receiving CHF 60 is larger than the probability of receiving CHF 20 from this alternative.

Alternatives with unknown probabilities



For some alternatives the exact probability of receiving the CHF 60 or CHF 20 amounts will be unknown when you choose. We will call these "alternatives with unknown probabilities." The figure to the right is an example of the visual layout for alternatives with unknown probabilities.

Note that the two possible amounts, CHF 60 and CHF 20, are written below and above the vertical bar, just like when probabilities are fully known.

The green block on the bottom is a visual representation of the minimum probability of receiving CHF 60 – For this alternative, the probability of receiving CHF 60 cannot be less than the probability represented by this green block. This minimum probability of receiving CHF 60 is specified exactly, in text, to the right of the green block. For this alternative it is 16%.

The blue block on the top is a visual representation of the minimum probability of receiving CHF 20 – For this alternative, the probability of receiving CHF 20 cannot be less than the probability represented by this blue block. This minimum probability of receiving CHF 20 is specified exactly, in text, to the right of the blue block. For this alternative it is 24%.

The grey block in the center covers part of the alternative so that you (and the experimenters) do not know the exact probabilities of receiving CHF 60 or CHF 20. So, for the alternative depicted above, the probability of receiving CHF 60 could be as low as 16% or, it could be as high as 76% (16%+60%). Conversely, the probability of receiving CHF 20 could be as high as 84% (24%+60%) or, it could be as low as 24%. The question marks (??%) in text, on top of this grey block serve as an indicator that this part of the probabilities is unknown. I will explain more about these unknown probabilities shortly.

The heights of the blocks are all proportional to the known and unknown probabilities. In the figure above, the height of the green block is smaller than the height of the blue block because the minimum probability of receiving CHF 60 is smaller than the minimum probability of receiving CHF 20. The height of the grey block is larger than both the green and blue blocks because the total amount of unknown probability in this alternative is larger than the minimum probabilities for the CHF 60 and CHF 20 amounts. The number to the left of the grey block gives the exact amount of unknown probability for this alternative: 60%. Note that the minimum probabilities and missing probability amounts add up to exactly 100%.

Please wait to turn the page until you are instructed to do so.

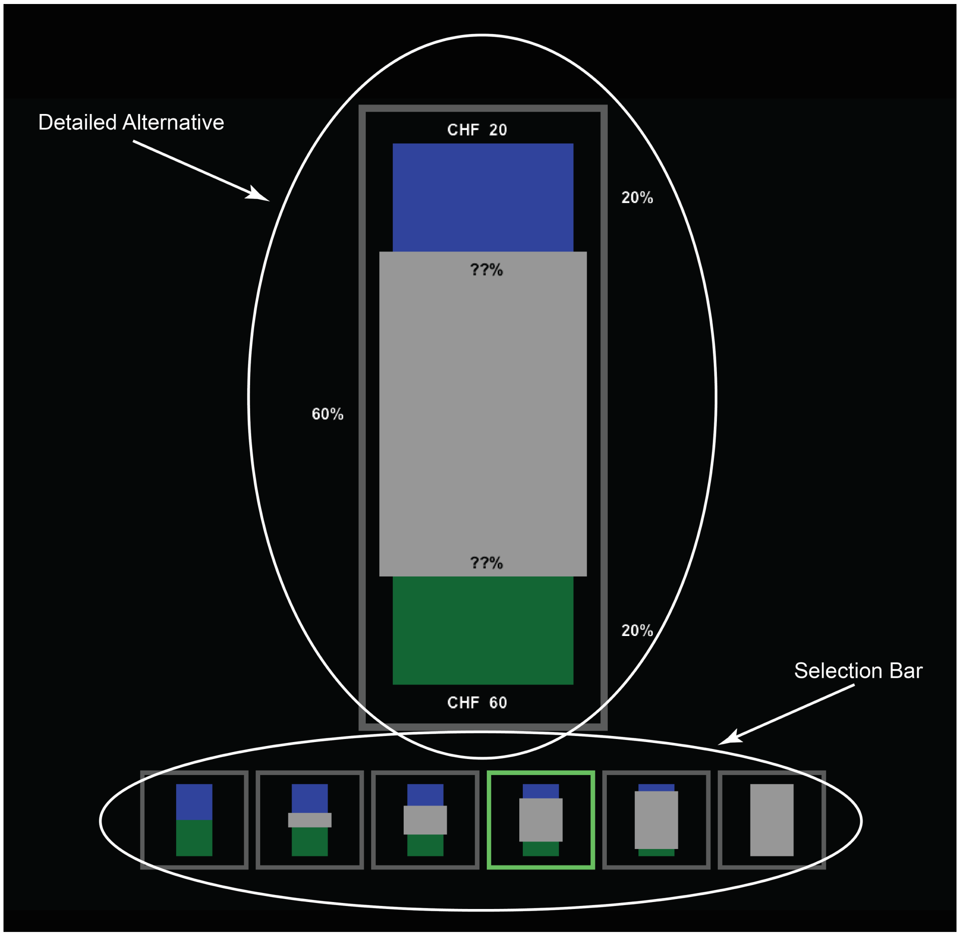
Instructions Page 4

The Graphical Interface

In the experiment you will be presented with a series of choice-situations. In each of these choice-situations there will be a list of alternatives. In each choice-situation your job is to select the alternative that you want most. To make your job easier the information in each choice-situation will be presented with a graphical interface. This section explains how the graphical interface displays all of the information in each choice-situation.

The Graphical Interface has two principle components:

1. *Selection Bar*: This shows thumbnail images of all the alternatives in the choice-situation.
2. *Detailed Alternative*: This shows a zoomed view of the alternative highlighted in the Selection Bar.



Instructions Page 5

Detailed Alternative

At the center of the graphical interface is a large, *Detailed Alternative* that shows all of the amount and minimum probability information, in text, just like the alternatives detailed above.

Selection Bar

At the bottom of each screen is a row of boxes. In the center of each of the boxes is a small image of an alternative that the box represents. Note that these thumbnail images do not have text on them. To see the exact minimum probability and amount information for any of the alternatives just move your mouse over the box with the small image on it. When you move your mouse over one of the boxes, a green outline will surround the box and the thumbnail alternative will be depicted as the Detailed Alternative so that you can see the exact amount and minimum probability information.

Making a Choice

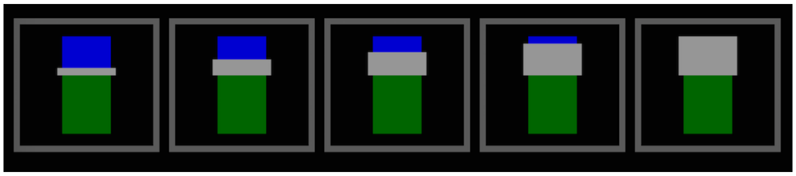
To indicate which alternative you want most from those available move your mouse so that the green outline surrounds the box with the small image of your most preferred alternative, so that it is represented as the Detailed Alternative. Then click your mouse.

When you click your mouse the green outline and Detailed Alternative will be locked and a “Confirm” button and a “Cancel” button will appear. If you want to unlock the green outline just click on the “Cancel” button. If this is the alternative that you want most in this choice-situation, click on the “Confirm” button. Once you click “Confirm” you will not be able to change your selection.

Please wait to turn the page until you are instructed to do so.

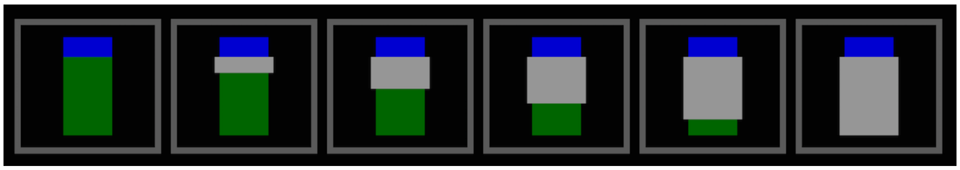
The Experiment: Your Choices

Each choice-situation in this experiment will have five alternatives with unknown probabilities. Some choice-situations will also include a sixth alternative that has fully known probabilities. In the experiment your job is to select the alternative that you want most from these lists of five or six alternatives. To make your job easier, the lists of alternatives are always arranged in a systematic way in the Selection Bar.



The Selection Bar depicted just above represents a choice-situation in which there are five alternatives, each of which has unknown probabilities. For each alternative the minimum probability of receiving CHF 60 is the same. This is easy to see because the height of the green block at the bottom of each alternative, the block that represents the minimum probability you will get CHF 60, is the same height across the entire Selection Bar. But, as you move from left to right across the Selection Bar, the minimum probability of receiving CHF 20 decreases while the unknown probability increases. This is easy to see because the height of the blue block at the top of each alternative, the block that represent the minimum probability you will get CHF 20, decreases when moving from left to right, while the height of the grey block, the block that represents the unknown probabilities, increases.

The Selection Bar depicted at the bottom of this page represents a choice-situation in which there are six alternatives, five of which have unknown probabilities and one that has fully known probabilities. For the five alternatives with unknown probabilities the minimum probability of receiving CHF 20 is the same. This is easy to see because the height of the blue block at the top of each alternative, the block that represents the minimum probability you will get CHF 20, is the same across the entire Selection Bar. But, as you move from right to left across the Selection Bar, the minimum probability of receiving CHF 60 increases while the unknown probability decreases. This is easy to see because the height of the green block at the bottom of each alternative, the block that represent the minimum probability you will get CHF 60, increases when moving from right to left, while the height of the grey block, the block that represents the unknown probabilities, decreases.



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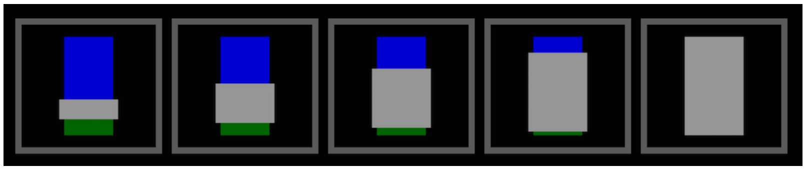
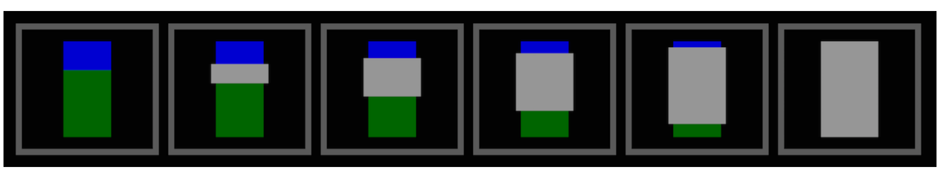


Most of the choice-situations in the experiment will be more complicated than the two just described. For example, in the Selection Bar depicted at the top of this page, there are five alternatives, all with unknown probabilities. For the right-most alternative the probabilities are completely unknown — the grey block covers the whole area. As you move from right to left across the Selection Bar the unknown probability amount decreases while the minimum probabilities for CHF 60 and CHF 20 both increase. The rate at which these values trade off against each other will vary from choice-situation to choice-situation. For the Selection Bar depicted at the top of this page, the tradeoff is tilted in favor of the minimum probability of CHF 20, relative to the minimum probability of CHF 60. You can see this as you move from right to left across the Selection Bar — the height of the blue block (which represents the minimum probability of getting CHF 20) increases at a faster rate than the height of the green block (which represents the minimum probability of getting CHF 60).

In contrast, the Selection Bar at the bottom of this page represents a choice-situation in which the tradeoff is tilted in the other direction. The right-most alternative once again has probabilities that are completely unknown. As you move from right to left across the selection bar the height of the unknown probability amount decreases while the minimum probabilities for CHF 60 and CHF 20 both increase. You can easily see this because the height of the green block (which represents the minimum probability of getting CHF 60) increases at a faster rate than the blue block (which represents the minimum probability of getting CHF 20). Also note that there are six alternatives in this choice-situation and that the left-most alternative has fully known probabilities.



Please wait to turn the page until you are instructed to do so.

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Payment

All payments will be in cash, today, here at the lab. All of your payment will come from the alternative you selected in the one choice-situation that counts for actual payment. Thus, only two payments are possible: CHF 60 or CHF 20.

During the experiment you will select the alternative you want most in each choice-situation. Once you are finished making your selections the computer will randomly select one choice-situation that will count for actual payment. The computer will display the choice-situation and the alternative you selected. When this happens please raise your hand to get an experimenter's attention. We will come over and write down the details of the alternative you selected in the choice-situation that counts for payment. You will verify that we wrote this down correctly and put your signature on the slip of paper. However, please remain seated until an experimenter instructs you to go receive your payment at the cashier's desk. When you are instructed to go to the cashier's desk please take your personal belongings with you. Once you arrive at the cashier's desk you will not be allowed to come back into this room.

When you arrive at the cashiers desk you will draw one ping-pong ball from the bingo-blower you saw on your way into the lab. That ping-pong ball will be used to determine your payment from the alternative you selected in the one choice-situation that counts for payment. Here is how that works.

Inside the bingo blower there are exactly 100 ping-pong balls. Each ping-pong ball inside the blower has a whole number (integer) written on it. The number written on each ping-pong ball is between 1 and 100. The set of 100 ping-pong balls in the bingo blower are set up in such a way so that the number of ping-pong balls between any odd-number and any larger, even-number is exactly the count between them. For example, there are exactly 10 ping-pong balls that can have numbers 1 through 10 written on them. However, it is unknown how many of those 10 ping-pong balls have an even-number written on them and how many have an odd-number written on them. Note that for this interval of 1 through 10 the even-numbers are 2, 4, 6, 8, and 10. The odd-numbers are 1, 3, 5, 7, and 9. To demonstrate how this works, and how we will use the ping-pong ball drawn from the bingo blower to determine your payment, we will go through three examples.

Even numbers end with a 2, 4, 6, 8, or 0.

Odd numbers end with a 1, 3, 5, 7, or 9.

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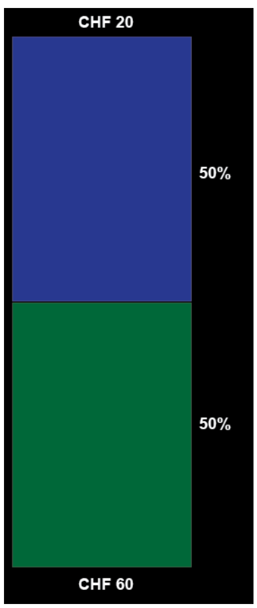
Example 1

Suppose your payment will be based on the alternative shown below. This alternative has fully-known probabilities: 50% for receiving CHF 20 and 50% for receiving CHF 60. Inside the bingo blower we know that there exactly 100 ping-pong balls and that 50 of the ping-pong balls can have numbers 1 through 50 written on them. So, the probability of a ping-pong ball being drawn with any number between 1 and 50 is exactly 50%. We also know that there are exactly 50 ping-pong balls that can have the numbers 51 through 100 written on them. So, the probability of a ping-pong ball being drawn with any number between 51 and 100 is exactly 50%. This lets us determine your payment from this alternative. If the number written on the ping-pong ball drawn from the bingo blower is any number between 1 and 50 you would receive CHF 20. If it is any number is between 51 and 100 you would receive CHF 60. The table next to the image provides a summary of how this works.

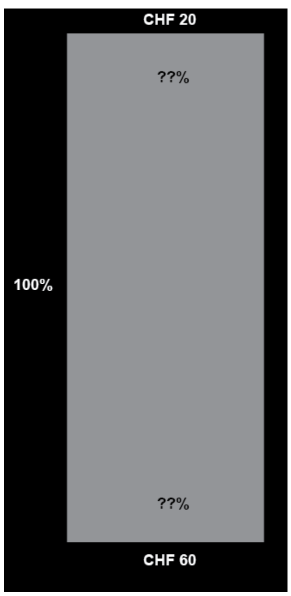


Number on ping-pong ball	Payment
1-50	CHF 20
51-100	CHF 60

Please wait to turn the page until you are instructed to do so.

Example 2

Suppose instead that your payment will be based on the alternative shown below. This alternative has probabilities that are completely unknown. Inside the bingo blower there are exactly 100 ping-pong balls. However, we do not know how many of the 100 ping-pong balls have an even-number written on them and how many have an odd-number written on them. It is possible that every ping-pong ball has an even-number written on it. So, the probability of drawing a ball with an even-number could be as large as 100%. It is also possible that no ping-pong ball has an even-number written on it. So, the probability of drawing a ping-pong ball with an even-number on it could be as small as 0%. This means that the probability of drawing an even-numbered ball could be anywhere between 0% and 100%. It could be 22%. It could be 78%. We just don't know. That the probability of drawing an even-numbered ball is unknown and between 0% and 100% lets us determine your payment from this alternative. If the ping-pong ball drawn from the bingo blower has an odd-number on it you would receive CHF 20. If the number on the ping-pong ball is even you would receive CHF 60. The table next to the image below provides a summary of how this works.



Number on ping-pong ball	Odd/Even	Payment
1-100	Odd	CHF 20
	Even	CHF 60

Please wait to turn the page until you are instructed to do so.

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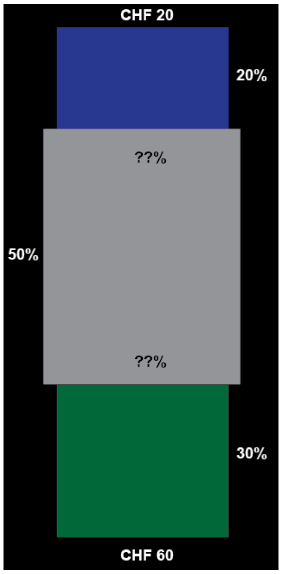
Example 3

This final example will demonstrate how we will determine your payment from alternatives that have both minimum probabilities and unknown probabilities. Suppose that your payment will be based on the alternative shown below. This alternative has a minimum probability of 20% for receiving CHF 20, a minimum probability of 30% for receiving CHF 60, and 50% of the probability is unknown. Inside the bingo blower there are are exactly 20 ping-pong balls (out of 100 total) that can have the numbers 1 through 20 written on them. So, the probability of drawing a ping-pong ball with any number between 1 and 20 written on it is exactly 20%. Also, there are exactly 30 ping-pong balls that can have the numbers 71 through 100 written on them. So, the probability of drawing a ping-pong ball with any number between 71 and 100 written on it is exactly 30%. We also know that there are exactly 50 ping-pong balls that can have the numbers 21 through 70 written on them. However, we don't know how many of these ping-pong balls have an even-number written on them — there could be any number between 0 and 50 ping-pong balls with even-numbers between 21 and 70. This means that the probability of drawing an even-numbered ball between 21 and 70 is unknown and could be anywhere between 0% and 50%.

This lets us determine whether you would receive CHF 20 from this alternative or CHF 60 — If the ping-pong ball drawn from the bingo blower has any number between 1 and 20 written on it you would receive CHF 20. If the number is between 71 and 100 you would receive CHF 60. If the number on the ping-pong ball drawn from the bingo blower is an odd-number between 21 and 70 you would receive CHF 20. If it is an even-number between 21 and 70 you would receive CHF 60. The table next to the image below provides a summary of how this works.



Number on ping-pong ball	Odd/Even	Payment
1-20	-	CHF 20
21-70	Odd	CHF 20
	Even	CHF 60
71-100	-	CHF 60

Please wait to turn the page until you are instructed to do so.

Instructions Page 12

Creating the ping-pong balls

The ping-pong balls that will be used to determine your payment were constructed using a procedure to insure that when you make your selections the unknown probabilities are unknown to both you and the experimenters. This section explains the procedure we used.

The numbers on the ping-pong balls were written by various students, staff, and faculty in the Department of Economics at the University of Zürich. These various people were handed an envelope in which there were two ping-pong balls and instructions. The box below provides an example of these instructions, using the numbers 1 and 2:

In this envelope there are 2 ping-pong balls.

Using the enclosed pen, please do one of the following:

1) *write a "1" on both ping-pong balls*
2) *write a "2" on both ping-pong balls*
3) *write a "1" on one ping-pong ball and a "2" on the other ping-pong ball*

When you are finished please put the ping-pong balls back into the envelope, ***seal it****, and place it in Dan Burghart's mailbox on the 2nd floor.*

Thank you.

A total of 50 such envelopes, using numbers 1-100, in sequential, two-digit increments, were created with this procedure. An assistant took these 50 sealed envelopes with the 100 ping-pong balls in them to a private location and put them into the bingo blower.

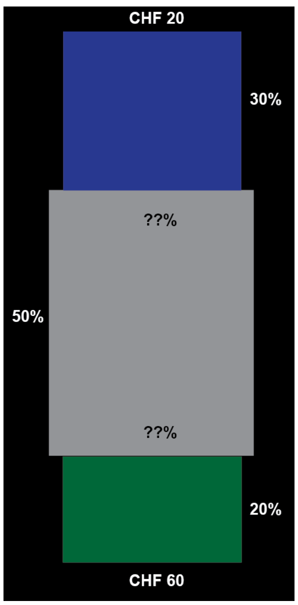
This procedure has several important features. First, the various people writing the numbers on the ping-pong balls did not know anything about the choices that can be made in this experiment.

A second feature of this procedure is that the assistant who put the ping-pong balls into the bingo blower did not know anything about the decisions that can be made in this experiment. Furthermore, that assistant is not involved with the experiment today.

We follow this procedure for two important reasons. First, when you make your selections in the experiment the unknown probabilities are, in fact, unknown to you. Second, we want to insure that the experimenters do not know anything about the unknown probabilities in this experiment – the experimenters know what you know about these unknown probabilities.

Quiz

1. For the alternative shown below, please fill-in the 3 cells in the table under "Number on ping-pong ball."



Number on ping-pong ball	Odd/Even	Payment
	-	CHF 20
	Odd	CHF 20
	Even	CHF 60
	-	CHF 60

2. Suppose the alternative shown above is the one that counts for payment and that a ping-pong ball with the number 43 written on it is drawn from the bingo blower. What would the payment be?

CHF ____________

The quiz given to subjects after the instructions before eliciting choices