EXPECTED REVENUE IN DISCRIMINATIVE AND UNIFORM PRICE SEALED-BID AUCTIONS

James C. Cox, Vernon L. Smith, and
James M. Walker

I. INTRODUCTION

This study uses laboratory experimental data to test the predictions of Nas equilibrium bidding theory for expected revenue in discriminative and unifor price sealed-bid auctions. The two auction institutions are defined as follows:

1. *Discriminative*—This is the auction in which each accepted bid is filled the prices specified by the bidder.
2. *Uniform price*—This is the auction in which all accepted bids are filled at common price that is equal to the highest rejected bid.

In the case of a single-unit auction the discriminative auction is referred to the *first-price auction* and the uniform price auction is referred to as the *secon price auction*.[1] The multiunit uniform price auction is also referred to as tl *competitive auction*. In addition, there is another variant of the uniform pri auction in which the common price is equal to the lowest accepted bid rath than the highest rejected bid.

The present paper is concerned with the limiting case of multiunit auctio

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where there is a perfectly inelastic supply of Q units of a homogeneous good and each of N bidders can submit a bid for a single unit. The other key feature of the auctions that we examine consists of bidder knowledge of the unit value of the auctioned object. Each bidder is assumed to know the value to him of a unit of the auctioned object before he submits his bid. A bidder does not know the values that his rivals place on a unit of the auctioned object but is assumed to know that each rival bidder's unit object value was drawn from a known probability distribution.

Nash equilibrium bidding theory for the auctions outlined in the preceding paragraphs has been developed by Vickrey (1962) and by Harris and Raviv (1981). The following section of our paper contains a derivation of some of the results in those papers and extensions of the theory for the case of uniform value distributions.

II. NASH EQUILIBRIUM BIDDING THEORY

Let v_i be the monetary value of a unit of the auctioned object to bidder i, where $i = 1, 2, \ldots, N$. The unit object value for each bidder is assumed to be drawn (with replacement) from the distribution with density function g and cumulative distribution function G. The support of G is assumed to be the nonempty interval $[0,\bar{v}]$. Let b_i be the amount bid by bidder i, and let p_i be the price that he pays for a unit of the auctioned object if his bid is accepted. In the event that his bid is accepted, the i^{th} bidder gains the money income $v_i - p_i$. Define $u(v_i - p_i)$ as the utility to bidder i of that money income and adopt the normalization that u(0) equals 0.[2] Further assume that the utility of a rejected bid is also 0. The utility function u is assumed to be increasing, differentiable, and concave.

In the uniform price auction the Q highest bidders each receive a single unit at a price equal to the highest rejected bid [which is the $(Q + 1)^{th}$ highest bid]. In this auction the dominant strategy for each bidder is to bid an amount equal to his object value. This result is implied by the following reasoning. Suppose that a bidder begins by tentatively setting his bid equal to the object value and then considers decreasing it. Any such decrease in his bid only decreases the probability of having the bid accepted without decreasing the amount paid in the event that the bid is accepted. In a similar vein, bidding more than value only exposes the bidder to the risk of having to pay more for the object than it is worth. Thus the dominant strategy function, b_U, for the uniform price auction is the identity map

$$b_U(v_i) = v_i. \tag{2.1}$$

Since all dominant strategy equilibria are Nash equilibria, (2.1) is the Nash equilibrium bid function for the uniform price auction.

We now index bidders from lowest to highest object value so that $v_1 \leq v_2 \leq \cdots \leq v_N$. The common price in the uniform price auction is the highest rejected bid which, given (2.1) and our indexing of bidders, is v_{N-Q}. Therefore, the seller's revenue is

$$R_U = Qv_{N-Q}. \tag{2.2}$$

When bidders' values are observable (as they are in the laboratory) the predicted revenue in the uniform price auction, conditional on the observed values, is given by (2.2). The unconditional expected revenue in the uniform price auction equals the product of Q and the expected value of v_{N-Q}. Note that v_{N-Q} is the $(N-Q)$th order statistic for a random sample of size N from the distribution G. Therefore, the unconditional expected revenue in the uniform price auction is

$$\begin{aligned} E(R_U) &= Q\int_0^{\bar{v}} \frac{N!}{(N-Q-1)!Q!} v[G(v)]^{N-Q-1}[1-G(v)]^Q g(v)\,dv \\ &= \frac{N!}{(N-Q-1)!(Q-1)!}\int_0^{\bar{v}} v[G(v)]^{N-Q-1}[1-G(v)]^Q g(v)\,dv. \end{aligned} \tag{2.3}$$

Now assume that the distribution of values is the uniform distribution. Then $g(v) = 1/\bar{v}$, $G(v) = v/\bar{v}$, and (2.3) can be rewritten as

$$E(R_U) = \frac{N!}{(N-Q-1)!(Q-1)!}\int_0^{\bar{v}} v[v/\bar{v}]^{N-Q-1}[1-v/\bar{v}]^Q[1/\bar{v}]\,dv. \tag{2.4}$$

Define

$$y = v/\bar{v} \tag{2.5}$$

and change variables in (2.4) to get

$$E(R_U) = \frac{N!}{(N-Q-1)!(Q-1)!}\,\bar{v}\int_0^1 y^{N-Q}[1-y]^Q\,dy. \tag{2.6}$$

The integral in (2.6) is a complete Beta function that can be integrated to yield

$$\begin{aligned} E(R_U) &= \frac{N!}{(N-Q-1)!(Q-1)!}\,\bar{v}\,\frac{(N-Q)!Q!}{(N+1)!} \\ &= \frac{Q(N-Q)}{(N+1)}\,\bar{v}. \end{aligned} \tag{2.7}$$

Statement (2.7) gives us the unconditional expected revenue in the uniform price auction when unit object values are drawn from the uniform distribution on $[0,\bar{v}]$.

In the discriminative auction each accepted bid is filled at the bid price. We will now derive the Nash equilibrium bidding strategy function for that auction.

Suppose that the i^{th} bidder believes that each of his rivals will use the differentiable bid function,

$$b_j = b_D(v_j), \tag{2.8}$$

that is increasing on $[0,\bar{v}]$. Let π denote the inverse of bid function (2.8); then

$$\pi(b_D(v_j)) = v_j. \tag{2.9}$$

The probability that any one of the $N - 1$ rivals of bidder i will bid less than some amount b in the range of (2.8) is the same as the probability that the rival will draw a value which, when substituted in (2.8), will yield a bid that is less than b. That probability is $G(\pi(b))$.

A bid by bidder i in the amount b will be accepted if at least $(N - 1) - (Q - 1)$ of the bids by his $N - 1$ rivals are less than b. The probability of that event is the same as the probability that at least $N - Q$ of the values drawn by the rivals are less than $\pi(b)$. That probability is given by the probability distribution function of the $(N - Q)^{th}$ order statistic for a random sample of size $N - 1$ from the distribution G, evaluated at $\pi(b)$:

$$F(\pi(b)) = \frac{(N-1)!}{(N-Q-1)!(Q-1)!}\int_0^{\pi(b)} [G(v)]^{N-Q-1}[1 - G(v)]^{Q-1} g(v)\, dv. \tag{2.10}$$

The expected utility-maximization problem for the ith bidder is

$$\max_{b_i} F(\pi(b_i))\, u(v_i - b_i). \tag{2.11}$$

The first-order condition for maximization problem (2.11) is

$$0 = u(v_i - b_i^o)\, f(\pi(b_i^o))\, \pi'(b_i^o) - u'(v_i - b_i^o)\, F(\pi(b_i^o)), \tag{2.12}$$

where f is the density function associated with F.

The bid function b_D, with inverse π, is a Nash equilibrium strategy function only if b_i^o equals $b_D(v_i)$. We now substitute $b_D(v_i)$ for b_i^o and $1/b'_D(v_i)$ for $\pi'(b_i^o)$ and v_i for $\pi(b_i^o)$ in statement (2.12) to get

$$0 = u(v_i - b_D(v_i))\, f(v_i)/b'_D(v_i) - u'(v_i - b_D(v_i))\, F(v_i). \tag{2.13}$$

If the solution of the differential equation (2.13) exists and is increasing as was initially assumed, then b_D is a Nash equilibrium bidding strategy function.

Now assume that all bidders are risk neutral and write the bid function fc this case as $b_{DN}(\cdot)$. Then (2.13) can be rewritten as

$$0 = -F(v_i) + [v_i - b_{DN}(v_i)]\, f(v_i)/b'_{DN}(v_i). \tag{2.14}$$

Statement (2.14) can be rewritten as

$$\frac{d}{dv_i}[F(v_i) b_{DN}(v_i)] = v_i\, f(v_i). \tag{2.15}$$

Since $b_{DN}(0)$ equals 0, the solution of (2.15) is

$$b_{DN}(v_i) = \frac{1}{F(v_i)}\int_0^{v_i} x\, f(x)\, dx. \tag{2.16}$$

Statement (2.16) defines the Nash equilibrium bidding strategy function for th discriminative auction when all bidders are risk neutral.

Now assume that unit object values are drawn from the uniform distribution Then statement (2.10) can be rewritten as

$$F(\pi(b)) = \frac{(N-1)!}{(N-Q-1)!(Q-1)!}\int_0^{\pi(b)} [v/\bar{v}]^{N-Q-1}[1 - v/\bar{v}]^{Q-1}[1/\bar{v}]\, dv. \tag{2.17}$$

We next use (2.5) to change variables in (2.17) and get

$$F(\pi(b)) = \frac{(N-1)!}{(N-Q-1)!(Q-1)!}\int_0^{\pi(b)/\bar{v}} y^{N-Q-1}[1 - y]^{Q-1}\, dy. \tag{2.18}$$

Statements (2.16) and (2.18) imply

$$b_{DN}(v_i) = \frac{\bar{v}\int_0^{v_i/\bar{v}} y^{N-Q}[1 - y]^{Q-1}\, dy}{\int_0^{v_i/\bar{v}} y^{N-Q-1}[1 - y]^{Q-1}\, dy}. \tag{2.19}$$

Noting that $(N - Q)$ and $(Q - 1)$ are positive integers and using the binomia expression for $(1 - y)^{Q-1}$, Eqs. (2.19) can be rewritten as:

$$b_{DN}(v_i) = \bar{v}\,\frac{\int_a^{v_i/\bar{v}} y^{N-Q}\sum_{k=0}^{Q-1}\frac{(-1)^k y^k (Q-1)!}{k!(Q-1-k)!}}{\int_a^{v_i/\bar{v}} y^{N-Q-1}\sum_{k=0}^{Q-1}\frac{(-1)^k y^k (Q-1)!}{k!(Q-1-k)!}}$$
$$= \bar{v}\,\frac{\sum_{k=0}^{Q-1}\frac{(-1)^k (v_i/\bar{v})^{N-Q+k+1}(Q-1)!}{(N-Q+k+1)\,k!\,(Q-1-k)!}}{\sum_{k=0}^{Q-1}\frac{(-1)^k (v_i/\bar{v})^{N-Q+k}(Q-1)!}{(N-Q+k)k!(Q-1-k)!}}. \tag{2.20}$$

In this form (2.20), the numerical solutions for the bid function can be found.

If all bidders are risk neutral and values are drawn from the uniform distribution, then (2.20) and our indexing of bidders imply that the seller's revenue in the discriminative auction is

$$R_{DN} = \sum_{j=N-Q+1}^{N} b_{DN}(v_j). \tag{2.21}$$

Now assume that a solution of (2.13) exists for some nonlinear function $u(\cdot)$. Write the resulting bidding strategy function for this risk averse case as $b_{DA}(\cdot)$. In the risk averse case the seller's revenue in the discriminative auction is

$$R_{DA} = \sum_{j=N-Q+1}^{N} b_{DA}(v_j). \tag{2.22}$$

Note that R_{DN} and R_{DA} can be ordered using a theorem proved by Harris and Raviv (1981, pp. 29–30). The theorem orders the bids as follows:

$$b_{DN}(v) < b_{DA}(v) < b_U(v) \text{ for all } v \in [0,\bar{v}]. \tag{2.23}$$

Statements (2.24) – (2.26) imply

$$R_{DN} < R_{DA} \text{ if } v_j > 0 \text{ for some j}. \tag{2.24}$$

The theory does not imply an ordering of R_U with respect to either R_{DN} or R_{DA}. Inspection of (2.2), (2.21), and (2.22) reveals the following: The relative size of R_U and R_{DN} (respectively R_{DA}) depends on the curvature of the bid function b_{DN} (respectively b_{DA}) and on the relative magnitudes of v_{N-Q} and all higher values.

Although the theory does not completely order the conditional revenues in the uniform and discriminative auctions, it does provide a complete ordering of the unconditional expected revenues. Define $E(R_{DN})$ as expected revenue in the discriminative auction when all bidders are risk neutral. Let $E(R_{DA})$ be expected revenue in the discriminative auction when all bidders are risk averse. We are interested in comparing $E(R_{DN})$ and $E(R_{DA})$ with each other and with expected revenue in the uniform price auction, $E(R_U)$. Harris and Raviv (1981, pp. 33–34) prove the following:

$$E(R_U) = E(R_{DN}) < E(R_{DA}). \tag{2.25}$$

Section IV of our paper uses data from laboratory auction market experiments to test the theory developed above. However, before reporting on the results of that test, we turn our attention to an explanation of our experimental design.

III. EXPERIMENTAL DESIGN

One feature of the design and execution of the experiments reported in this study was control of the procedures for conducting each experiment so that all exper-

iments—as far as possible—were conducted in the same manner. This desig strategy was implemented to minimize extraneous "noise" in the experimentε design, thus allowing for more powerful inferences. The key to implementin this control was use of the Plato computer system. Plato allows all subjects t see identical programed instructions and examples and also minimizes subject- experimenter interaction.

A second design feature was sequencing of auctions within experiments s that (a) paired comparisons of auctions under the two auction mechanisms coul be made, and (b) changes in the sequencing of treatment switchovers wer possible. The use of paired comparisons increases the power of tests betwee auction mechanisms. Allowing for the switchover from one auction mechanisn to another within a given experiment increases the credibility that any observe difference found between auctions is due to differences in the institutions an not to differences in the experimental subjects. Finally, changing the sequencin of treatment switchovers across studies allows for the investigation of effects o treatment sequencing; this makes possible an investigation of whether the orde in which the institutions are tested affects the observed differences betwee institutions.

A third feature of the experimental design was adjustment of the distributio from which values were chosen so that the expected gain per bidder from par ticipating in an auction was held constant as the number of bidders and th number of auctioned objects was varied. Our objective here was to minimiz any possible contamination of our results by changing bidder motivation cause by elements of the decision-making process that were excluded by the theor we were testing.

It is well known that any market (or other) decision task may have significan subjective costs of thinking, calculating, deciding, and transacting (Siegel, 1961 Marschak, 1968; Smith, 1976). The greater is the explicit monetary (or other reward which is obtained as an outcome of the decision relative to the subjectiv transactions cost, the more likely will maximization of the reward be the pre dominating influence in determining the decision. Since subjective transaction cost is not normally observable but may be a contaminating factor in testing theory, it can be important to attempt to control for this contamination.

It follows directly from the theoretical argument in Section II that individua bidder motivation depends on the number of bidders and on the number o auctioned objects. In either the uniform price or the discriminative auction th probability that a bid in any given amount will be accepted is a decreasin function of N and an increasing function of Q (given that the probability is no 0 or 1). Thus, for any given v_i, the expected gain to bidder i of a bid in an given amount b_i is a decreasing function of N and an increasing function of Q Therefore, for any given probability distribution of values, the expected gain t a bidder from participating in an auction is a decreasing function of N and a increasing function of Q. In order to hold constant the expected monetary gai from participating in auctions with different Q's and N's, and therefore to control for changing bidder motivation, we changed the probability distribution of values as we changed Q and N. We will next explain in detail the way in which the probability distribution of values was adjusted in order to accommodate changes in Q and N.

The control for bidder motivation was based on the expected gain per bidder in the uniform price auction. The expected revenue in that auction, $E(R_U)$, is given in statement (2.7). The bidders' expected cost per unit traded is $E(R_U)/Q$. The probability that an individual bidder will have a winning bid (i.e., draw one of the Q highest of the N values) is Q/N. Thus the expected monetary cost to an individual of bidding in the auction is

$$E(C_U) = \frac{Q}{N}\frac{1}{Q}E(R_U) = \frac{1}{N}\frac{Q(N-Q)}{(N+1)}\bar{v}. \tag{3.1}$$

We will next derive the expected return to an individual bidder from participating in the auction. An individual bidder has a 1/N probability of drawing the highest value, a 1/N probability of drawing the second highest value, and so on. Let $E(v_{N-j})$ denote the expectation of the $(N - j)$th highest value drawn. Then the expected return to an individual bidder from participating in the auction is

$$E(V_U) = \frac{1}{N}\sum_{j=0}^{Q-1} E(v_{N-j}), \tag{3.2}$$

where $E(v_{N-j})$ is the expected value of the $(N - j)$th order statistic for a random sample of size N from the uniform distribution on $[0,\bar{v}]$. It can be written as

$$E(v_{N-j}) = \frac{N!}{(N-j-1)!j!}\int_0^{\bar{v}} v[v/\bar{v}]^{N-j-1}[1 - v/\bar{v}]^j[1/\bar{v}]dv. \tag{3.3}$$

Now use (2.5) to change variables in (3.3) and proceed to integrate the resulting complete beta function to get

$$E(v_{N-j}) = \frac{N-j}{N+1}\bar{v}. \tag{3.4}$$

Statements (3.2) and (3.4) imply that the expected return to an individual bidder from participating in the auction is

$$E(V_U) = \frac{1}{N}\sum_{j=0}^{Q-1}\frac{N-j}{N+1}\bar{v}. \tag{3.5}$$

Finally, we use (3.1) and (3.5) to find the expected gain to an individual bidder from participating in the auction:

$$E(V_U) - E(C_U) = \frac{1}{N}\left[\sum_{j=0}^{Q-1}\frac{N-j}{N+1}\bar{v} - \frac{Q(N-Q)}{N+1}\bar{v}\right] = \frac{Q(Q+1)}{2N(N+1)}\bar{v}. \tag{3.6}$$

The control for bidder motivation was based on (3.6). As N and Q were varie $\bar{v}$ was varied so as to hold constant the expected gain per bidder in the unifor price auction. The experiments reported in this paper were conducted using tl two sets of experimental design parameters that are listed in Table 1.

We do not argue that making expected gain per bidder a design consta guarantees equal motivation across our experiments. Rather, we argue that th procedure should yield more uniform motivation that if we ignored the issu Ideally, we want the utility of the monetary rewards relative to the nonmoneta factors to be invariant across experiments, but neither utility nor the nonmoneta factors are observable.

We will conclude this section of the paper by briefly noting several oth feature of our experiments. The experiments used both inexperienced and "e: perienced" subjects, the latter having previously participated in at least o experiment with the discriminative and uniform price auctions. Using exper enced subjects minimizes any effects on bidding behavior that might be attribute to subject learning, in the sense of gaining understanding of the auctions t participating in the experiments.

As noted above, before an auction began each bidder learned his value f the commodity being auctioned. A table was displayed to each bidder givir his bids, values, and gains in all preceding auctions. Also provided were tl highest accepted bid and highest rejected bid in the immediately preceding au tion. If the subject wished he could also request a table from the computer th summarized all preceding auctions in terms of highest rejected bids, highe accepted bids, and whether or not his own bid was accepted.

It should be emphasized that in our experimental design the values drawn f each subject varied between auctions. This design is significantly different fro that used by Miller and Plott (1985). In their experiments, individual subje values rotated between auctions in a given sequence but total demand (the ordere values) was held constant. That design makes possible an investigation of tl adjustment of bids over time when the induced total demand curve is stationar In our design the induced-demand curve varied between periods depending o sampling variation from a linear demand curve (uniform density of values).

Finally, the only restriction that we placed on subjects' bids was that they ha

Table 1. Design Parameters

	Design 1	*Design 2*
Starting capital	\$1.00	\$1.00
Minimum resale value	\$0.00	\$0.00
Maximum resale value	\$0.80	\$2.24
Number of subjects	10	10
Units offered	7	4
$E(R_U) = E(R_{DN})$	1.527	4.887

Table 2. Observed Mean (Variance) of Revenue Across Auctions by Institution and Subject Classification

					Theoretical Revenue Conditional Upon Values Actually Drawn	
Experimental Session (1)	*Statistic (2)*	*Discriminative (3)*	*Uniform (4)*	*Discriminative (5)*	*Uniform (6)*	*Discriminative (Risk Neutral) (7)*
1DU1	Mean	1.942	.870		1.372	1.514
	(Variance)	(.225)	(.095)		(.301)	(.018)
1UD1′	Mean		1.750	1.230	1.558	1.537
	(Variance)		(.854)	(.0456)	(.815)	(.014)
1UD2x	Mean		1.743	1.282	1.558	1.537
	(Variance)		(.869)	(.029)	(.815)	(.014)
1DU2′x	Mean	1.444	1.212		1.372	1.514
	(Variance)	(.053)	(.127)		(.301)	(.018)
1UD3	Mean		2.012	1.807	1.558	1.537
	(Variance)		(.731)	(.097)	(.815)	(.014)
1DU3′	Mean	1.889	1.063		1.372	1.514
	(Variance)	(.247)	(.135)		(.301)	(.018)
1UD8x	Mean		1.211	1.375	1.558	1.537
	(Variance)		(.357)	(.018)	(.815)	(.014)
1DU8′Sx	Mean	1.484	1.271		1.372	1.514
	(Variance)	(.017)	(.156)		(.301)	(.018)
2DU4	Mean	4.672	4.494		4.544	4.814
	(Variance)	(.315)	(1.179)		(1.528)	(.113)
2UD4′	Mean		5.237	4.730	4.818	4.693
	(Variance)		(1.038)	(.531)	(.532)	(.230)
2DU5x	Mean	4.467	4.224		4.544	4.814
	(Variance)	(.095)	(.735)		(1.528)	(.113)
2UD5′x	Mean		4.876	4.695	4.818	4.693
	(Variance)		(.764)	(.386)	(.532)	(.230)
2DU6G	Mean	4.562	4.286		4.544	4.814
	(Variance)	(.220)	(.917)		(1.528)	(.113)
2UD6′G	Mean		4.642	4.649	4.818	4.693
	(Variance)		(.503)	(.275)	(.532)	(.230)
2UD7x	Mean		4.810	4.216	4.818	4.693
	(Variance)		(.943)	(.145)	(.532)	(.230)
2DU7′Sx	Mean	4.592	4.458		4.544	4.814
	(Variance)	(.126)	(1.406)		(1.528)	(.113)

to be within the maximum and minimum values for the specified distribution from which values were drawn. Thus subjects could make bids which were less than, equal to, or greater than their values. This was true for both the discriminative and uniform price markets.

IV. EXPERIMENTAL RESULTS: TESTS OF REVENUE HYPOTHESES IN THE TWO AUCTIONS

The first column of Table 2 identifies the treatment conditions for each experiment. Thus, the experiment designated 1DU1 indicates the following: the first "1" denotes that the design 1 parameters of Table 1 were used; "D" indicates that the first 23 auction periods were conducted under discriminative rules; "U" means that the next 22 (for a total of 45) auctions, were conducted using the uniform price rules; the final "1" denotes that the session was a member of the first matched pair of experiments. The experiment labeled 1UD1′ is matched with 1DU1 to form pair number 1. Hence 1UD1′ used design 1; a group of subjects distinct from those who participated in 1DU1; and a sequence of 45 valuations for each subject that is identical to the randomly drawn assignments used in 1DU1, but with the first sequence of 23 auctions conducted under uniform price rules and the last sequence of 22 auctions conducted under discriminative rules. The second matched pair of experiments are designated 1UD2x and 1DU2′x, with the "x" denoting that only experienced subjects participated in these two sessions. The "Sx" appearing in 1DU8′Sx and 2DU7′Sx denotes that experienced subjects were also screened to eliminate those who in previous experiments had deviated most strongly from dominant strategies in the uniform price auction. Almost all the subjects were undergraduates enrolled in various business or economics courses. An exception is the matched pair 2DU6G and 2UD6′G, which used Masters and Ph.D. graduate students in economics and business administration. The design 2 experimental sessions used the same treatment procedures except that a different set of random value sequences was used in these experiments than in design 1. In both experimental designs individual subjects were randomly assigned to particular bidder conditions. Hence, except by chance, an experienced subject would not get the same sequence of 45 values even if he/she participated twice in a design 1 (or 2) experiment. Also the sessions were conducted days and often weeks apart, making it virtually impossible for any individual to perceive two experiments as parametric equivalents.

A total of 8 (Table 2) matched pairs, or 16 experiments consisting of 6 inexperienced sessions, 8 experienced sessions, and 2 graduate student sessions, constitute the complete list of experiments to be reported. Columns 3, 4, and 5 of Table 2 report the observed mean (variance) of auction revenue for each treatment (institution or subject) condition.[3] The last two columns (6 and 7) report the theoretical value-conditional revenues that result from applying the

uniform and risk neutral discriminative models to the values actually realized in each session. Thus, for column 6 the dominant strategy bid function (2.1) is applied to the values, $v_i(t)$, realized for each particular auction, t, and the corresponding theoretical realized revenue, $R_U(t)$, is computed using $v_{N-Q}(t)$ in Eq. (2.2). Column 6 then lists the mean and variance of $R_U(t)$ across all the auctions in each sequence. For column 7, a much more laborious calculation applies the bid function (2.19) to the realized values $v_i(t)$; the theoretical risk-neutral discriminative revenue, $R_{DN}(t)$, is then calculated from (2.21) for each t, and finally the mean and variance of the $R_{DN}(t)$ is computed across each sequence. Note in particular that although $E(R_U) = E(R_{DN})$, the random variables R_U and R_{DN} have *distinct distributions*, with particular sample realizations of R_U and R_{DN} being quite different even for the same realization of values. Comparisons of the population parameter values for $E(R_U) = E(R_{DN})$, for each of the two designs in Table 1, with the particular mean theoretical realizations in Table 2 indicate how close the latter sample means are to their population values. For example, in design 1 the sample theoretical means in the uniform price auction were 1.372 and 1.558 while the theoretical population mean was 1.527.

From (2.7) and (2.25) the theory of bidding in the two types of auctions implies that

$$E(R_U) = E(R_{DN}) = \frac{Q(N-Q)\bar{v}}{(N+1)} < E(R_{DA}). \tag{4.1}$$

Consequently, if we let r_D be the unknown population mean revenue in the discriminative auction, the research hypothesis is $r_D \geq E(R_{DN})$; that is, the population mean cannot be less than its theoretical value if all bidders are risk neutral. The null alternative hypothesis which the theory proposes for rejection is

$$H_o^D: \quad r_D - E(R_{DN}) < 0, \tag{4.2}$$

where $E(R_{DN})$ is given in (4.1). If the distribution of revenue is approximately normal and we observe an experimental sample of size T with mean $\bar{R}_D$ and standard deviation S_D, then a t-test with $t = \sqrt{T}\,(\bar{R}_D - E(R_{DN}))/S_D$ is appropriate. The results of such a t-test of H_o^D are shown in Table 3 for all experimental sessions. Note that (1) only 3 of the 16 experiments yield results (a positive t) consistent with the research hypothesis; (2) in all three of these design 1 cases subjects were inexperienced; (3) in 9 of the 16 cases we would *reject* the *research* hypothesis in favor of the null hypothesis if that had been the prediction of the theory. These results are underlined if we examine the pooled t values for the discriminative auction revenues in the third column of Table 4. In both designs the bids of experienced and graduate student subjects yield revenues significantly *below* the expected risk-neutral revenue. For inexperienced subjects, revenues were significantly above this expected level in design 1 and insignificantly below

Table 3. Revenue Comparisons: Observed Mean vs. Theoretical Mean

Experimental session	$\bar{R}_D - E(R_{DN})$	S_D	T	t	$\bar{R}_U - E(R_U)$	S_U	T	t
1DU1	.417	.474	20	3.934*	−.657	.308	20	−9.540**
1UD1′	−.297	.161	20	−8.250†	.223	.924	20	1.079
1UD2x	−.245	.170	20	−6.445†	.216	.932	20	1.036
1DU2′x	−.083	.230	20	−1.612	−.315	.356	20	−3.957**
1UD3	.280	.311	20	4.026*	.485	.855	20	2.537**
1DU3′	.362	.497	20	3.527*	−.464	.367	19	−5.364**
1UD8x	−.152	.134	20	−5.073†	−.316	.597	20	−2.367**
1DU8′Sx	−.043	.130	20	−1.479	−.256	.395	20	−2.898**
2DU4	−.215	.561	20	−1.714	−.393	1.179	20	−1.491
2UD4′	−.157	.729	20	−.963	.350	1.019	20	1.536
2DU5x	−.420	.308	20	−6.098†	−.663	.857	20	−3.460**
2UD5′x	−.192	.621	20	−1.383	−.011	.874	20	−.056
2DU6G	−.325	.469	20	−3.099†	−.601	.958	20	−2.806**
2UD6′G	−.238	.524	20	−2.031†	−.245	.709	20	−1.545
2UD7x	−.671	.381	20	−7.876†	−.077	.971	20	−.355
2DU7′Sx	−.295	.355	20	−3.716†	−.429	1.186	20	−1.618

*H_o^D is rejected at a significance level $p < .001$ (one-tailed test).
**Significantly different from zero, $p < .05$ (two-tailed test).
†Result is significant ($p < .025$) but in the direction contrary to the research hypothesis.

Table 4. Pooled Revenue Comparisons

	Mean Difference from Expected Revenue				*Paired Sample Revenue Difference*	
	Discriminative		*Uniform Price*			
Treatment Conditions	$\bar{R}_D^{\bullet} - E(R_{DN})$	t	$\bar{R}_U^{\bullet} - E(R_U)$	t	$(R_D^{\bullet} - R_U^{\bullet})$	t
Design 1: N = 10; Q = 7						
(a) Experienced S_S	−.131	−7.06*	−.168	−2.636	.037	.57
(b) Inexperienced S_S	.190	4.69	−.103	−1.50	.286	3.73*
Design 2: N = 10; Q = 4						
(a) Experienced S_S	−.395	−5.99*	−.295	−2.70*	−.099	−.91
(b) Inexperienced S_S	−.186	−1.82	−.022	−.126	−.165	−.92
(c) Graduate students	−.281	−3.59*	−.423	−3.21*	.143	1.07

*Significant, $p < .01$

it in design 2. Hence, the more experienced or "advanced" the bidders, the les consistent are their bids with the theory.

In general outline these results are not in disagreement with previous laborator experiments reported by Smith (1967) and Miller and Plott (1985). They ar also consistent with the analysis of U.S. Treasury bond auction data reporte by Tsao and Vignola (1980). The evidence from several independent empirica studies using distinct experimental paradigms, in both laboratory and field, sug gests that in a wide range of circumstances we must reject the hypothesis tha discriminative auction revenue will be at least as great as revenue in the uniforn price auction.

Letting r_U be the unknown population mean revenue for the uniform pric auction, from (4.1) the research hypothesis is that $r_U = E(R_U)$, with nul alternative,

$$H_o^U: \quad r_U - E(R_U) \neq 0. \qquad (4.3$$

Since in this case the theory yields an extreme (point) hypothesis which is no testable in the classical sense, we follow the usual procedure and report t value conditional upon the research hypothesis being true. Hence, a low t value i "good" in terms of the theory, with no prediction as to sign (two-tailed test) The resulting t values for all experiments are listed in Table 3. In four of th experiments (1UD1′, 1UD2x, 1UD3, 2UD4′), the observed mean revenue ex ceeded the theoretical expected revenue. This is a consequence of several subject bidding in excess of their assigned values in several auctions. It was this phe nomenon that induced us to conduct the two experiments (2DU7′Sx and 1DU8′Sx with subjects who had exhibited the most consistent tendency to submit dominan strategy bids equal to individual values.[4] In Table 3 the bids of these subjects provided mean revenues below expected revenue (significantly below in experiment 1DU8'Sx). In Table 4, the pooled comparisons for the uniform price auction show that sample mean revenues are below the theoretical prediction for all groups, and these results are significant for all but the inexperienced.

Hence, in both auction institutions the behavior of revenue is consistent with the theory only for *inexperienced* subjects in design 1. Experience clearly has an important effect, and in a direction unfavorable to the theory. On average, experienced subjects bid less than predicted by the noncooperative models of behavior in the two auction systems. This suggests some tendency toward "co-operative-like" bidding under the parameter conditions in the two experimental designs that were employed.

The last two columns on the right of Table 4 provide pooled comparisons of the sample revenue differences between the discriminative and uniform price auctions. These differences fail to be significant for all groups except the inexperienced subjects in design 1. Although the theory performs poorly in predicting the outcomes for each type of auction, empirically the two auctions do not yield significantly different revenue characteristics.

V. EXPERIMENTAL RESULTS: EFFECT OF SUBJECT AND SEQUENCE CONDITIONS

The data of Tables 3 and 4 suggest that revenues are influenced by subject condition and whether the discriminative auction came first or second (after the uniform) in the switch sequence. Thus in Table 4 both the experienced and the graduate student subjects generated lower revenue (relative to expected revenue) in both auctions than did the inexperienced subjects. Also note in Table 3 that the design 1 discriminative revenues (relative to expected) tend to be higher when the discriminative rules are applied first in the sequence. To determine the effect, if any, of these subject and sequencing conditions, we report in Table 5 estimates of a linear regression of various measured revenue differences, Y(t), on dummy variables representing subject inexperience (I), selection (S), and graduate standing (G), and the order sequence (D) of the discriminative and uniform price auction rules. Measures of Y(t) are based (1) on the observed revenue in each auction t, namely, $R_D^*(t)$ in discriminative auctions and $R_U^*(t)$ in uniform price auctions; and (2) on the theoretical revenue predicted for auction t given the valuation assignments actually realized in auction t, namely, $R_{DN}(t)$ computed from (2.21) for risk-neutral discriminative auction bidders and $R_U(t)$ computed from (2.2) for dominant strategy bidders in the uniform price auction. By using measures based on $R_{DN}(t)$ and $R_U(t)$ we allow sampling variation from the uniform distribution of individual values to impinge on the theoretical revenue predictions. Variability in the realized or observed revenues, $R_U^*(t)$ and $R_D^*(t)$, will reflect sampling variability in the values assigned individual bidders, as well as individual variability in bidding behavior or strategy. Consequently, in allowing the theoretical revenues to reflect sampling variability in individual value assignments, we sharpen the paired comparisons $R_U^*(t) - R_U(t)$ and $R_D^*(t) - R_{DN}(t)$. Based on *expected* theoretical revenue the "comparisons" would be $R_U^*(t) - E(R_U)$ and $R_D^*(t) - E(R_{DN})$, in which the measures $E(R_U)$ and $E(R_{DN})$ have the property of suppressing, by the expectation operation, sampling variability in the valuations.

The regression variables for the paired comparisons are defined as follows:

$$Y(t) = \begin{cases} R_D^*(t) - R_{DN}(t), \text{ for the discriminative auction theoretical comparisons;} \\ R_U^*(t) - R_U(t), \text{ for the uniform price auction theoretical comparisons;} \\ R_D^*(t) - R_U^*(t), \text{ for comparison of behavior in the two auctions.} \end{cases}$$

$$I = \begin{cases} 1, \text{ if subjects were inexperienced;} \\ 0, \text{ if subjects were experienced.} \end{cases}$$

$$S = \begin{cases} 1, \text{ if subjects were selected (i.e., exhibited tendency to follow dominant strategy bidding behavior in previous session);} \\ 0 \text{ if subjects were not selected.} \end{cases}$$

$$G = \begin{cases} 1, \text{ if subjects were graduate students (inexperienced);} \\ 0, \text{ if subjects were not graduate students.} \end{cases}$$

$$D = \begin{cases} 1, \text{ if the auctions were in periods 4–23;} \\ 0, \text{ if the auctions were in periods 26–45.} \end{cases}$$

The regression equation for the estimates shown in Table 5 is

$$Y(t) = \alpha + \beta_I I + \beta_S S + \beta_G G + \beta_D D + \epsilon(t). \quad (5.1)$$

In interpreting the results of Table 5, it should be noted that when $I = S = G = D = 0$ we have the paired revenue differences that result from using nonselected, experienced, undergraduate subjects in auctions which were in periods 24–45. This is the base condition from which the marginal effects of the various dummy variables (treatment conditions) are measured.

The estimates of β_I in Table 5 indicate that one should be hesitant in making predictions based on inexperienced subjects in multiunit auctions. In both designs and in both auction institutions, inexperienced subjects provided consistently higher paired theoretical revenue comparisons than did experienced subjects (see Table 4; also Table 5, rows 1, 2, 4, 5). But the experience factor does not appear to have a consistent *marginal* effect on the observed revenue differences in the

Table 5. Regression Coefficients (t values) for Paired Revenue Comparison

Design Condition	Y(t), Paired Revenue Difference	α	β_I	β_S	β_G	β_D	Adjusted R^2	F Value
Design 1: N = 10; Q = 7	$R_D^*(t) - R_D(t)$	−.262 (−6.01)**	.311 (5.84)**	−.067 (−.775)		.298 (5.60)	.325	26.51**
	$R_U^*(t) - R_U(t)$	−.419 (−4.771)	.145 (1.647)	.318 (2.217)*		.469 (5.319)**	.141	9.672**
	$R_D^*(t) - R_U^*(t)$	.039 (.362)	.404 (3.44)**	.303 (2.24)*		−.306 (−3.19)**	.106	7.35**
Design 2: N = 10; Q = 4	$R_D^*(t) - R_D(t)$	−.221 (−3.657)**	.249 (2.822)**	.158 (1.312)	.153 (1.737)	−.159 (−2.154)*	.045	2.888*
	$R_U^*(t) - R_U(t)$	−.280 (−2.63)**	.305 (2.41)*	.194 (1.12)	−.099 (−.778)	.318 (2.99)**	.080	4.47**
	$R_D^*(t) - R_U^*(t)$	.362 (2.20)*	−.195 (−.937)	−.055 (−.262)	.114 (.547)	−.663 (−4.51)**	.105	5.64**

*Significant, $p < .05$.
**Significant, $p < .01$.

two auction systems. In design 1 inexperience yields a significant increase in discriminative relative to uniform price revenue, whereas in design 2 inexperience reduces (insignificantly) discriminative relative to uniform price revenue (Table 5, rows 3, 6).

The effect of selective screening of experienced subjects (β_S), as against the base of experienced subjects, is to increase the observed revenue in the uniform auctions when compared to the theoretical revenue in this auction. However the effect of selective screening in comparisons of observed revenues for the uniform and discriminative auctions is not consistent across designs. In comparison with experienced undergraduate subjects, the marginal effect (β_G) of inexperienced graduate students was to increase observed revenue in the discriminative auction relative to theoretical by a small amount.

The effect of the switch sequence order (β_D) was significant in every regression but not in a consistent direction for the discriminative revenue prediction. That is, when the discriminative treatment came first, it increased discriminative revenue in design 1 but lowered it in design 2.[5] When the uniform auctions came first, it increased the uniform price revenues with respect to the theoretical prediction. Finally, when we compare observed discriminative revenue to observed uniform revenue, the sequencing variable (β_D) is negative. This indicates that the observed revenue in the uniform auctions is increased relative to the revenue generated in the discriminative auctions when the auctions are first in the sequencing within a given experiment.

Tables 6 and 7 report data on the incidence of individual overbids, i.e., bids in excess of assigned value, under various subject and treatment conditions. Again the effect of experience is evident. In Table 6, overbidding, which is fairly common in the uniform price auction, is nevertheless much lower with

Table 6. Incidence of Overbids in Discriminative and Uniform Price Auction

	*Number of Bids in Excess of Valuation (Number of Such Bids Providing a Loss)/ Total Number of Bids**	
	Discriminative	*Uniform*
Design 1: N = 10; Q = 7		
Subject condition		
Experienced	3(1)/800	106(14)/800
Inexperienced	29(23)/800	231(48)/800
Design 2: N = 10; Q = 4		
Subject condition		
Experienced	11(3)/800	139(16)/800
Inexperienced	15(2)/400	161(28)/400
Graduate Students	5(5)/400	67(10)/400

*Based on all 10 subject bids in the 20 auction sequences: 4–23; 26–45.

Table 7. Incidence of Overbids in Uniform Price Auctions Under Each Switch Sequence

	Number of Bids in Excess of Valuation/Total Number of Bids: Sequence	
	DU	*UD*
Design 1: N = 10; Q = 7		
Subject Condition		
Experienced	20/400	86/400
Inexperienced	51/400	180/400
Design 2: N = 10; Q = 4		
Subject condition		
Experienced	15/200	112/400
Inexperienced	47/200	114/200
Graduate students	7/200	60/200
Selected experienced	12/200	

experienced and with graduate students than with subjects drawn from the inexperienced undergraduate pool. The extent to which overbidding is associated with values that are "safely" in excess of the uniform price is indicated in parentheses by the number of overbids that provide a loss to the bidder. Thus, in design 1, 106 out of 800 bids submitted by experienced subjects were in excess of value, but in only 14 of these cases did the bidder lose money. Overbidding in discriminative auctions is quite insignificant among experienced (3 or 11 in 800) or graduate subjects (5 in 400) and is likely to be accounted for by typing errors that are not corrected before the bid is entered (occasionally subjects indicated that they had made such mistakes).[6]

Table 7 provides a more detailed breakdown of overbids in uniform price auctions. In particular, Table 7 shows the considerable effect of institution order on overbids in the uniform price auction. When the uniform price auction follows a discriminative auction, overbids are only about 1/4 as frequent with experienced subjects as when the order is reversed. This tendency for greater overbidding when the uniform auction is not preceded by the discriminative auction can be used to account for the observed result that revenues generated in uniform price auctions increase relative to revenues generated in the discriminative auctions when the auctions are first in the sequencing.

VI. TRENDS OVER TIME IN DISCRIMINATIVE AUCTIONS

In Figure 1 we examine the possibility of any sequential effects on revenues generated in the multiperiod environment of these experiments. In particular, Figure 1 displays data on the difference between theoretical revenue for the risk

neutral case, conditional on resale values actually drawn [$R_{DN}(t)$] and the co responding observed revenue for the discriminative auction. As in the analysi of Section V, computing theoretical revenue conditional on actual resale value controls for variation in revenues due solely to bidders having different assigne valuations. The data in Figure 1 give mean observed revenue for each perio with experiments using experienced subjects.

The results in Figure 1 suggest that there was some tendency for observe revenue to fall relative to $R_{DN}(t)$ in later periods of experiments in design 1 However, for experiments in design 2 this conclusion is supported only for thos experiments in which the discriminative auctions followed the uniform pric auctions. In a statistical analysis of the first four periods of each experimer compared to the last four, the results from above are supported. Hence, ther seems to be no general pattern for describing any sequential effects of multiperio discriminative auctions.

VII. REVENUE COMPARISONS IN RELATION TO MILLER AND PLOTT

As noted in Miller and Plott (1985), the difference in revenues generated in th discriminative and uniform price auctions can be significantly affected by th elasticity of demand of the induced-demand curve. Increasing the elasticity c demand at the point of intersection of the induced-demand curve and the suppl schedule was shown by Miller and Plott to increase revenues in uniform pric auctions relative to discriminative ones. The Miller–Plott results can be relate to our experiments by examining the effects of (1) the slope of the inducec demand curve and (2) the height of the induced-demand curve at the Q + highest resale value. In our experimental design, both of these values varie because of sampling variation in the resale values drawn for successive auctions Table 8 shows the statistical results of a linear regression analysis conducted t test the effect of these factors. The model tested can be described as

$$D(t) = \alpha_o + B_1 SL_t + B_2 H_t + e_t, \quad (5.2$$

where

$D(t)$ = $R^*_D(t) - R^*_U(t)$, for comparing observed behavior in the tw auctions; $R^*_D(t) - R_U(t)$, for comparing observed behavior in the discrim inative auction to theoretical for the uniform auction;

$SL(t)$ = the absolute value of the slope of the induced demand curve fc period t, using the Q + 1 highest resale values for computin the slope;

$H(t)$ = the value of the induced demand curve at the Q + 1 highest resal value for period t;

e_t = random error term, with $e_{it} \sim N(0,\sigma^2)$..

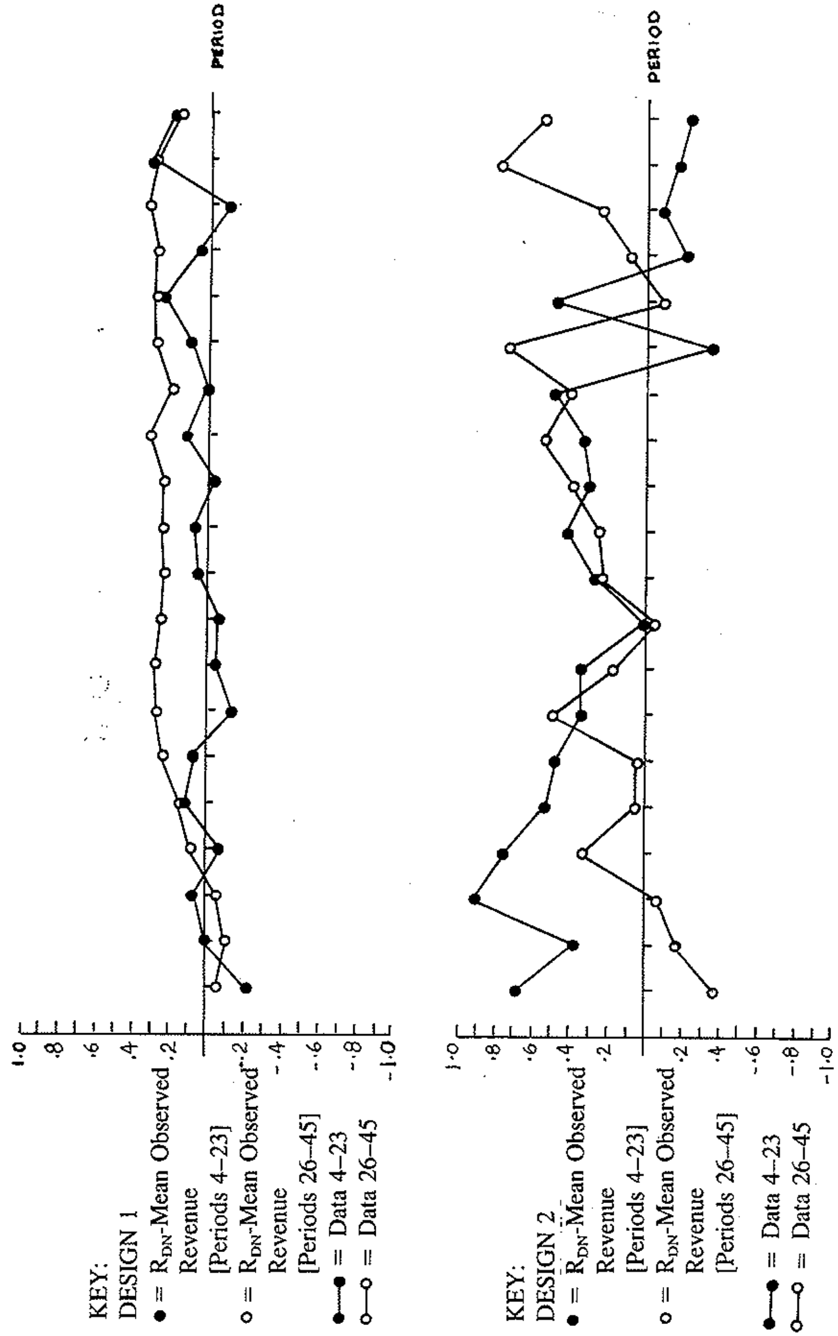

Figure 1. Mean revenue in auction sequence.

Table 8. Effects of Parametric Changes on Induced Demand Curve[a]

	Dependent Variable	*Independent Variables* α_0	SL_t	H_t	*F*	*Adj.* R^2	*N*
Design 1	$R^*_D(t) - R^*_U(t)$	1.039	3.345	−1.589	31.35	.43	80
		(2.001)	(2.809)	(5.136)			
	$R^*_D(t) - R_U(t)$	0.038	9.254	−3.643	71.91	.64	80
		(.093)	(5.079)	(5.861)			
Design 2	$R^*_D(t) - R^*_U(t)$	0.422	3.786	−3.081	41.01	.50	80
		(1.047)	(0.945)	(5.078)			
	$R^*_D(t) - R_U(t)$	1.995	2.382	−2.316	73.52	.65	80
		3.867	(2.199)	(7.809)			

[a]The t statistics are in parentheses.

Table 9. Mean Efficiency of Discriminative and Uniform Price Auctions

	Mean Efficiency *Discriminative*	*Uniform Price*	T
Design 1: N = 10; Q = 7			
Subject condition			
Experienced	98.3	97.8	80
Inexperienced	97.2	95.6	80
Design 2: N = 10; Q = 4			
Subject condition			
Experienced	97.6	98.8	80
Inexperienced	98.4	95.3	40
Graduate students	96.5	97.9	40

Increasing the absolute value of the slope of the demand curve and controlling for height, we increase the revenue generated in the discriminative auction relative to the uniform. Also, increasing the height of the demand curve at the Q + 1 highest resale value and controlling for slope, we increase the revenue generated in the uniform price auction relative to that of the discriminative auction.

VIII. ALLOCATIVE EFFICIENCY

If we let A = A(t) be an index set of those bidders whose bids were accepted in auction t, and P = P(t) be an index set of the Pareto optimal allocations in auction t (i.e., the individuals who were assigned the Q highest valuations), then a measure of efficiency can be defined as

$$E(t) = 100 \frac{\sum_{i \in A} v_i(t)}{\sum_{j \in P} v_j(t)}. \quad (6.1)$$

Efficiency is 100% if and only if the Q highest valuation bidders also constitute the set of winning bidders. Table 9 records the mean efficiency for T auctions, $(1/T) \sum_{t=1}^{T} E(t)$, under each treatment condition. Uniform price auctions are more efficient than discriminative auctions only in design 2 with the experienced and the graduate student subjects. These results are in contrast with those reported for single-object auctions (Coppinger et al., 1982; Cox et al., 1982) in which, under all design conditions (Q = 1, with N varying from 3 to 9), the second-price sealed-bid auction was more efficient than either the first-price or Dutch auctions.

IX. SUMMARY AND DISCUSSION

Received models of bidding in discriminative and uniform price sealed-bid auction institutions are not supported by the revenue results from experiments using N = 10 bidders, two different supply offerings, Q = 7 and Q = 4, and using subjects with different levels of experience and formal education. In particular Nash equilibrium theory provides its poorest performance when the subjects are experienced or are graduate students. This suggests the possibility of a tendency toward "cooperative-like" bidding with more sophisticated bidders. However, a cooperative interpretation is speculative since communication is not permitted among the subjects except that following each auction the highest rejected and highest accepted bids are reported to each bidder. Hence, at most, we could have some type of tacit cooperation.

The fickle character of the results vis-à-vis the theory also is manifest in a measure of internal consistency: The order in which the discriminative and uniform price rules are presented (in blocks of 23 followed by 22 auctions) has a significant effect on observed relative to theoretical revenue, but the direction of this effect is not consistent. Thus, when the discriminative sequence is first, this increases the discriminative revenue comparison when N = 10, Q = 7, but lowers it when N = 10, Q = 4. When the discriminative sequence is first, observed relative to theoretical revenue in the subsequent sequence of uniform price auctions is decreased. This sensitivity of the results to parameters which are external to the requirements of Nash equilibrium theory is consistent with a

tacit "collusion" interpretation; i.e., one might expect attempts at tacit cooper tion to exhibit highly erratic success.

For all the cases examined so far, revenues in the uniform price (or secon price) auction tend to be below the levels predicted by the noncooperativ dominant strategy equilibrium. However, in second-price auctions there is ev dence of learning by many individual subjects in whom the proportion of dor inant strategy choices increases in successive auctions (Coppinger et al., 1980

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NOTES

1. First and second price sealed-bid auctions are examined in Cox et al. (1982).

2. We here follow a common procedure in assuming that all bidders have the same risk preference See Cox et al. (1982) for a critique of this approach and development of a model for single-u auctions that admits differential risk preferences.

3. The first three periods of each experiment were dropped, as well as the first two periods aft a treatment switchover. This was done to eliminate any learning effects associated with the first fe trials.

4. Overbidding value is of course not consistent with any theory known to us, but underbiddi is consistent with cooperative behavior. It should be noted, however, that the nonoptimality overbidding is not transparent, and indeed is not an unsafe action when one's value is comfortab in excess of the average or expected price. Smith (1967) reported several cases in which individu submitted outrageously high bids ($100, $1000) in multiunit uniform price auctions. This led Belovi (1979) to introduce a ceiling bid rule in such experimental auctions. Bidding above value in th manner is like placing a buy order "at market" on a stock exchange, except that in such cases t institution has rules under which trading is suspended if market price becomes indeterminate subject to unusually large changes.

5. Testing the linear model above with interaction terms shows significant effects in most instance However, in the case of design 1 for the variable $[R_D^*(t) - R_{DN}]$, the interaction between inexp rienced subjects and the discriminative auction being first in the sequencing is highly significar With the interaction term, the sequencing variable is no longer significantly different from zer Thus, at least part of the difference between design 1 and design 2 for the sequencing variable due to the particular bidding pattern of inexperienced subjects in design 1.

6. Overbidding in the discriminative auction occurred at rates of about 30 per 800 bids amo inexperienced subjects and 10 per 800 bids among experienced subjects. It might be thought th such overbidding represents some sort of aberrant or uninformed behavior peculiar to the subjec in laboratory experiments. Several observations weigh against such a naive interpretation: (1) 1975, when the U.S. Treasury auctioned gold for the first time in over 40 years, in one of t auctions, using *discriminative rules*, several of the bids submitted exceeded the selling price of go in the London bullion market on the day the bids were submitted. (2) There are many auctic markets in which various observers have claimed that people get "caught up in the bidding" (e.g in the auction of grazing leases, see Miklius et al., 1980) or fall victim to the "winner's curse (discussed in Lohrenz and Waller, 1983) in oil exploration lease bidding, and pay "too much" f particular items. (3) Buying "at market" on organized exchanges is very common among investor who always have the option of specifying a limit price.

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Appendix follows

APPENDIX:

Instructions For Discriminative-Uniform Sequence Design 2

INSTRUCTIONS

Before you make any market decisions in this experiment, you will be given a starting capital credit balance of $1.ØØ. Any profit earned by you in the experiment will be added to your starting capital, and any losses incurred by you will be subtracted from your starting capital. At the end of the experiment your net balance will be calculated and paid to you in real money.

For example, if you make market decisions in the experiment that earn you profits of say $5.35, you will be paid $ 6.35. But, if you make market decisions that earn losses, these losses will be subtracted from your starting capital. If your losses exceed your starting capital, you will be paid nothing at the end of the experiment.

Press -NEXT- to Continue

In this experiment we will create a market in which you will act as buyers in a sequence of trading periods. Before trading begins an announcement is made indicating the quantity of the commodity that is for sale.

In each period, your task is to attempt to buy units of the commodity by submitting bids for it along with the other buyers in the experiment. Each unit that you are able to purchase is then resold by you at a price whose determination will be explained to you shortly. The procedure for determining whether a bid is accepted, and the price that is paid for an accepted bid will be explained later.

If one of your bids is not accepted then your profit for that bid is zero. If one of your bids is accepted, you make a profit equal to the difference between your selling price and your purchase price. If this difference is negative it represents a loss. It is possible for all of your bids to be accepted, or only part of your bids to be accepted.

Press -NEXT- to Continue
Press -BACK- to Review

Before you can start making market decisions in the experiment, you still need to know:

1) For all accepted bids, how the resale price of the unit is determined.
2) How it is determined whether your bid is accepted.
3) If a bid is accepted, what price you will have to pay for the unit you bid on.
4) Finally, how many bids you are able to make and how you make these bids in the market.

Instructions and examples explaining each of the above four items will now be given.

Press -NEXT- to Continue
Press -BACK- to Review

Determining the Resale Value of a Purchased Unit

For all bids that are accepted, the resale value(price) of each unit is determined by a random drawing from the 225 numbers that come in $.Ø1 increments and fall in the range of $ Ø.ØØ to $ 2.24. In other words the resale value will be drawn at random from the values of $ Ø.ØØ, $ Ø.Ø1,$ Ø.Ø2,.....,$ 2.23,$ 2.24.

Each of these prices is equally likely to be drawn in each market period. Since there are 225 prices, this means there is a 1/225 chance that any one price will be drawn in any given market period.

For example, if $Ø.Ø3 is drawn in one period, this has no effect on the 1/225 chance that $Ø.Ø3 will be drawn in any later market period.

The resale value for each period will be drawn independently for each buyer and can therefore be different for each buyer. For Example, if your resale value for Period(1) is $ 2.Ø4 then the resale values for the other buyers could possibly equal $ 2.Ø4 in Period(1) but they could just as easily be any other value between $ Ø.ØØ and $ 2.24.

You will be given your resale value before you make your bids.You will not know the resale values of the other buyers in the market.

Press -NEXT- to Continue
Press -BACK- to Review

Acceptance of a Bid and the Price Paid for a Unit

Whether a bid is accepted and the price that would be paid for an accepted bid is determined as follows:

Suppose X units are offered for sale at the beginning of a market period. Also, suppose for this example that each bidder can submit only two bids. Each bid specifies a price for a single unit of the commodity. The bid prices must be in dollars and cents for example $ 1.99,$1.08, or $0.02. The bids from all buyers will then be collected by "Plato" and then be arranged in descending order from the highest to the lowest. With X units offered for sale, the first X of these bids (starting with the highest) will be accepted, and the remaining will be rejected. In the case of ties at the lowest of the accepted bids, random numbers will be used to determine which of the accepted bids will be accepted.

Press -NEXT- to Continue
Press -BACK- to Review

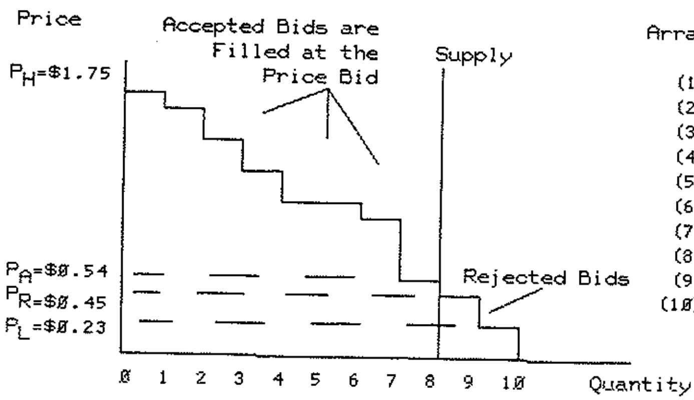
Once "Plato" orders the bids from highest to lowest and determines which bids will be accepted, the highest accepted bid and the highest rejected bid will be shown to each buyer. "Plato" will also signal to each player which of his bids were accepted.

Each accepted bid will represent the purchase of one unit of the commodity at a purchase price equal to the buyers bid price for that unit. Therefore, the greater your bid price the smaller is your profit on that bid if the bid is accepted. But, the greater your bid the more likely it will be accepted.

You must weigh these considerations carefully in deciding upon each bid price to be submitted. A bid price of $0.00 is acceptable and is essentially equivalent to not entering a bid or "standing pat".

The following graph and numerical example illustrate the previous instructions on market transactions for one market period. In this example, X=8 (there are 8 units offered for sale). There are 5 buyers and each buyer is allowed to bid for two units. After all bids are made, they are arranged in descending order as shown, where:

P_H= $1.75 represents the highest bid
P_L= $0.23 represents the lowest bid
P_A= $0.54 represents the lowest bid accepted
P_R= $0.45 represents the highest rejected bid



Array of Bids

(1) $1.75
(2) $1.66
(3) $1.47
(4) $1.25
(5) $1.02
(6) $1.02
(7) $0.95
(8) $0.54
(9) $0.45
(10) $0.23

Press -NEXT- to Continue
Press -BACK- to Review

NUMERICAL EXAMPLE

Array of Bids

(1) $1.75
(2) $1.66
(3) $1.47
(4) $1.25
(5) $1.02
(6) $1.02
(7) $0.95
(8) $0.54
(9) $0.45
(10) $0.23

Current Standing - Buyer X

Profit Period 1	$0.10
Previous Profits	$0.00
Starting Capital	$1.00
Current Balance	$1.10

Results (Period 1) - Buyer X

	Unit1	Unit2
Bid	$1.75	$1.25
Accepted	Yes	Yes
Price Paid	$1.75	$1.25
Resale Price	$1.55	$1.55
Profit	-$0.20	$0.30

In this example, we are looking at an imaginary player (Buyer X), in the first period of an experiment. For this experiment, there were 5 buyers each having 2 bids. There were eight units of the commodity for sale and for the example the starting capital was $1.00. The bid of $0.54 was the eighth highest bid, and therefore the last bid accepted.

Press -NEXT- to Continue the Example
Press -BACK- to Review

The preceding table is displayed while the subject reads the following segments of information:

From the table labeled (Results (Period 1) - Buyer X), we see that Buyer X made bids of $1.75 and $1.25. Since both bids were higher than the highest rejected bid of $0.45 they were both aceppted. Buyer X therefore pays a price of $1.75 for unit one and $1.25 for the second unit of the commodity.

Now assume the drawing for the resale price yields a price of $1.55 . For Period 1, Buyer X incurs a profit of $1.55-$1.75=-$0.20 for unit 1 and a profit of $1.55-$1.25=$0.30 for unit 2. His net profits for Period 1 equal -$0.20+$0.30=$0.10.

The table labeled (Current Standings - Buyer X) shows the current money standings for Buyer X. Since this is the first period, he has no previous profits. Therefore, his current balance equals $0.10 (profit for Period 1) + $1.00 (starting capital) = $1.10.

Press -NEXT- for an Example in which YOU Participate
Press -BACK- to Review

MARKET PERIOD ILLUSTRATION

In this example, you will be given the same type of information you will receive in the experiment, except you will be told what bids to make (this is for illustrative purposes).

Market Information

Number of units offered	= 7
Number of Buyers	= 4
Number of Bids Per Buyer	= 3
Resale Price Maximum	= $2.95
Resale Price Minimum	= $1.50
Starting Capital	= $2.00

The information above is what you would be given before entering the experiment, except the actual numbers may vary. You now know how many units will be offered for sale, the number of buyers in the market and the number of bids each buyer can make.

You also know that if one of your bids is accepted, the price at which you can sell the unit will be drawn at random from the numbers: $2.95, $2.94, $2.93, $2.92 $2.91, ...,$1.53.$1.52,$1.51,$1.50.

Press -NEXT- to Continue
Press -BACK- to Review

EXAMPLE MARKET PERIOD

This example is just like the experiment in which you will participate, except for:

(1) the numbers may be different from what they will be in the experiment.

(2) you are asked to make the following bids: Unit 1=$1.95, Unit 2=$1.80, Unit 3=$0.73

In each market period you will be given tables, as shown below, for entering your bids.

Type Bid 1:	Type Bid 2:	Type bid 3:

After all bids are made, you will be shown the market results of your bids. During the experiment, there will also be a data sheet available at all times with summary information.

Press -NEXT- to enter Market Period (Trial1)
Press -BACK- to Review

MARKET PERIOD (TRIAL1)

(Remember to bid: Unit1=$1.95, Unit2=$1.80, Unit3=$0.73)
Resale Value Period (Trial1) = $2.05

After entering each bid press -NEXT-

Type Bid 1:	Type Bid 2:	Type Bid 3:
1.95 ok	1.8 ok	.73 ok

Press -DATA- to see Data Summary Sheet

Press -NEXT- to see Market Results
Press -BACK- to Review

Results - Market Period(Trial1)

Current Standing - Buyer 1

Profit Period(Trial1)	$Ø.35
Previous Profits	$Ø.ØØ
Starting Capital	$2.ØØ
Current Balance	$2.35

Bid Information

Highest Accepted Bid = $2.63
Highest Rejected Bid = $1.45

Results(Trial Period) - Buyer 1

	Unit1	Unit2	Unit3
Bid	$1.95	$1.8Ø	$Ø.73
Accepted	Yes	Yes	No
Resale Price	$2.Ø5	$2.Ø5	$2.Ø5
Price Paid	$1.95	$1.8Ø	$Ø.ØØ
Profit	$Ø.1Ø	$Ø.25	$Ø.ØØ

Press -NEXT- for Explanation of Tables
Press -BACK- to Review

The preceding table is displayed while the subject reads the following segments of information:

Summary of Period(Trial1)

1) Highest accepted bid was $2.63 and the highest rejected bid was $1.45.
2) Your two Highest bids were accepted.
3) The random drawing for resale price yielded $2.Ø5.
4) You paid $1.95 for Unit1 and $1.8Ø for Unit2.
5) Profit on Unit1 was $2.Ø5-$1.95=$Ø.1Ø, and on Unit2 profit was $2.Ø5-$1.8Ø=$Ø.25.
6) Total profit for Period(Trial1) = $Ø.35.
7) If this were not an example, your current balance would be $2.ØØ+$Ø.3Ø=$2.35.

Press -NEXT- to Continue
Press -Back- to Review

Example 2

This is the final example before entering the experiment. You will face the same the market situation as in the previous example, except now you are asked to make the following bids:
Unit1 Bid=$2.35, Unit2 Bid=$1.8Ø, Unit3 Bid=$Ø.92

Press -NEXT- to enter Market Period(Trial2)
Press -DATA- to see Data Summary Sheet
Press -BACK- to Review previous example

MARKET PERIOD(TRIAL2)

(Remember: Unit1 Bid=$2.35, Unit2 Bid=$1.8Ø, Unit3 Bid=$Ø.92)
Resale Value Period(Trial2) $2.25

After entering each bid Press -NEXT-

Type Bid 1:	Type Bid 2:	Type Bid 3:
2.35 ok	1.8 ok	.92 ok

Press -DATA- to see Data Summary Sheet
Press -NEXT- to see results of Market Period(Trial2)
Press -BACK- to Review

Results - Market Period(Trial2)

Current Standing - Buyer 1

Profit Period(Trial2)	$Ø.35
Previous Profits	$Ø.35
Starting Capital	$2.ØØ
Current Balance	$2.7Ø

Highest Accepted Bid = $2.52
Highest Rejected Bid = $1.75

Results - Period(Trial2) - Buyer 1

	Unit1	Unit2	Unit3
Bid	$2.35	$1.8Ø	$Ø.92
Accepted	Yes	Yes	No
Resale Price	$2.25	$2.25	$Ø.ØØ
Price Paid	$2.35	$1.8Ø	$Ø.ØØ
Profit	-$Ø.1Ø	$Ø.45	$Ø.ØØ

Press -NEXT- for Explanation of Tables
Press -BACK- to Review
Press -DATA- for Data Summary Page

The preceding table is displayed while the subject reads the following segments of information:

Summary of Period(Trial2)

1) Highest accepted bid was $2.52 and the highest rejected bid was $1.75.
2) Your two Highest bids were accepted.
3) The random drawing for the resale price yielded a price of $2.25.
4) You paid $2.35 for Unit1 and $1.8Ø for unit 2.
5) Profit on Unit1=$-Ø.1Ø and profit on Unit2=$Ø.45.
6) Total profit for Period(Trial2)=$Ø.35.
7) Profit from Period(Trial1)=$Ø.35.
8) Current Balance (for example purposes) =$2.7Ø.

Press -NEXT- to Enter Experiment
Press -BACK- to Review

Experiment Market Parameters

Number of Units Offered	= 4
Number of Buyers	= 10.
Number of Bids Per Buyer	= 1
Maximum Resale Price	= $ 2.24
Minimum Resale Price	= $0.00
Starting Capital	= $1.00

The information above describes the market in which you will be participating. If you have any questions concerning how the market operates please raise your hand or Press -HELP- for a Summary. If you enter the market you will not be able to go back for a review.

Press -NEXT- to Enter the Experiment
Press -BACK- to review previous material
Press -HELP- for a Summary

SUMMARY

1) You begin the experiment with a starting capital of $1.
2) Any accepted bid will be resold by you at a resale price whose value is chosen at random and lies between $0.00 and $ 2.24.
3) Plato orders all bids from highest to lowest. For this experiment the highest 4 bids will be accepted.
4) If your bid is accepted the price you pay is the price you bid.
5) Profit on an accepted bid equals the difference between the resale value drawn for that period and the price you pay for your bid.
6) Profits or losses are added to your starting capital to give your current balance.

Press -BACK-

After a sequence of 23 discriminative auctions, the subjects see the following:

Market Change

In the periods to follow, the market will operate as before except for one change. In the following periods the price you pay for an accepted bid will no longer be equal to the value of your bid. Now, the price you pay for an accepted bid will be equal to the value of the highest rejected bid, NOT the price you bid.

Thus, your profit is not decreased by submitting a bid above the highest rejected bid. The greater your bid the more likely it will be above the highest rejected and therefore accepted. But of course, the greater the value of the highest rejected bid the lower your profit on a bid if that bid is accepted.

Press -NEXT- to Continue

Consider two possible bids that you might make of say \$ 2.Ø4 or \$ 1.74 and a resale value of say \$ 1.84. For the first example assume the highest rejected bid was \$ 1.64. Both the bid of \$ 2.Ø4 and the bid of \$ 1.74 would have been accepted. Also each bid would give a profit of \$ 1.84-\$ 1.64=\$.2Ø.

Now consider a second example where this time the highest rejected bid is \$ 1.94. In this case only the bid of \$ 2.Ø4 would be accepted. Let the resale value be equalto \$ 1.84, just as in the first example. The profit on the accepted bid of \$ 2.Ø4 is \$ 1.84-\$ 1.94= -\$.1Ø, since profit on an accepted bid equals the resale value minus the value of the highest rejected bid.

Notice from the examples above that if you were to bid no higher than the resale value of \$ 1.84 there would be no chance of losing money. But, as shown in the second example, if you bid higher than the resale value and your bid is accepted and at the same time the highest rejected bid is above the resale value you would lose money on that bid.

Press -NEXT- to Return to the Market
Press -HELP- For a Summary of How the Market Works

SUMMARY

1) You begin the experiment with a starting capital of $1.
2) Any accepted bid will be resold by you at a resale price whose value is chosen at random and lies between $0.00 and $ 2.24.
3) Plato orders all bids from highest to lowest. For this experiment the highest 4 bids will be accepted.

4) If your bid is accepted the price you pay is the amount of the highest rejected bid.
5) Profit on an accepted bid equals the difference between the resale value drawn for that period and the price you pay for your bid.
6) Profits or losses are added to your starting capital to give your current balance.

Press -BACK-